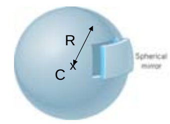
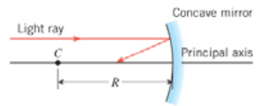
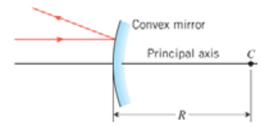
Spherical Mirrors

Spherical Mirrors

Spherical mirrors are special curved mirrors that can be considered as cutout from the surface of a sphere. So for any spherical mirror, one can identify the **<u>center, C,</u>** and **<u>radius of curvature, R,</u>** of the sphere associated with the mirror. The **<u>principal axis</u>** is the straight line connecting C and the mid-point of the mirror. **<u>Concave mirrors</u>** have C in front of the mirror. **<u>Convex mirrors</u>** have C behind the mirror.

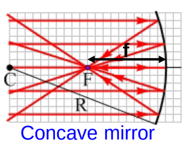




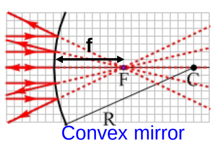


Concave vs. Convex Mirrors

The main difference between concave and convex mirrors is the most easily seen by <u>considering how they reflect light rays parallel and close to the principal axis</u>.



Concave mirror

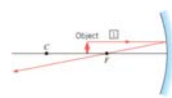


Convex mirror

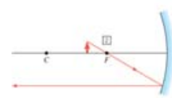
For concave mirror, the reflected rays **<u>converge</u>** to the **<u>focal point</u>** F located at **<u>f = +R/2</u>** on the principal axis. Positive sign means that the rays actually converge at and subsequently emanate from the focal point.
For convex mirror, **<u>f = -R/2</u>** and is located on the principal axis **<u>behind the mirror.</u>** Negative sign means that rays "appears" to, but actually not, converge to and then emanate from the focal point

Image Formation by Concave Mirrors

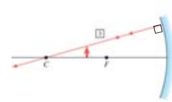
Based on the law of reflection, three convenient ways of drawing reflected rays from a spherical mirror are established:



This ray, coming from the object, is initially parallel to the principal axis. After reflection, it emerges from the point where it hits mirror and travels along the straight line passing through the focal point.



This ray, coming from the object, initially passes through the focal point. After reflection, it emerges parallel to the principal axis.



This ray, coming from the object, travels along a line that passes through the center, C. After reflection, it emerges along the same path it came from.

Image Formation by Concave Mirrors

By drawing the reflected rays by using any two of the above three ways, we can locate the image, which is either the direct intercept of the reflected rays (if the image is **<u>real</u>**) or the intercept of their extrapolations behind the mirror (if the image is **<u>virtual</u>**). An image is real if light actually converge at the image position (and so the image can be recorded by a photographic plate or visualized by a projector screen placed at the image position). Converse is true for a virtual image.

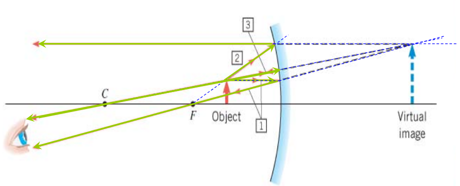
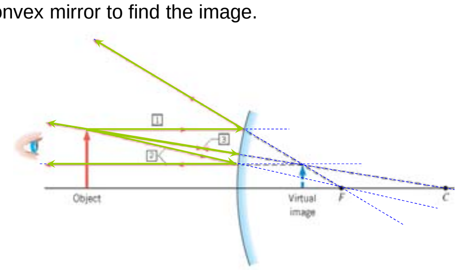


Image Formation by Convex Mirrors

We can establish three similar ways to draw reflected rays for a convex mirror. It turns out the image formed by a convex mirror is always virtual, upright and diminished. So one always needs to extrapolate the reflected rays behind a convex mirror to find the image.



The Mirror Equation

Drawing a ray diagram is a great way to get an idea about what's going on. But to find the various distances precisely, it's better to use the mirror equation.

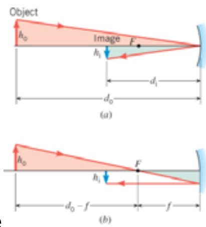
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

This can be rewritten as:

$$d_i = \frac{d_o \times f}{d_o - f}$$

d_o = object distance, d_i = image distance

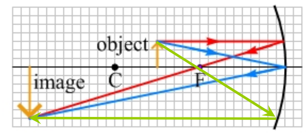
Magnification, $m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$



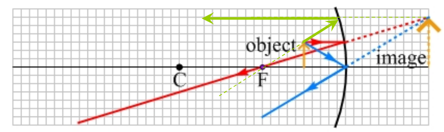
The above diagrams are used to derive the mirror equation.

Sign Convention of Magnification

A negative *m* means that the image is inverted.



Positive *m* means an upright image.



Sign Convention of the Mirror Equation

The parameters, *f*, d_o and d_i can be positive or negative. Below lists the sign convention.

	In front of the mirror	Behind the mirror
Object position	d_o is +	d_o is –
Image position	d_i is +	d_i is –
Focal point, F	f is + (true for concave mirrors)	f is – (true for convex mirrors)

Image characteristics for a convex mirror

- virtual and upright
- smaller than the object

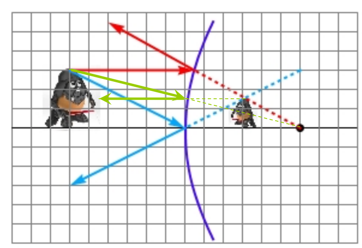
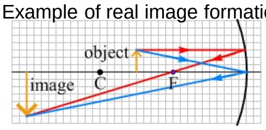


Image characteristics for a concave mirror

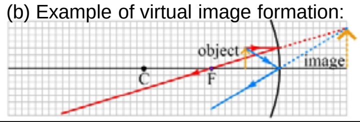
Depending on the distance of the object from the mirror, the image can be:

- real and inverted, or virtual and upright
- larger than, smaller than, or equal in size to the object

(a) Example of real image formation:



(b) Example of virtual image formation:



Equation for a plane mirror?

Does the mirror equation apply to a plane mirror? $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

1. Yes, using the focal length for a plane mirror of zero.
2. Yes, using the focal length for a plane mirror of infinity.
3. Yes, but do we really need an equation for a plane mirror??
4. No.

Example 1 (Convex Mirror)

A Star Wars action figure, 15 cm tall, is placed 30 cm in front of a convex mirror that has a radius of curvature of 60 cm.

Where is the image?

How tall is the image?

What are the characteristics of the image?

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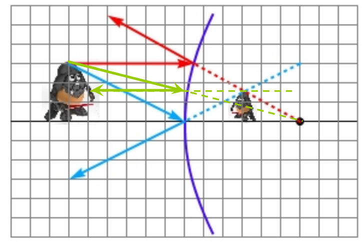
Example 1 (Convex Mirror)

First, sketch a ray diagram. The focal point is 30 cm behind the mirror.

1 grid unit = 5 cm.



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Example 1 (Convex Mirror)

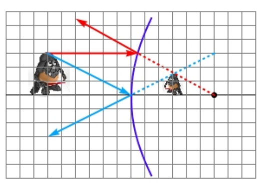
Where is the image?

$$d_o = 30 \text{ cm}, \ f = -30 \text{ cm}$$

$$d_i = \frac{d_o \times f}{d_o - f}$$

$$= \frac{(30 \text{ cm}) \times (-30 \text{ cm})}{(30 \text{ cm}) - (-30 \text{ cm})}$$

$$= -15 \text{ cm}$$



This agrees with the ray diagram.

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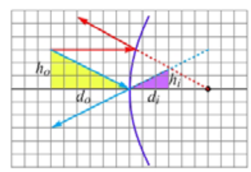
Example 1 (Convex Mirro)

How tall is the image?

$$d_o = 30 \text{ cm}, \ d_i = -15 \text{ cm},$$

$$h_o = 15 \text{ cm}$$

The similar triangles tell us that the image is half the height of the object. Or, h_i = 7.5 cm.



Alternatively, we can find the image height from the magnification equation.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = 7.5 \text{ cm}$$

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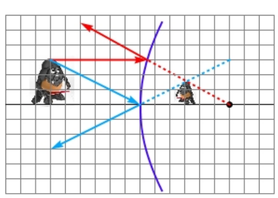
Example 1 (Convex Mirror)

What are the image characteristics?

The image is:

- virtual (since d_i<0)
- upright (since m>0)
- smaller than the object (since |m|<1)



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Example 2 (Concave Mirror)

A Star Wars action figure, 15 cm tall, is placed 60 cm in front of a concave mirror that has a radius of curvature of 60 cm.

Where is the image?

How tall is the image?

What are the characteristics of the image?

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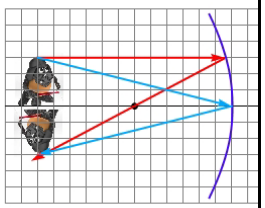
Example 2 (Concave Mirror)

Where is the image?

$d_o = 60 \text{ cm}, \ f = +30 \text{ cm}$

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(60 \text{ cm}) \times (30 \text{ cm})}{(60 \text{ cm}) - (30 \text{ cm})} = +60 \text{ cm}$$



This answer agrees with the ray diagram.

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Example 2 (Concave Mirror)

How tall is the image?

$d_o = 60 \text{ cm}, \ d_i = 60 \text{ cm}, \ h_o = 15 \text{ cm}$

We can find the image height from the magnification equation.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -15 \text{ cm}$$

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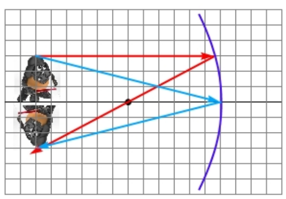
Example 2 (Concave Mirror)

What are the image characteristics?

The image is:
- real
- inverted
- the same size as the object



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Example 3 (Concave Mirror)

A Star Wars action figure, 15 cm tall, is placed 15 cm in front of a concave mirror that has a radius of curvature of 60 cm.

Where is the image?

How tall is the image?

What are the characteristics of the image?

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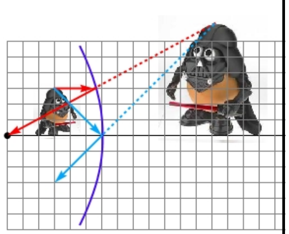
Example 3 (Concave Mirror)

Where is the image?

$d_o = 15 \text{ cm}, \ f = +30 \text{ cm}$

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(15 \text{ cm}) \times (30 \text{ cm})}{(15 \text{ cm}) - (30 \text{ cm})} = -30 \text{ cm}$$



This answer is close to the ray diagram. The discrepancy is due to limitation of the graphical method.

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Example 3 (Concave Mirror)

How tall is the image?

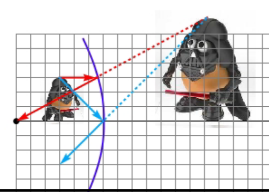
$d_o = 15 \text{ cm}, \ d_i = -30 \text{ cm}, \ h_o = 15 \text{ cm}$

By the magnification equation,

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \ \Rightarrow \ h_i = +30 \text{ cm}$$

What are the image characteristics?

The image is:
- virtual
- upright
- larger than the object



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