**Reading:** Brown and Churchill (Chapters 4-6) .

**Problems:** Due Monday Oct 1.

1. Evaluate

$$\int_0^\infty \frac{\ln x}{(a^2 + x^2)^2} dx.$$

2. Evaluate

$$\int_0^\infty \frac{dx}{1 + x^{2n+1}}$$

for $n \geq 1$ .

3. For $a^2 > b^2 + c^2$, show that

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta + c\sin\theta} = \frac{2\pi}{\sqrt{a^2 - b^2 - c^2}}.$$

Why is this condition on $a$, $b$, and $c$ needed?
Hint: Rewrite integral as a contour integral in terms of $z = e^{i\theta}$.

4. Show that
$$\int_0^\infty \frac{x^{-1/2}\, dx}{(1 + x^2)} = \frac{\pi}{\sqrt{2}}.$$
In this problem, you have to pay special attention in integrating around a branch point.

5. Show that

(a) $\displaystyle\sum_1^\infty \frac{1}{n^6} = \frac{\pi^6}{945}$;   (b) $\displaystyle\sum_1^\infty \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$;   (c) $\displaystyle\sum_{-\infty}^\infty \frac{1}{1 + n^2 + n^4} = \frac{\pi}{\sqrt{3}} \tanh \frac{\pi\sqrt{3}}{2}.$