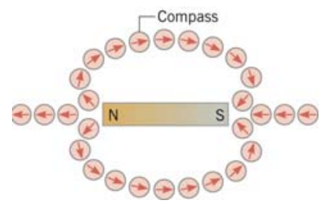
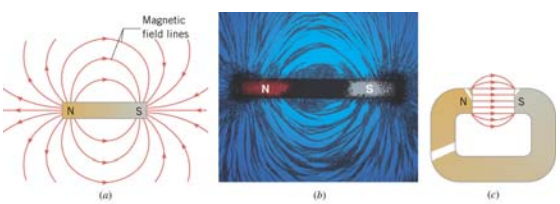
Magnetic Fields
and
Forces on Moving Charges

Permanent Magnets



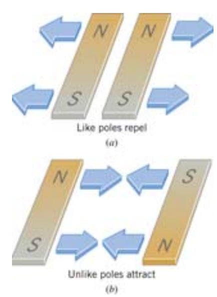
We encounter ***magnetic fields*** frequently in daily life from those due to a ***permanent magnet***. Each permanent magnet has a ***north*** pole and a ***south*** pole. If we put small compasses (***the needle of a compass is a permanent magnet itself***) around a permanent magnet, the north pole of each compass needle would point in the direction of the ***local*** magnetic field.

Permanent Magnets



Besides small compasses, we can use iron filings to indicate the magnetic field lines in a region as shown in Fig. (b). The iron filings get magnetic ***polarized*** by the local magnetic field and thus align with the magnetic field lines. Fig. (a) and (c), respectively, shows the pattern of the magnetic field lines in the vicinity of a ***bar magnetic*** and in the gap of a ***horseshoe magnet***.

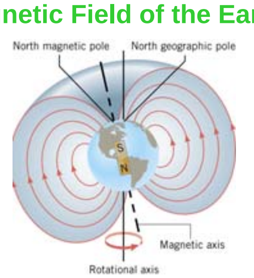
Interactions between magnetic north and south poles



The behavior of magnetic poles is similar to that of like and unlike electric charges.

But different from electric charges, no magnetic monopoles (*i.e.,* isolated single magnetic charges) has ever been observed. Magnetic poles always come in pairs, comprising the north and the south poles at the same time.

Magnetic Field of the Earth



The earth produces a magnetic field like that of a bar magnetic. The field is as such that it attracts the North pole of a compass needle to its ***north magnetic pole***. The magnetic axis of the earth's magnetic field is varying with time. Currently, it is at an angle of about 11.5º from the earth's geographic north-south axis.

Forces on a Moving Charge due to a Magnetic Field

For a charge to experience a force when placed in a magnetic field,

1. the charge must be moving.
2. the velocity of the charge must have a component that is perpendicular to the magnetic field.

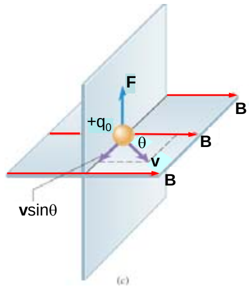
This force is called ***Lorentz force*** in physics, but is also commonly called ***magnetic force***.

Forces on a Moving Charge Particle due to a Magnetic Field

The Lorentz or magnetic force is given by:

$$F = qvB\sin\theta$$

where q is the charge of the particle, v is the particle's speed, B is the magnetic field and θ is the angle between the ***B*** and ***v*** vector. In SI units, magnetic fields are in ***Tesla***. As an example, the earth's magnetic field is about 10^{-4} Tesla in the vicinity of its surface.



The direction of the Lorentz force is determined by a ***right-hand rule*** (RHR). In this class note, we refer to this right-hand rule as ***RHR-1***. Later in this chapter, we will encounter another RHR!

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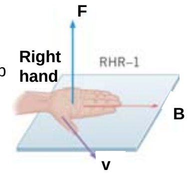
Right Hand Rule No. 1

Right Hand Rule No. 1. (RHR-1) Extend the right hand so the fingers point along the direction of the magnetic field and the thumb points along the velocity of the charge. The palm of the hand then faces in the direction of the magnetic force that acts on a positive charge.



If the moving charge is negative, the direction of the force is opposite to that predicted by RHR-1.

Notes:
(1) ***F*** is always perpendicular to both ***B*** and ***v***.
(2) This rule works even when ***v*** is not exactly perpendicular to ***B***.
(3) Details of this RHR is different from that adopted in Essential Physics but it gives the same result.

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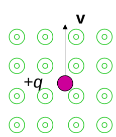
Direction of Lorentz Force I

A positive charge moves upward in a magnetic field that points out of the paper. What is the direction of the Lorentz force?



1. Right
2. Left
3. Into the screen
4. Out of the screen
5. Up
6. Down

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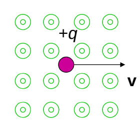
Direction of Lorentz Force II

A positive charge moves to the right in a magnetic field that points out of the paper. What is the direction of the Lorentz force?



1. Right
2. Left
3. Into the screen
4. Out of the screen
5. Up
6. Down

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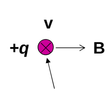
Direction of Lorentz Force III

A positive charge moves into the paper in the presence of a magnetic field that points right. What is the direction of the Lorentz force?



This means that **v** points into the paper

1. Right
2. Left
3. Into the screen
4. Out of the screen
5. Up
6. Down

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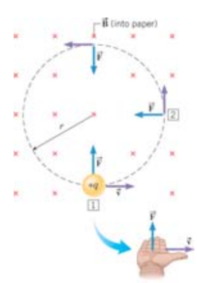
Motion of charges moving perpendicular to B

If **v** is perpendicular to **B**, the charge follows a circular path (see figure at right). The radius of the circular path can be derived as follows:

$$F = qvB = \frac{mv^2}{r}$$

So, the radius of the circular path is:

$$r = \frac{mv}{qB}$$



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Motion of charges moving perpendicular to B

The time for the object to go once around the circle (the **period, *T***) is:

$$\boxed{T=\frac{2\pi r}{v}}=\frac{2\pi mv/qB}{v}=\boxed{\frac{2\pi m}{qB}}$$

Interestingly, the time is independent of the speed. The faster the speed, the larger the radius, but the period is unchanged.

If the object doesn't go around a complete circle, but instead only a fraction, *f*, of it, the time it takes would be *fT*.

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Kinetic energy of charges moving in a magnetic field, B

Case 1: $\mathbf{v} \perp \mathbf{B}$

Because the magnetic force is perpendicular to the velocity vector, **v**, the magnitude of the velocity is not changed. So, the kinetic energy, K is not changed.

Case 2: **v** // **B**

There is no magnetic force, the magnitude of **v** is still not changed. So, K is also not changed.

Case 3: **v** makes an arbitrary angle with **B**

Decompose **v** into $\mathbf{v}_\perp$ and $\mathbf{v}_{//}$ perpendicular and parallel to B, respectively. Since both $|\mathbf{v}_\perp|$ and $|\mathbf{v}_{//}|$ are not changed, $|\mathbf{v}|^2 = |\mathbf{v}_\perp|^2 + |\mathbf{v}_{//}|^2$ is also not change. So K is generally not changed!

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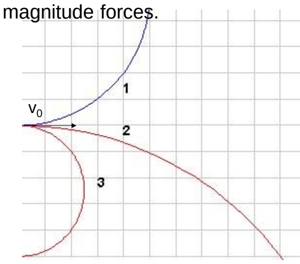
Circular paths

Three charged objects with the same mass and the same magnitude charge have initial velocities directed right. Here are the trails they follow through a region of uniform magnetic field. Rank the objects based on their speeds and magnitude forces.

1. 1 > 2 > 3
2. 2 > 1 > 3
3. 3 > 2 > 1
4. 3 > 1 > 2
5. None of the above



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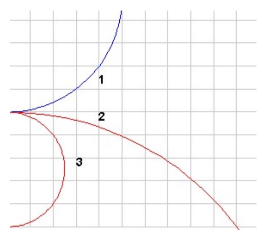
Circular paths

$$r=\frac{mv}{qB}$$

The radius of the path is proportional to the speed, so the correct ranking by speed is choice 2, 2 > 1 > 3.



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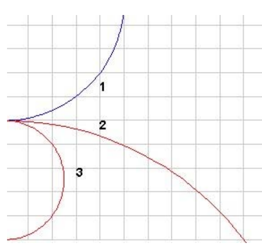
Circular paths, II

Three charged objects with the same mass and the same magnitude charge have initial velocities directed right. Rank the objects based on the magnitude of the force they experience as they travel through the magnetic field.

1. 1 > 2 > 3
2. 2 > 1 > 3
3. 3 > 2 > 1
4. 3 > 1 > 2
5. None of the above



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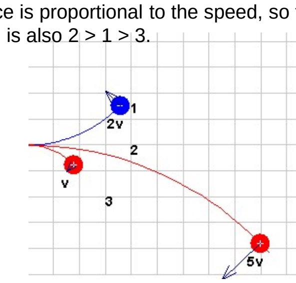
Circular paths, II

$F = qvB\sin\theta$

The force is proportional to the speed, so the correct ranking by force is also 2 > 1 > 3.



The arrows illustrate the the Lorentz force vectors (with both magnitudes and directions) acting on the three objects.

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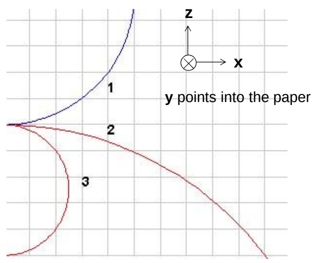
Circular paths, III

If the objects tracing paths 2 and 3 are positive, what is the direction of the magnetic field?

1. Right
2. Left
3. Into the screen
4. Out of the screen
5. Up
6. Down



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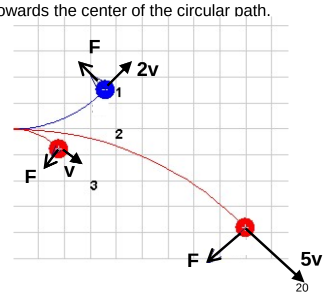
Circular paths, III

Note that:

- V is tangential to the circular path and directs away from the starting point.
- The Lorentz force points towards the center of the circular path.

To apply the RHR-1, orient your right hand such that the palm is facing the direction of **F**. Extend your thumb so that it makes 90º with the other fingers. Rotate your right hand (with the palm still facing **F**) until the thumb points in the direction of **v**. The direction of the other fingers would then be pointing in the direction of **B.**



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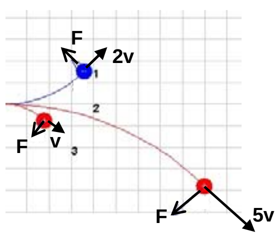
Circular paths, IV

If the velocities of the three objects with same mass and magnitude charge are as shown in the drawing (in units of v), rank the time taken for the particles to complete one-quarter of the circular path.

1. 1 > 2 > 3
2. 2 > 1 > 3
3. 3 > 2 > 1
4. 3 > 1 > 2
5. 1 = 2 = 3
6. None of the above



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Circular paths, IV

Recall: $T = \frac{2\pi m}{qB}$

To complete a quarter-circle, the time taken is $T/4$. Since T is independent of v, the time taken is the same for the three objects. Note that if the objects have different masses and/or charges, we can use the above equation to rank the times: For example, doubling the mass m would require twice the time, doubling the charge q would require half the time, etc.

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Possible paths of a charge in a magnetic field

If the velocity of a charge is parallel to the magnetic field, the charge moves with constant velocity because there's no net force.

If the velocity is perpendicular to the magnetic field, the path is circular because the force is always perpendicular to the velocity.

What happens when the velocity is not one of these special cases, but has a component parallel to the field and a component perpendicular to the field?

The parallel component produces straight-line motion. The perpendicular component produces circular motion. The net motion is a combination of these, a spiral. Simulation

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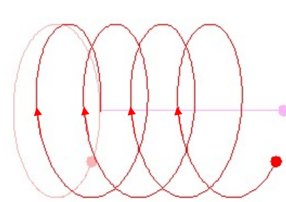
Which way is the field?

The charge always spirals around the magnetic field. Assuming the charge in this case is positive, which way does the field point in the simulation?

1. Left
2. Right



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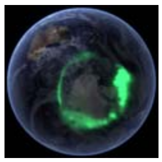
Spiraling charges

Charges spiral around magnetic field lines.

Charged particles near the Earth are trapped by the Earth's magnetic field, spiraling around the Earth's magnetic field down toward the Earth at the magnetic poles.

The energy deposited by such particles gives rise to the aurora borealis (northern lights) and the aurora australis (southern lights). The colors are usually dominated by emissions from oxygen atoms.



Photos from Wikipedia

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A mass spectrometer

A mass spectrometer is a device for separating particles based on their masses. There are different types – we will investigate one that exploits electric and magnetic fields.

Step 1: Accelerate charged particles via an electric field.

Step 2: Use an electric field and a magnetic field to select particles of a particular velocity.

Step 3: Use a magnetic field to separate particles based on mass.

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Step 1: The Accelerator

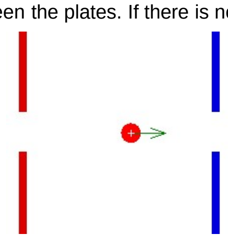
Simulation

The simplest way to accelerate ions is to place them between a set of charged parallel plates. The ions are repelled by one plate and attracted to the other. If we cut a hole in the second plate, the ions emerge with a kinetic energy determined by the potential difference between the plates. If there is no energy loss,

K = | q ΔV |

So, the maximum speed of a particle is:

$$v = \sqrt{\frac{2q\,|\,\Delta V\,|}{m}}$$



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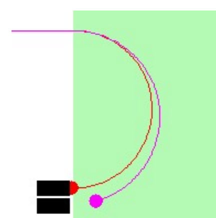
Step 3: The Mass Separator

Simulation

In the last stage, we consider ions entering a region of uniform magnetic field B' with the same velocity *v*. We will discuss later how we can make sure of this. The field is perpendicular to the velocity. Everything is the same for the ions except for their masses, so the radius of each circular path depends only on the ion mass.

$$F = qvB' = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB'}$$

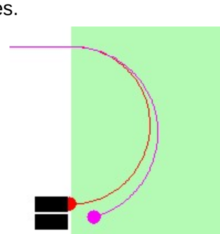


Step 3: The Mass Separator

$$F = qvB' = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB'}$$

The ions are collected after traveling through half-circles, with the separation *s* between two ions is equal to the difference in the diameters of their respective circles.

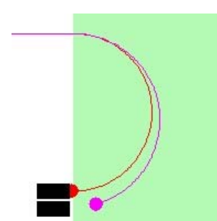
$$s = 2r_2 - 2r_1 = \frac{2(m_2 - m_1)v}{qB'}$$



Magnetic field in the mass separator

In what direction is the magnetic field in the mass separator? The paths shown are for positive charges.

1. up
2. down
3. left
4. right
5. into the screen
6. out of the screen



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Step 2: The Velocity Selector

Simulation

To ensure that the ions arriving at step 3 have the same velocity, the ions pass through a velocity selector, a region with uniform electric and magnetic fields.

The electric field comes from a set of parallel plates, and exerts a force of $\vec{F}_E = q\vec{E}$ on the ions.

The magnetic field is perpendicular to both the ion velocity and the electric field. The magnetic force, $F_M = qvB$, exactly balances the electric force when:

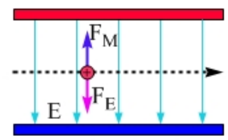
$$qE = qvB \quad \Rightarrow \quad E = vB$$

Ions with a speed of $v = \frac{E}{B}$ pass straight through.

Magnetic field in the velocity selector

In what direction is the magnetic field in the velocity selector, if the positive charges pass through undeflected? The electric field is directed down.

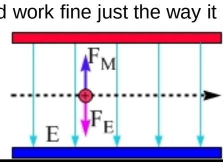
1. up
2. down
3. left
4. right
5. into the screen
6. out of the screen



Negative ions in the velocity selector

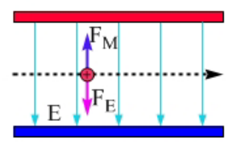
If the charges passing through the velocity selector were negative, what (if anything) would have to be changed for the velocity selector to allow particles of just the right speed to pass through undeflected?

1. reverse the direction of the electric field
2. reverse the direction of the magnetic field
3. reverse the direction of one field or the other
4. reverse the directions of both fields
5. none of the above, it would work fine just the way it is



Negative ions in the velocity selector

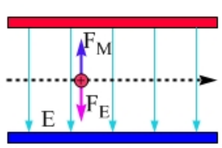
If the charges are negative, both the electric force and the magnetic force reverse direction. The forces still balance, so we don't have to change anything.



Faster ions in the velocity selector

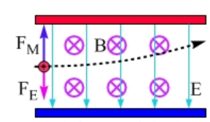
Let's go back to positive ions. If the ions are traveling faster than the ions that pass undeflected through the velocity selector, what happens to them? They get deflected …

1. up
2. down
3. into the screen
4. out of the screen

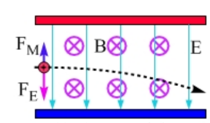


Faster ions in the velocity selector

For ions with a larger speed, the magnetic force exceeds the electric force and those ions are deflected up of the beam. The opposite happens for slower ions, so they are deflected down out of the beam.



(a) fast particles



(b) slow particles