

Electromagnetic Waves

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Maxwell's equations

James Clerk Maxwell wrote down a set of four equations that are a great summary of what we have done in this course so far. The equations also predict that electromagnetic (EM) waves with certain characteristics that we will discuss here can exist.

Maxwell's equations:

Equation 1: electric fields come from charges.
 Equation 2: magnetic field lines are continuous loops.
 Equation 3: a changing magnetic flux can produce an electric field.
 Equation 4: magnetic fields come from currents, or from changing electric flux (this was what Maxwell added).

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Prediction of EM Waves

Following the Maxwell equations, an EM wave carries simultaneously oscillating electric E and magnetic B field, which can be written as:

$$E(x,t) = E_0 \cos(kx - \omega t) \quad B(x,t) = B_0 \cos(kx - \omega t)$$

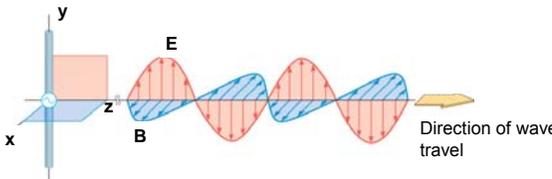
This means that the E and B fields are in phase. The equations also predict that the EM wave travels at the speed, c given by:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8 \text{ m/s}$$

Soon it is realized that (optical) light is a kind of EM wave so c is the speed of light.

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Electromagnetic waves



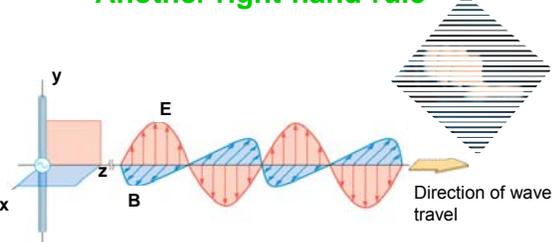
- For EM waves that are linearly polarized (i.e., the E field vectors are aligned in one particular direction), Maxwell predicted that the B field vectors must be aligned in a direction perpendicular to that of the E field vectors, and both fields perpendicular to the direction of propagation of the wave according to a right-hand-rule (see next page).

- In open space, EM waves are transverse waves.

[Simulation](#)

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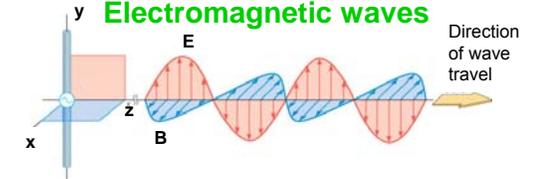
Another right-hand-rule



To apply the right-hand-rule, point your fingers in the direction of the E vector then curl them towards the direction of the B vector. The direction your thumb pointing is the direction of travel of the wave.

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Electromagnetic waves



Another consequence of Maxwell's equations is:

$$c = \frac{E}{B}$$

EM waves do not need a medium. In some sense they keep themselves going - the oscillating electric field generates an oscillating magnetic field, and the oscillating magnetic field in turn generates an oscillating electric field.

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Detecting EM waves

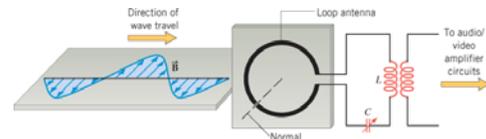
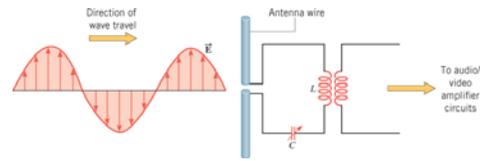
In an EM wave, the energy is divided equally between the electric fields and the magnetic fields. Usually, an antenna detects either the electric fields or the magnetic fields. Antennas are generally either conducting rods or conducting loops. Which would be used to detect the different fields?

1. Conducting rods are good for detecting electric fields and conducting loops are good for detecting magnetic fields
2. Conducting rods are good for detecting magnetic fields and conducting loops are good for detecting the electric fields
3. You can use either type of antenna for either type of field - you just need to orient the antenna appropriately



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Detecting EM waves



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Root-Mean-Square Average of the E and B Field

It is common to refer to the root-mean-square (r.m.s.) average of the electric and magnetic fields of an EM wave. They are the square root of the time average of $E(t)^2$ and $B(t)^2$. We are interested in the average of $E(t)^2$ and $B(t)^2$ rather than the average of $E(t)$ and $B(t)$ because their average value is always zero. (Why?)

$$E_{rms} = \sqrt{\langle E(t)^2 \rangle} = \sqrt{\langle (E_0 \sin \omega t)^2 \rangle} = \sqrt{E_0^2 / 2} = \frac{E_0}{\sqrt{2}}$$

$$B_{rms} = \sqrt{\langle B(t)^2 \rangle} = \sqrt{\langle (B_0 \sin \omega t)^2 \rangle} = \sqrt{B_0^2 / 2} = \frac{B_0}{\sqrt{2}}$$

In the above, $\langle \dots \rangle$ means average over time t . Also note that $\langle \sin^2 \omega t \rangle = 1/2$. E_{rms} and B_{rms} serves as the average value of the E and B field of the EM wave.

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Average Energy Density carried by an EM Wave

Average energy density:

$$u = \frac{\text{Total energy}}{\text{Volume}} = \frac{1}{2} \epsilon_0 \langle E(t)^2 \rangle + \frac{1}{2\mu_0} \langle B(t)^2 \rangle$$

time average

Recall: $\frac{E}{B} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \Rightarrow \epsilon_0 E^2 = \frac{B^2}{\mu_0}$

Thus, $u = \epsilon_0 \langle E(t)^2 \rangle = \frac{1}{\mu_0} \langle B(t)^2 \rangle$
 $= E_0^2 \langle \sin^2(\omega t) \rangle = E_0^2 / 2 = B_0^2 \langle \sin^2(\omega t) \rangle = B_0^2 / 2$

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Average Energy Density carried by an EM Wave

$$\text{Or, } u = \frac{\epsilon_0}{2} E_0^2 = \frac{1}{2\mu_0} B_0^2$$

where E_0 and B_0 is respectively, the amplitude of the E and B field of the EM wave.

With the above equation, if one knows the energy density, u , of an EM wave, the amplitudes, E_0 and B_0 can be deduced.

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Intensity carried by an EM Wave in air or vacuum

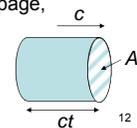
Average Intensity (i.e., average power carried across a unit area):

Volume illuminated by the EM wave in time t over an area A

$$I = \frac{P}{A} = \frac{\text{Average total energy}}{tA} = \frac{uctA}{tA} = cu$$

From the expressions of u in a previous page,

$$I = c\epsilon_0 E_0^2 / 2 = cB_0^2 / (2\mu_0)$$



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Radiation Pressure of an EM Wave

Besides energy, EM waves carry momentum, p , too.

For objects with a mass,

$$E = mv^2/2 \quad \text{and} \quad p = mv \quad \Rightarrow \quad p = 2E/v$$

For EM waves, which doesn't have a mass,

$$p = E/c \quad (\text{a result of relativity})$$

When an EM wave impinges on an object, it transfers momentum to the object. As a result, the object "feels" a pressure from the wave, which we refer to as the radiation pressure, P .

If the object is 100% absorbing,	$P = \frac{I}{c}$
If the object is 100% reflecting,	$P = \frac{2I}{c}$

The factor 2 in the second case can be understood because if the EM wave is 100% bounced off the object, the change in momentum is twice as large than when there is no bouncing off at all.

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Doppler effect of EM waves

Unlike sound waves, the speed of EM waves as seen by an observer is always the same irrespective of how he/she is moving. This has to do with relativity. As a result, there is no Doppler effect for EM waves associated with motions of the observer.

But if the source moves, the apparent wavelength would still be changed as in sound waves. This leads to the following equation for the Doppler effect of EM waves:

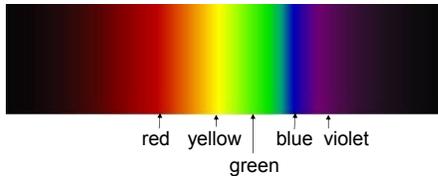
$$f' = f \left(1 \pm \frac{v_{rel}}{c} \right)$$

Where v_{rel} is the relative velocity of the source to the observer. The + (-) sign applies when the source is approaching (receding from) the observer.

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The electromagnetic spectrum

Visible light is a kind of electromagnetic wave. Below is a section of the **electromagnetic spectrum** encompassing the visible spectrum. Which side (left or right) is the wavelength longer?



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The electromagnetic spectrum

Our eyes are particularly sensitive to visible light, but EM waves cover a huge range of frequencies (see the Appendix). We split them up depending on their frequency and on how they are produced. As you move from right to left along the spectrum, the energy and frequency decrease, and the wavelength increases.

- gamma rays are produced by radioactive decays and nuclear decays.
- X-rays are produced by decelerating electrons.
- ultraviolet (UV), visible, and infrared (IR) waves are produced by electron transitions (usually in solid materials) and by vibrating atoms.
- microwaves and radio and TV waves are produced by charges accelerated in antennas.
- waves with even lower energy are known as long waves, because of their very long wavelength.

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Interactions with the human body

Gamma rays and X-rays are known as ionizing radiation. They can cause chemical changes as well as mutations of DNA.

Ultraviolet light is associated with suntans, sunburns, and cataracts.

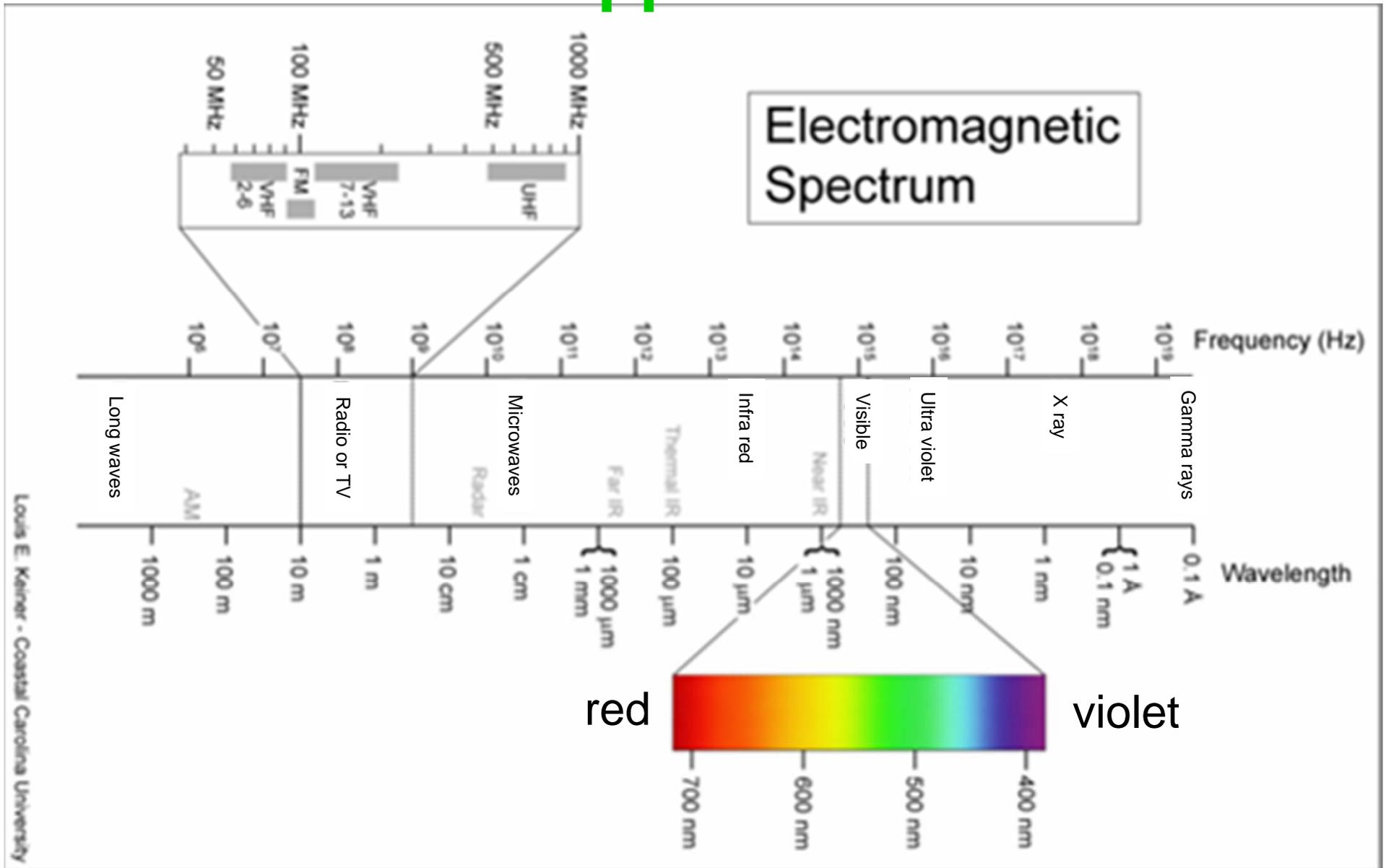
Rods and cones in our eyes are sensitive to visible light, which is why we can see.

Heat sensors in our skin can detect infrared waves.

We're not particularly sensitive to anything with a longer wavelength.

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Appendix



Louis E. Keiner - Coastal Carolina University

Image by Louis Keiner of Coastal Carolina University