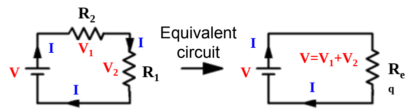
Resistor Combinations

Resistors in series



When resistors are in series they are arranged in a chain, the current has only one path to take.

So, the current is the same through each resistor in series.

At the same time, the sum of the potential drops across each resistor must equal to the total potential drop of the voltage supply across the whole chain, i.e., the voltage supply's emf, V. Thus, $V = V_1 + V_2 \Rightarrow IR_{eq} = IR_1 + IR_2 \Rightarrow R_{eq} = R_1 + R_2$.

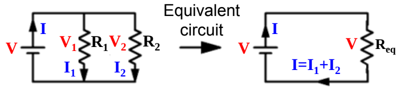
We can generalize the above result to any number of resistors, so the equivalent resistance of resistors in series is: $\boxed{R_{eq} = R_1+R_2+R_3+\ldots}$ and the loop rule is: $\boxed{V = V_1+V_2+V_3+\ldots}$

Resistors in parallel

When resistors are arranged in parallel, the current has multiple paths to take. Nevertheless, the currents of different paths must add to equal the total current entering the parallel combination, i.e., $\boxed{I_1 + I_2 = I}$. At the same time, each resistor is connected completely across the voltage supply. So, the potential difference across each resistor is the same equal to the emf of the voltage supply, $V_1 = V_2 = V$.

$$I = I_1 + I_2$$

$$\Rightarrow \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$



For more than two resistors connected in parallel,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$

Light bulbs in parallel

A 100-W light bulb is connected in parallel with a 40-W light bulb, and the parallel combination is connected to a standard electrical outlet. The 40-W light bulb is then unscrewed from its socket. What happens to the 100-W bulb?

1. It turns off
2. It gets brighter
3. It gets dimmer (but stays on)
4. Nothing at all – it stays the same

Light bulbs in series

A 100-W light bulb is connected in series with a 40-W light bulb and a standard electrical outlet. Which bulb is brighter?

1. The 40-watt bulb
2. The 100-watt bulb
3. Neither, they are equally bright

Light bulbs in series

The brightness is related to the power (not the power stamped on the bulb, the power actually being dissipated in the bulb). The current is the same through the bulbs, so consider:

$$P = I^2 R$$

We already showed that the resistance of the 100 W bulb is 144 Ω and the 40 W bulb has a resistance of 360 Ω at 120 volts. Neither bulb has 120 volts across it, so they are not going to dissipate the amount of power stated by their power rating. Instead, they will dissipate according to $P = I^2R$. Since the resistance of the 40 W bulb is larger, it would dissipate more power and thus be brighter.

Light bulbs in series, II

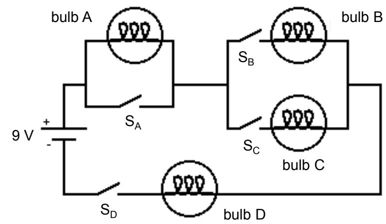
A 100-W light bulb is connected in series with a 40-W light bulb and a standard electrical outlet. The 100-W light bulb is then unscrewed from its socket. What happens to the 40-W bulb?

1. It turns off
2. It gets brighter
3. It gets dimmer (but stays on)
4. Nothing at all – it stays the same

7

Example I: Bulbs and switches

Four identical light bulbs are arranged in a circuit. What is the minimum number of switches that must be closed for at least one light bulb to come on?

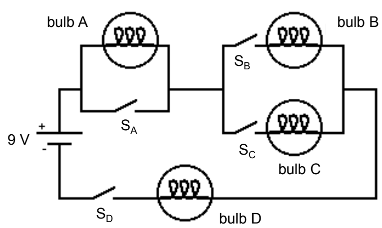


8

Bulbs and switches

What is the minimum number of switches that must be closed for at least one light bulb to come on?

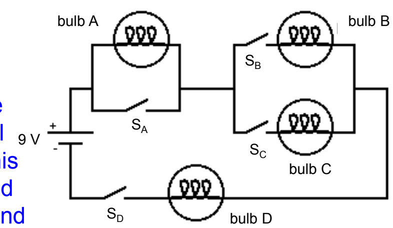
1. 1
2. 2
3. 3
4. 4
5. 0



9

Example I: Bulbs and switches

For there to be a current, there must be a complete path through the circuit from one battery terminal to the other. This can be achieved by closing S_D and S_B or S_C at the minimum.



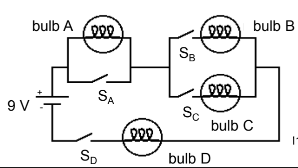
10

Bulbs and switches, II

Which switches should be closed to maximize the brightness of bulb D?

1. All four switches.
2. Switch D and either switch B or switch C
3. Switch D and both switches B and C
4. Switch A, either switch B or switch C, and switch D
5. Only switch D.

Let's discuss the answer in the following slides.



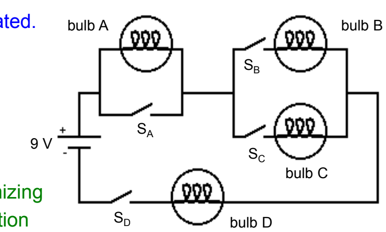
11

Example 2: Bulbs and switches, II

What determines the brightness of a bulb?

The power dissipated.

Since $P = I^2R = V^2/R$, for a bulb with fixed resistance, maximizing the power dissipation in the bulb means maximizing the current through it, or the voltage across it.

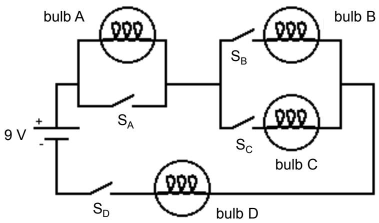


12

Example 2: Bulbs and switches, II

We need to close switch D, and either switch B or switch C, for bulb D to come on. Do the remaining switches matter?

Consider this. How much of the current that passes through the battery passes through bulb D?

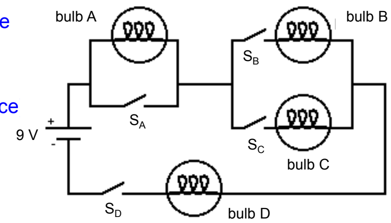


All of it.

Example 2: Bulbs and switches, II

If we open or close switches, does it change the total current in the circuit?

Absolutely, because it changes the total resistance (the equivalent resistance of the circuit.

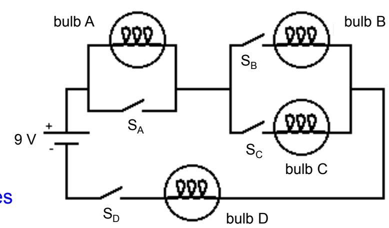


$$I_{total} = \frac{V_{battery}}{R_{eq}}$$

Example 2: Bulbs and switches, II

Does it matter whether just one of the switches B or C is closed, or both of them are closed?

Yes. Closing both switches B and C decreases the resistance of that part of the circuit where B is parallel to C. That increases the current in the circuit and hence the brightness of bulb D.

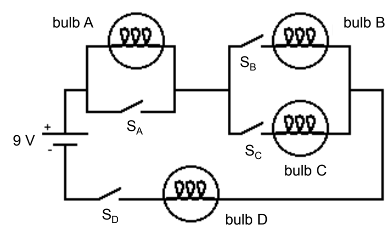


Example 2: Bulbs and switches, II

What about switch A?

An open switch is a path of _infinite_ resistance.

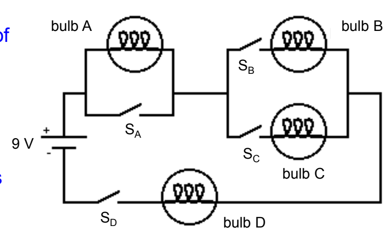
A closed switch is a path of _zero_ resistance.



Example 2: Bulbs and switches, II

What about switch A?

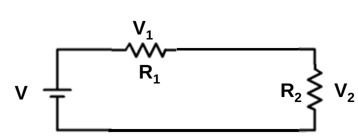
Closing switch A shorts bulb A out of the circuit. That decreases the total resistance, so increases the current and makes bulb D brighter.



So to maximize the brightness of D, close all 4 switches.

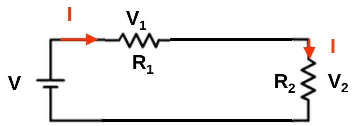
Comparing Voltages in Series Connections

In the series connection shown below, if $R_1 = 2R_2$, what is V_1 and V_2 in terms of V?



Answer: $V_1 = (2/3)V$ and $V_2 = (1/3)V$

Comparing Voltages in Series Connection



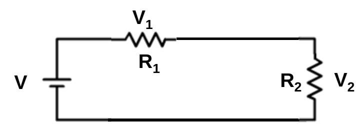
By Ohm's law, $V_1 = IR_1$ and $V_2 = IR_2$

So, $\boxed{V_1/V_2 = R_1/R_2}$ (i.e., $V \propto R$)

If $R_1 = 2R_2$, $V_1 = 2V_2$.

Finally, use the fact that $V = V_1 + V_2$. We can readily see that $V_1 = (2/3)V$ and $V_2 = (1/3)V$

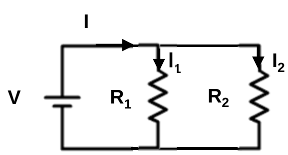
Comparing Voltages in Series Connection



This example shows that in series connections, the total voltage across the resistors connected in series is divided in proportion to the value of individual's resistance. The bigger the resistance, the bigger its share of the total voltage.

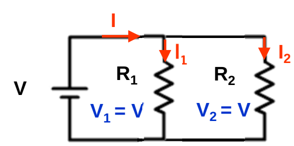
Comparing Currents in Parallel Connections

In the parallel connection shown below, if $R_1 = 2R_2$, what is I_1 and I_2 in terms of I?



Answer: $I_1 = (1/3)I$ and $I_2 = (2/3)I$

Comparing Currents in Parallel Connections



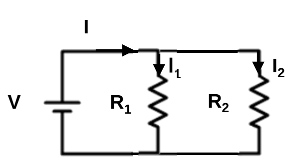
By Ohm's law, $I_1 = V/R_1$ and $I_2 = V/R_2$

So, $\boxed{I_1/I_2 = R_2/R_1}$ (i.e., $I \propto 1/R$)

If $R_1 = 2R_2$, $I_2 = 2I_1$.

Finally, use the fact that $I = I_1 + I_2$. We can readily see that $I_2 = (2/3)I$ and $I_1 = (1/3)I$

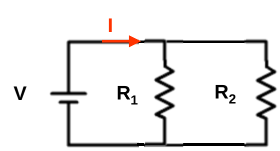
Comparing Currents in Parallel Connections



This example shows that in parallel connections, the total current in the parallel connection (i.e., the sum of the currents in all the parallel branches) is divided according to the inverse proportion of individual's resistance. The bigger the resistance, the smaller its share of the total current.

Total Current in Parallel Connections I

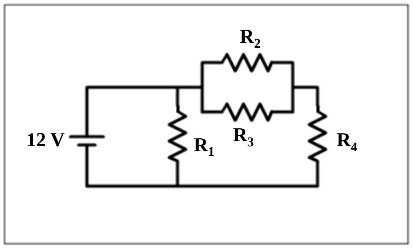
In the parallel circuit discussed above, what is the total current I in terms of V, R_1 and R_2?



Answer: $I = I_1 + I_2 = V/R_1 + V/R_2 = V(1/R_1 + 1/R_2)$

Note that this is the same as $I = V/R_{eq}$ where $1/R_{eq} = 1/R_1 + 1/R_2$.

Example 3: A combination circuit

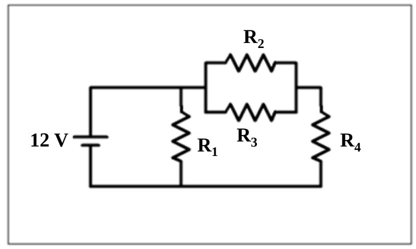


How do we analyze a circuit like this, to find the current through, and voltage across, each resistor?

$R_1 = 6\ \Omega$ $R_2 = 36\ \Omega$ $R_3 = 12\ \Omega$ $R_4 = 3\ \Omega$

25

Example 3: A combination circuit



First, replace two resistors that are in series or parallel by one equivalent resistor. Keep going until you have one resistor. Find the current in the circuit. Then, expand the circuit back again, finding the current and voltage at each step.

26

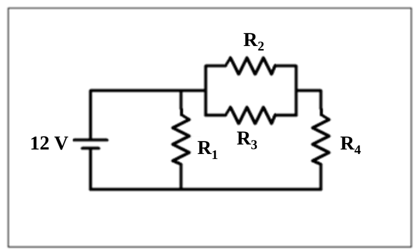
Combination circuit: rules of thumb

Two resistors are in series when the same current that passes through one resistor goes on to pass through another.

Two resistors are in parallel when they are directly connected together at the two ends, and the current splits, some passing through one resistor and some through the other, and then re-combines.

27

Example 3: A combination circuit



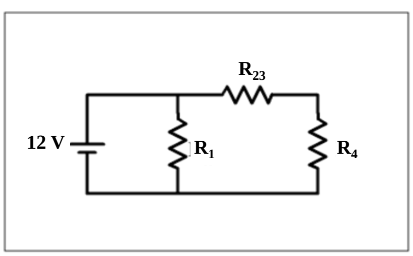
Where do we start?

$R_1 = 6\ \Omega$ $R_2 = 36\ \Omega$ $R_3 = 12\ \Omega$ $R_4 = 3\ \Omega$

Step 1: Resistors 2 and 3 are in parallel. Find their equivalent resistance R_{23}.

28

Example 3: A combination circuit

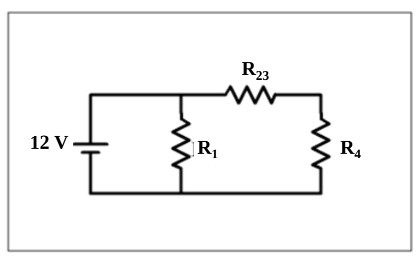


$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{36\ \Omega} + \frac{1}{12\ \Omega} = \frac{1}{36\ \Omega} + \frac{3}{36\ \Omega} = \frac{4}{36\ \Omega}$$

$$R_{23} = \frac{36\ \Omega}{4} = 9\ \Omega$$

29

Example 3: A combination circuit



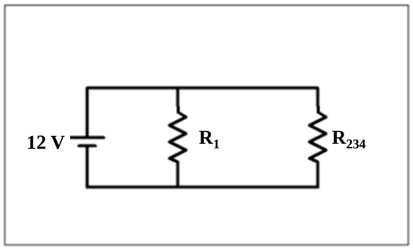
What next?

$R_1 = 6\ \Omega$ $R_{23} = 9\ \Omega$ $R_4 = 3\ \Omega$

Step 2: Resistors 2-3 and 4 are in series. Find their equivalent resistance R_{234}.

30

Example 3: A combination circuit



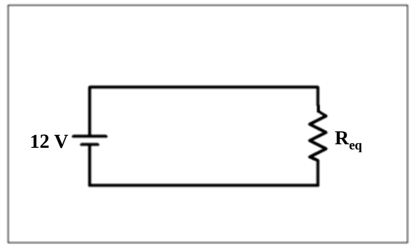
$$R_{234} = R_{23} + R_4 = 9\,\Omega + 3\,\Omega = 12\,\Omega$$

R_1 = 6 Ω R_{234} = 12 Ω

Step 3: R_1 and R_{234} are in parallel. Find their equivalent resistance, R_{eq}.

31

Example 3: A combination circuit

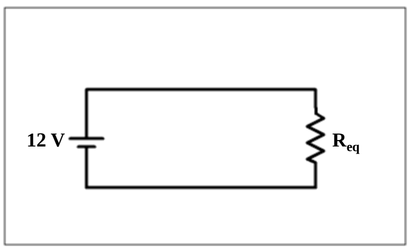


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{234}} = \frac{1}{6\,\Omega} + \frac{1}{12\,\Omega} = \frac{2}{12\,\Omega} + \frac{1}{12\,\Omega} = \frac{3}{12\,\Omega}$$

$$R_{eq} = \frac{12\,\Omega}{3} = 4\,\Omega$$

32

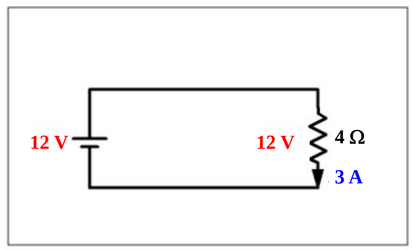
Example 3: A combination circuit



Now, find the current in the circuit.

33

Example 3: A combination circuit

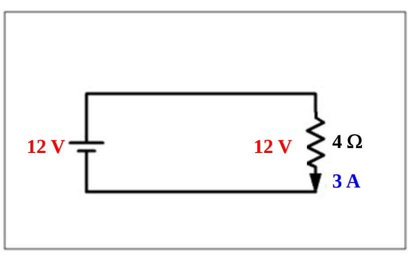


Now, find the current in the circuit.

$$I_{total} = \frac{V_{battery}}{R_{eq}} = \frac{12\text{ V}}{4\,\Omega} = 3\text{ A}$$

34

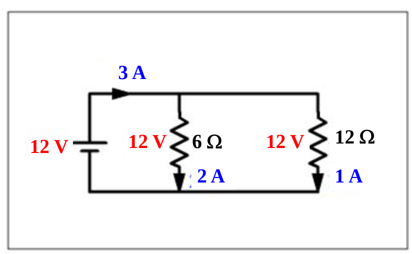
Example 3: A combination circuit



Expand the circuit back, in reverse order.

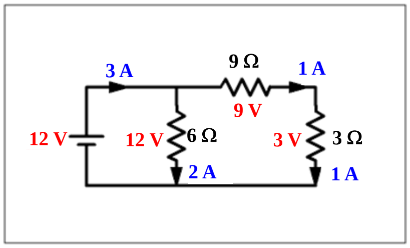
35

Example 3: A combination circuit



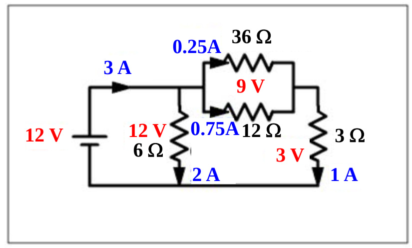
When expanding an equivalent resistor back to a parallel pair, the voltage is the same, and the current splits. Apply Ohm's Law to find the current through each resistor. Make sure the sum of the currents is the current in the equivalent resistor. 36

Example 3: A combination circuit



When expanding an equivalent resistor back to a series pair, the current is the same, and the voltage divides. Apply Ohm's Law to find the voltage across each resistor. Make sure the sum of the voltages is the voltage across the equivalent resistor. 37

Example 3: A combination circuit

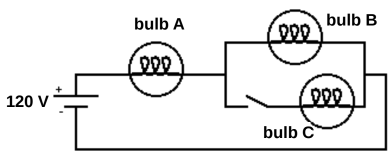


The last step:
R_2 = 36 Ω, R_3 = 12 Ω and they are in parallel. So, $I_2/I_3 = R_3/R_2$ = 1/3. This, plus the fact that I_{23}= 1A gives I_2 = 0.25A and I_3 = 0.75A 38

Three identical bulbs

Three identical light bulbs are connected in the circuit shown. When the power is turned on, and with the switch beside bulb C left open, how will the brightness of the bulbs compare?

1. A = B = C
2. A > B > C
3. A > B = C
4. A = B > C
5. B > A > C

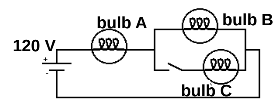


39

Three identical bulbs, II

When the switch is closed, bulb C will turn on, so it definitely gets brighter.

What about bulbs A and B?



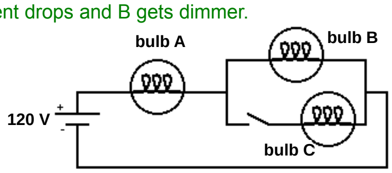
1. Both A and B get brighter
2. Both A and B get dimmer
3. Both A and B stay the same
4. A gets brighter while B gets dimmer
5. A gets brighter while B stays the same
6. A gets dimmer while B gets brighter
7. A gets dimmer while B stays the same
8. A stays the same while B gets brighter
9. A stays the same while B gets dimmer

40

Three identical bulbs, II

Closing the switch brings C into the circuit - this reduces the overall resistance of the circuit, so the current in the circuit increases.

Increasing the current makes A brighter. Because $\Delta V = IR$, the potential difference across bulb A increases. This decreases the potential difference across B, so its current drops and B gets dimmer.



41