Waves & Sound

Waves

What is a wave?

A wave is a periodic disturbance that carries energy from one place to another.

Classifying waves

1. **Mechanical Waves** - e.g., water waves, sound waves, and waves on strings. The wave requires a medium through which to travel, but there is no net flow of mass though the medium, only a flow of energy. We'll study these this week.

2. **Electromagnetic Waves** - e.g., light, x-rays, microwaves, radio waves, etc. They're the same kind of waves, just with different frequency ranges, and they don't need a medium. We'll look at these a little later.

3. **Matter Waves** - waves associated with things like electrons, protons, and other tiny particles. We'll do these toward the end of this course.

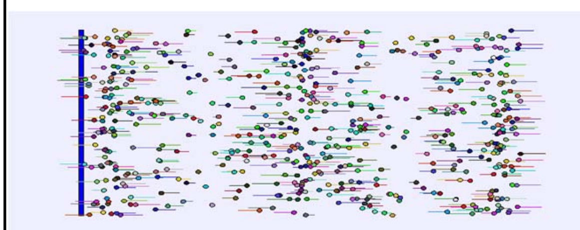
Another way to classify waves

Transverse Waves - the wave disturbance oscillates in a direction perpendicular to the way the wave is traveling. A good example is a wave on a string or electromagnetic waves traveling in free space.

Longitudinal Waves - the wave disturbance oscillates along the same direction as the way the wave is traveling. Sound waves are longitudinal waves.

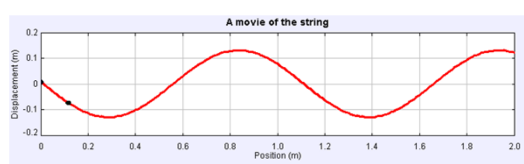
Simulation

Longitudinal Waves



Note: In longitudinal waves, compressions and rarefactions occur about points where the displacement of the particles in the medium is momentarily zero.

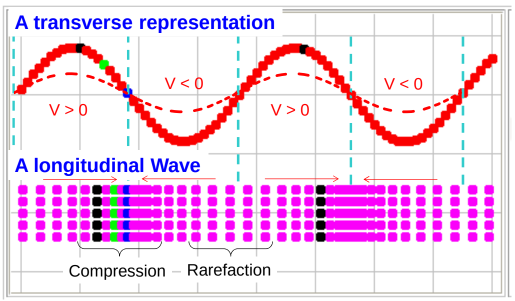
Transverse Waves



Simulation

Transverse representation of a longitudinal wave

Figure legend: The dashed sine-wave in the upper drawing denotes the transverse representation found at a slightly earlier time. The red arrows in the lower drawing denote the direction of the disturbance velocity in different parts of the wave.



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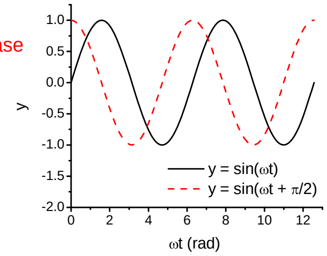
Connection with simple harmonic motion

Consider a single-frequency transverse wave.
Each particle experiences simple harmonic motion in the *y*-direction. The motion of any particle is given by:

Angular frequency Phase

$$y(t) = A\sin(\omega t + \phi)$$

Amplitude



Simulation

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Describing the motion

For the simulation, we could write out 81 equations, one for each particle, to fully describe the wave. Which parameters would be the same in all 81 equations and which would change?

1. The amplitude is the only one that would stay the same.
2. The angular frequency is the only one that would stay the same.
3. The phase is the only one that would stay the same.
4. The amplitude is the only one that would change.
5. The angular frequency is the only one that would change.
6. The phase is the only one that would change.
7. All three parameters would change.

$$y(t) = A\sin(\omega t + \phi)$$

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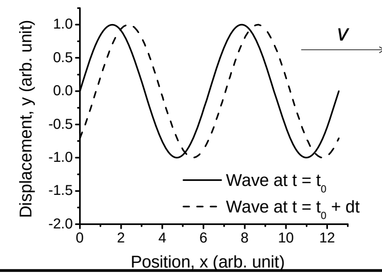
Describing the motion at ALL points of a wave

Each particle oscillates with the same amplitude and frequency, but with its own phase angle.

For a wave traveling right, particles to the right lag behind particles to the left. The phase difference is proportional to the distance between the particles.



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Describing the motion at ALL points of a wave

So, for a wave traveling right, if we say the motion of the particle at *x* = 0 is given by:

$$y(0,t) = A\sin(\omega t)$$

The motion of a particle at another *x*-value is:

$$\boxed{y(x,t) = A\sin(\omega t - kx) \quad \text{(Right-traveling wave)}}$$

where *k* is a constant known as the wave number.
Note: this *k* is not the spring constant.

This one equation describes the whole wave.

Similarly, for a wave traveling left, the motion of the particle at position x is:

$$\boxed{y(x,t) = A\sin(\omega t + kx) \quad \text{(Left-traveling wave)}}$$

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What is *k*?

A particle a distance of one wavelength away from another particle would have a phase difference of 2π.

$kx = 2\pi$ when $x = \lambda$, so the wave number is $k = \dfrac{2\pi}{\lambda}$

The wave number is related to wavelength the same way the angular frequency is related to the period.

The angular frequency: $\omega = \dfrac{2\pi}{T}$

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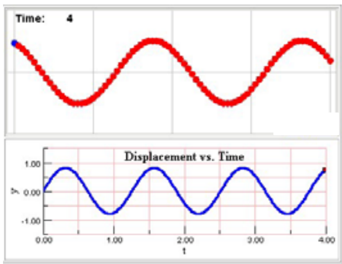
Wavelength and period

The top picture is a photograph of a wave on a string at a particular instant. The graph underneath is a plot of the displacement as a function of time for a single point on the wave.

To determine the wavelength, do we need the photograph, the graph, or both?



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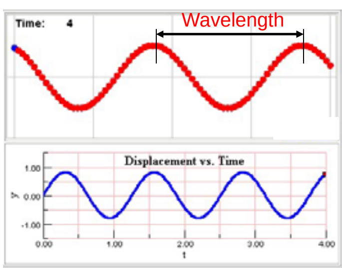

Ans. The photograph.

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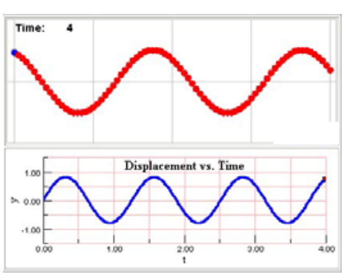

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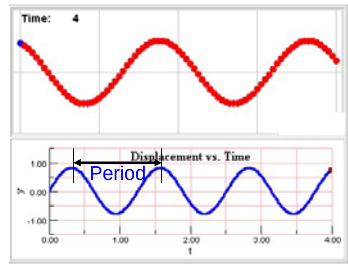

Ans. The graph.

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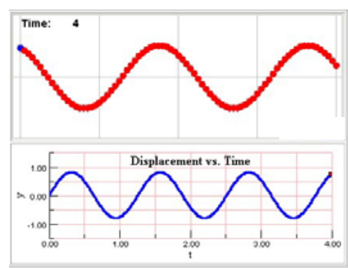
Wavelength and period

The top picture is a photograph of a wave on a string at a particular instant. The graph underneath is a plot of the displacement as a function of time for a single point on the wave.

To determine the maximum speed of a single point in the medium, do we need the photograph, the graph, or both?



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Maximum speed of a single point

Each point experiences simple harmonic motion, so we think back to last semester:

$$u_{\max} = A\omega = A(2\pi f) = A\left(\frac{2\pi}{T}\right)$$

We can get both the amplitude and the period from the graph.

Note that <u>the maximum speed of a single point (which oscillates in the *y*-direction) is quite a different thing from the speed of the wave (which travels in the *x*-direction).</u>

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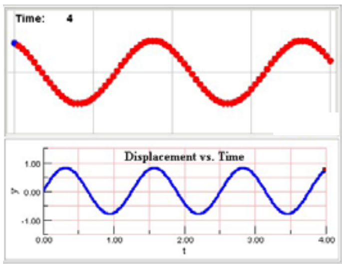
Wavelength and period

The top picture is a photograph of a wave on a string at a particular instant. The graph underneath is a plot of the displacement as a function of time for a single point on the wave.

To determine the speed of the wave, do we need the photograph, the graph, or both?



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A wave on a string

What parameters determine the speed of a wave on a string?

Ans. Properties of the medium: the tension in the string, and how heavy the string is.

$$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}}$$

where μ is the mass per unit length of the string.

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Wave speed

The wave travels a distance of one wavelength in a time of one period, so we need both the photograph and the graph:

$$v = \frac{\lambda}{T}, \quad \text{but } f = \frac{1}{T}, \quad \text{so } \boxed{v = f\lambda}$$

In general:

- frequency is determined by whatever excites the wave
- wave speed is determined by properties of the medium.

Given the frequency of the excitation source and properties of the medium (and hence wave speed), the wavelength is determined by the equation above: $\lambda = \dfrac{v}{f}$

Simulation

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Speed of a wave on a rope

A rope, supported at the top, hangs vertically down. The bottom end is free to move. If a pulse is sent down the rope from the top, what does it do?

1. Speeds up
2. Slows down
3. Travels at a constant velocity

Ans. The gravitational force causes the tension in the rope, which determines the speed of the pulse, $v = (T/\mu)^{1/2}$.

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Speed of a wave on a string?

Which of the following determines the wave speed of a wave on a string?

1. the frequency at which the end of the string is shaken up and down
2. the coupling between neighboring parts of the string, as measured by the tension in the string
3. the mass of each little piece of string, as characterized by the mass per unit length of the string.
4. Both 1 and 2
5. Both 1 and 3
6. Both 2 and 3
7. All three.

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Making use of the mathematical description

The general equation describing a transverse wave moving in one dimension, in the positive x-direction is:

$$y(x,t) = A\sin(\omega t - kx)$$

Sometimes a cosine is appropriate, rather than a sine.

If the wave goes in the negative x-direction, we use:

$$y(x,t) = A\sin(\omega t + kx)$$

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Making use of the mathematical description

Here's a specific example:

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(a) Determine the wave's amplitude, wavelength, and frequency.

(b) Determine the speed of the wave.

(c) If the string has a mass/unit length of μ = 0.012 kg/m, determine the tension in the string.

(d) Determine the direction of propagation of the wave.

(e) Determine the maximum transverse speed of the string.

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Making use of the mathematical description

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(a) Determine the wave's amplitude, wavelength, and frequency.

The amplitude is the coefficient that is multiplying the sine.
A = 0.9 cm

The wavenumber *k* is the coefficient that is multiplying the *x*:
k = 1.2 m^{-1}. The wavelength is: $\lambda = \frac{2\pi}{k} = 5.2\text{ m}$

The angular frequency ω is the coefficient that is multiplying the *t*.
ω = 5.0 rad/s. The frequency is: $f = \frac{\omega}{2\pi} = 0.80\text{ Hz}$

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Making use of the mathematical description

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(b) Determine the speed of the wave.

The wave speed can be found from the frequency and wavelength:

$$v = f\lambda = (0.80\text{ Hz})(5.2\text{ m}) = 4.2\text{ m/s}$$

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Making use of the mathematical description

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(c) If the string has a mass/unit length of μ = 0.012 kg/m, determine the tension in the string.

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$F_T = \mu v^2 = (0.012\text{ kg})(4.17\text{ m/s})^2 = 0.21\text{ N}$$

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Making use of the mathematical description

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(d) Determine the direction of propagation of the wave.

To find the direction of propagation of the wave, just look at the sign between the *t* and *x* terms in the equation. In our case we have a minus sign.

A negative sign means the wave is traveling in the +x direction.

A positive sign means the wave is traveling in the -x direction.

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Making use of the mathematical description

$$y(x,t) = (0.9\text{ cm})\sin\left[(5.0\text{ s}^{-1})t - (1.2\text{ m}^{-1})x\right]$$

(e) Determine the maximum transverse speed of the string.

All parts of the string are experiencing simple harmonic motion (SHM). In SHM, the maximum speed is:

$$u_{max} = A\omega$$

In this case we have *A* = 0.9 cm and ω = 5.0 rad/s, so:

$$u_{max} = A\omega = (0.9\text{ cm})(5.0\text{ rad/s}) = 4.5\text{ cm/s}$$

This is quite a bit less than the 4.2 m/s speed of the wave!

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Sound

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Speed of sound

Sound waves are longitudinal waves consisting of periodic compression and rarefaction of the medium in which the sound wave is traveling. Sound waves are created by a vibrating source.

In which medium does sound travel faster, air or water?

1. Sound travels faster through air
2. Sound travels faster through water

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Speed of sound

In general, the speed of sound is the highest in solids, then liquids, then gases. Sound propagates by molecules passing the wave on to neighboring molecules, and the coupling between molecules is the strongest in solids.

Medium	Speed of sound
Air (0°C)	331 m/s
Air (20°C)	343 m/s
Helium	965 m/s
Water	1400 m/s
Steel	5940 m/s
Aluminum	6420 m/s

Speed of sound in air:

$v = (331 \text{ m/s}) + (0.6 \text{ m/(s °C)}) \times T_C$ (T_c is temperature in °C)

The range of human hearing

Humans are sensitive only to a particular range of frequencies, typically from 20 Hz to 20000 Hz. Whether you can hear a sound also depends on its intensity - we're most sensitive to sounds of a couple of thousand Hz, and considerably less sensitive at the extremes of our frequency range.

We generally lose the top end of our range as we age.

Other animals are sensitive to sounds at lower or higher frequencies. Anything less than the 20 Hz that marks the lower range of human hearing is classified as infrasound - elephants, for instance, communicate using low frequency sounds. Anything higher than 20 kHz, our upper limit, is known as ultrasound. Dogs, bats, dolphins, and other animals can hear sounds in this range.

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Biological applications of ultrasound

- imaging, particularly within the womb
- breaking up kidney stones
- therapy, via the heating of tissue
- navigation, such as by dolphins (natural sonar)
- prey detection, such as by bats

In imaging applications, high frequencies (typically 2 MHz and up) are used because the small wavelength provides high resolution. More of the ultrasound generally reflects back from high-density material (such as bone), allowing an image to be created from the reflected waves.

Picture from Wikipedia.

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Sound intensity

The intensity of a sound wave is its power/unit area. $I = \frac{P}{A}$

In one dimension, the intensity is constant as the wave travels. In two or three dimensions, the intensity decreases as one gets farther from the source. In three dimensions, for a source emitting sound uniformly in all directions, the intensity drops off as $1/r^2$, where r is the distance from the source.

To understand the r dependence, imagine a sphere of radius r surrounding the source. All the sound, emitted by the source with power P, passes through the sphere. When the sound reaches the sphere, its intensity is: $I = \frac{P}{4\pi r^2}$

That's the surface area of a sphere in the denominator.

Double the distance and the intensity drops by a factor of 4.

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The decibel scale

The decibel scale is logarithmic, much like the Richter scale for measuring earthquakes. Sound intensity in decibels is given by:

$$\beta = (10\ \text{dB})\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity in W/m^2 and I_0 is a reference intensity known as the threshold of hearing. I_0 = 1 x 10^{-12} W/m^2 .

Every 10 dB represents a change of one order of magnitude in intensity. 120 dB, 12 orders of magnitude higher than the threshold of hearing, has an intensity of 1 W/m^2. This is the threshold of pain.

A 60 dB sound has ten times the intensity of a 50 dB sound, and 1/10th the intensity of a 70 dB sound. 38

Relative decibels

An increase of x dB means that the sound has increased in intensity by some factor. For instance, an increase by 5 dB represents an increase in intensity by a factor of 3.16 (= 10$^{(5/10)}$).

The decibel equation can also be written in terms of a change. A change in intensity, in dB, is given by:

$$\boxed{\Delta\beta = (10\ \text{dB})\log_{10}\left(\frac{I_f}{I_i}\right)}$$

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