Superposition of Waves

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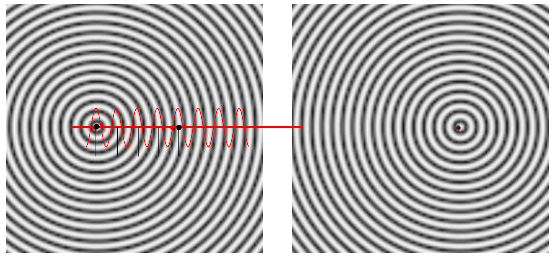
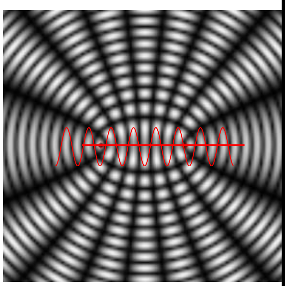
Interference from two sources

Below are the sound wave patterns that are emitted by two speakers one at a time. How would the pattern look like when they superpose?



Interference from two sources

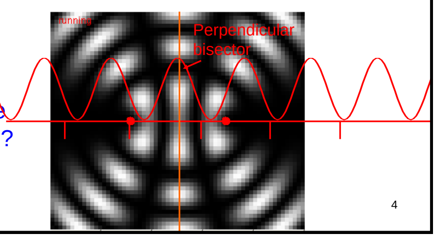
When two <u>in-phase, coherent</u> sources broadcast sound waves with the same wavelength (or frequency) at the same time, an interference pattern like the one shown results. In the picture, the dark and bright bands correspond to the regions with zero and maximum intensity, respectively. Since the local intensity is proportional to the <u>square</u> of the local displacement of the wave, the bright bands correspond to alternating positive and negative maximum displacements of the wave. [Simulation]



Interference from two sources

In the example shown in the previous page, the separation between the two sources is 4 wavelengths. In the picture shown below, the distance between the two sources is a little less than 1.5 wavelengths. Notice that the pattern is always symmetric about the perpendicular bisector of the line joining the two sources.

How do these patterns form?



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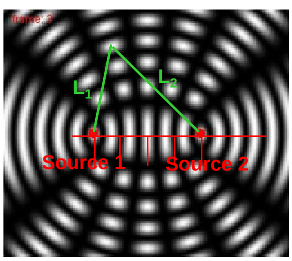
Interference from two sources

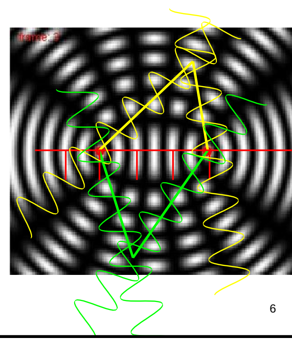
When there is interference, what happens (i.e., whether there is a maximum or zero resultant intensity) at any point depends on the path length difference ΔL ($=L_1 - L_2$), the distance from one source to the point minus the distance from the other source to the point.



Interference from two sources

Condition for constructive interference: $\Delta L = n\lambda$, where n is any integer.

Condition for destructive interference:
$\Delta L = (n + ½)\lambda$, where n is any integer.



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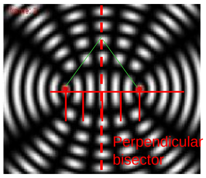
Interference in 2-D

When the two sources are in phase you always get constructive interference along the perpendicular bisector of the line joining the sources. This is because the path length difference at any point along this line is:

1. zero
2. half a wavelength
3. one wavelength
4. an integer number of wavelengths



If you think more than one answer is possible, choose the most precise answer.

Example 1

Two sources are broadcasting identical single-frequency waves, in phase. You stand 3 m from one source and 4 m from the other. What is the lowest frequency at which constructive interference occurs at your location? Take the speed of sound to be 340 m/s.

Example 1

First, find the path length difference - how much further are you from one source than the other? In here, it is ΔL = 4 m – 3 m = 1 m. Constructive interference occurs if $\Delta L = n\,\lambda$, where n = 0, 1, 2, ...

So, 1 meter = $n\,\lambda$

The lowest frequency corresponds to the largest wavelength, which occurs when the value of n is the smallest or n = 1. This gives a wavelength of 1 m.

With a wavelength of 1 m and a speed of 340 m/s, the frequency is:

$$f = \frac{v}{\lambda} = \frac{340\ \text{m/s}}{1\ \text{m}} = 340\ \text{Hz}$$

Example 1

What are the next two smallest frequencies at which constructive interference occurs?

Just take the next two smallest n values, 2 and 3 in $\Delta L = n\lambda$, which gives $\Delta L/n = \lambda$.

Substituting $\Delta L/n$ for λ gives:

$$f_n = \frac{v}{\lambda} = \frac{nv}{\Delta L} = n \times \frac{340\ \text{m/s}}{1\ \text{m}} = n \times 340\ \text{Hz}$$

so:

$$f_2 = 2 \times 340\ \text{Hz} = 680\ \text{Hz}$$

$$f_3 = 3 \times 340\ \text{Hz} = 1020\ \text{Hz}$$

Example 1

What are the lowest three frequencies giving destructive interference at your location?

Example 1

Destructive interference occurs when $\Delta L = (n + ½)\,\lambda$, where n = 0, 1, 2, ...

Solving this for λ gives:

$$\lambda = \frac{\Delta L}{n + 0.5}$$

Substituting this in $f = v/\lambda$ gives:

The corresponding frequencies are:

$$f_n' = (n+0.5)\frac{v}{\Delta L} = (n+0.5) \times \frac{340\ \text{m/s}}{1\ \text{m}} = (n+0.5) \times 340\ \text{Hz}$$

Example 1

Frequencies giving destructive interference are

$$f_n' = (n+0.5)\frac{v}{\Delta L} = (n+0.5)\times\frac{340\ \text{m/s}}{1\ \text{m}} = (n+0.5)\times 340\ \text{Hz}$$

The lowest *n* that makes sense is *n* = 0.

$$f_0' = (0.5)\times 340\ \text{Hz} = 170\ \text{Hz}$$

$$f_1' = (1.5)\times 340\ \text{Hz} = 510\ \text{Hz}$$

$$f_2' = (2.5)\times 340\ \text{Hz} = 850\ \text{Hz}$$

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