Resistance and Power

Ohm’s Law

Ohm’s law states that the current $I$ flowing through a piece of material is proportional to the voltage $\Delta V$ applied across the material. The resistance $R$ is defined as the ratio of $\Delta V$ to $I$.

$$\frac{\Delta V}{I} = R = \text{constant}$$

$$\Delta V = IR$$

The SI unit of resistance is Volts/Ampere (V/A) or Ohms ($\Omega$).

Circuit diagram of the flashlight, and illustration of the variations in Ohm’s Law:

Table 20.4 Resistivities of Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity $\rho$ ((\Omega\cdot\text{m}))</th>
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<th>Resistivity $\rho$ ((\Omega\cdot\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductors</td>
<td></td>
<td>Semiconductors</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>$2.82 \times 10^{-9}$</td>
<td>Carbon</td>
<td>$3.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$1.72 \times 10^{-9}$</td>
<td>Germanium</td>
<td>$0.5$</td>
</tr>
<tr>
<td>Gold</td>
<td>$2.44 \times 10^{-9}$</td>
<td>Silicon</td>
<td>$20$–$2100$</td>
</tr>
<tr>
<td>Beryllium</td>
<td>$4.7 \times 10^{-9}$</td>
<td>Mica</td>
<td>$10^4$–$10^6$</td>
</tr>
<tr>
<td>Mercury</td>
<td>$95.8 \times 10^{-9}$</td>
<td>Nickel</td>
<td>$10^2$–$10^4$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$1.29 \times 10^{-9}$</td>
<td>Teflon</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Resistance Values</td>
<td></td>
<td>Wood (maple)</td>
<td>$3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

*The values pertain to temperatures near 20 °C.

**Depending on purity.

Resistivity values cover an incredibly wide range, from $10^{-8}$ to $10^{12}\Omega\cdot\text{m}$!

Temperature Dependence of Resistance

Resistance changes with temperature.

$$\rho = \rho_0 \left[ 1 + \alpha(T - T_0) \right]$$

$\alpha$ is the temperature coefficient of resistivity, and can be positive or negative. For heating elements like wires in an electric stove or a light bulb, $\alpha$ is positive. At higher temperatures, the intrinsic random motions of the electrons in these materials are more vigorous, impeding the drift motion of the electrons induced by the potential difference across the material. Because the resistivity increases with current – which causes the temperature of the material to increase, Ohm’s law doesn’t apply.

Electric Power

Suppose some charge $q$ emerges from a battery and the potential difference between the battery terminals is $\Delta V$.

Recall that power, $P$, is the rate of change of energy ($= \Delta U/\Delta t$).

We can derive the following:

$$\frac{\Delta U}{\Delta t} = q \Delta V = I \Delta V \Rightarrow P = I \Delta V$$

Units of Power: Joules/sec or Watts

The fact that power is dissipated when a current flows through a resistor is commonly called “Joule heating”. If the power dissipation is significant, it can cause the temperature of the resistor to rise. If the temperature of the resistor reaches about 4000 K, the black-body radiation it emits is visible. Heating wires usually have high melting temperatures above this value.
Electric Power (cont’d)
The power derived above, \( P = I \Delta V \) is the power that must be delivered to an object in order to support the flow of current \( I \) through that object while maintaining a voltage of \( \Delta V \) across it. Below are alternative expressions for power, depending on whether the voltage or the current is known:

\[
P = I \Delta V = \frac{\Delta V^2}{R} \]

\[
P = I^2 R = l(lR) = l^2 R
\]

Understanding your electric bill
The electric company bills you for the amount of energy you use each month.

They measure this in units of kilowatt-hours (kW-h)

How much does 1 of these units cost?

Approximately 18 cents.

How many joules is 1 kW-h?

\[ 1 \text{ kW-h} = 1000 \text{ W} \times 1 \text{ h} = 1000 \text{ J/s} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J} \]

The cost of energy

Here’s how to find the total cost of operating something electrical:

\[ \text{Cost} = (\text{Power rating in kW}) \times (\text{number of hours it’s running}) \times (\text{cost per kW-h}) \]

The cost of watching TV

The average household in the U.S. has a television on for about 3 hours every day. About how much does this cost every day?

1. 0.1 cents
2. 1 cents
3. 10 cents
4. $1

The cost of watching TV

Solution:

In doing this calculation, use 200 W for the power rating of a TV. (This corresponds to the power rating of an average 42" LCD TV. Note that plasma TV has a higher power rating.)

\[
\text{Cost} = (\text{Power rating in kW}) \times (\text{number of hours it’s running}) \times (\text{cost per kW-h})
\]

\[
= (0.2 \text{ kW}) \times (3 \text{ h}) \times ($0.16 \text{ per kW-h})
\]

\[
= $0.1
\]
Wiring in the wall
The figure below shows how electrical appliances are typically wired in the wall. The appliances are said to be in a parallel connection. From the figure, we can see that each appliance in a parallel connection has the same potential difference as that of the mains supply, which is 120 V in this country.

Power Rating of an Electrical Appliance
100 WATT A19/FROST TURBO LIFE LIGHT BULB 20,000 HOURS

Resistance of a Light Bulb and its “Power Rating”

Power rating specifies the power an electrical appliance will consume if it is connected parallel to the mains supply ($\Delta V = 120 \text{ V}$).

Question: Which light bulb, one with a 100 W power rating or one with a 40 W power rating has a higher resistance?

Solution:
To fulfill the power rating, $P$, the resistance, $R$, must satisfy:

$$P = \frac{\Delta V^2}{R} = \frac{(120 \text{ V})^2}{R}$$

For the “100 W” bulbs:
$$R = \frac{120^2}{100} = 144 \Omega$$

For the “40 W” bulbs:
$$R = \frac{120^2}{40} = 360 \Omega$$

Note that if we wire an appliance in series with something else across the mains supply, the power dissipated by the appliance will be different than its power rating. Will the power dissipation be higher or lower than the power rating?

Ans. Lower because the mains voltage is divided between the appliance and that something else.