Capacitors with Dielectrics

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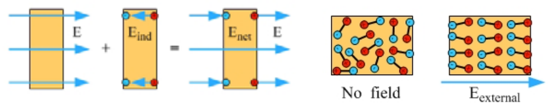
Dielectrics

When a material (generally an insulator) is inserted into a capacitor, we call the material a dielectric. Adding a dielectric allows the capacitor to store more charge for a given potential difference.



When a dielectric is inserted into a charged capacitor, the dielectric is polarized by the field. The electric field from the dielectric will partially cancel the electric field from the charge on the capacitor plates. If the capacitor is connected to a battery at the time, the battery is able to store more charge in the capacitor, bringing the field back to its original value.

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Dielectrics (cont'd)

Thus, when a dielectric is inserted in a charged capacitor (not connected to a power supply), the electric field would be decreased and so would the voltage (= *Ed*). Since *C* = *Q*/*V*, this means that *C* must be bigger when a dielectric is inserted.

For a parallel-plate capacitor containing a dielectric, the capacitance is:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

where κ is the dielectric constant.

In general, adding a dielectric to a capacitor increases the capacitance by a factor of κ.

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The dielectric constant

Every material has a dielectric constant κ, which as discussed above tells you how effective the dielectric is at increasing the amount of charge a capacitor can store for given voltage difference applied across it. Alternatively, we can think of the dielectric constant as telling you how effectively the dielectric is in reducing the electric field of a charged capacitor not connected to a power supply.

$$\kappa = \frac{E_0}{E_{net}} > 1$$

E_0 is the field without the dielectric.
E_{net} is the field with the dielectric.

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The dielectric constant of a conductor

$$\kappa = \frac{E_0}{E_{net}}$$

What is the dielectric constant of a conductor?

1. Zero
2. Infinity
3. This question makes no sense – a dielectric is an insulator, so a conductor does not have a dielectric constant.

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Playing with a dielectric – With the power supply disconnected, insert a dielectric

A capacitor is charged by connecting it to a power supply. Then the connections to the power supply are removed, and a piece of dielectric is inserted between the plates. What happens to the charge and the potential difference of the capacitor?

1. The charge on the plates increases, as does the potential difference.
2. The charge on the plates increases, while the potential difference stays constant.
3. The charge on the plates stays the same, while the potential difference increases.
4. The charge on the plates stays the same, while the potential difference decreases.
5. Neither the charge nor the potential difference changes.

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Playing with a dielectric – With the power supply disconnected, insert a dielectric

Does anything stay the same?

Ans. Because the charge is stranded in the capacitor plates, the charge cannot change.

Adding the dielectric increases the capacitance by a factor of κ.

To see what happens to the potential difference, look at $Q = C\,\Delta V$.

Increasing C while keeping the charge the same means that the potential difference decreases.

We can also get that from $\Delta V = Ed$, with the field being reduced upon inserting the dielectric.

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Energy and dielectrics

The energy stored in a capacitor is still given by:

$$U = \frac{1}{2}Q\,\Delta V = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

Consider a capacitor with nothing between the plates. The capacitor is charged by connecting it to a battery. Afterward the connections to the battery are removed.

When a dielectric is added to the capacitor, what happens to the stored energy?

1. The energy increases
2. The energy decreases
3. Energy is conserved! The energy stays the same.

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Energy and dielectrics

With the battery connections removed, the charge on the capacitor is stranded on the capacitor plates so remains constant. Adding the dielectric increases the capacitance. From the equation, $U = \frac{Q^2}{2C}$

the energy is decreased. Where does the energy go?

By the same token, if you then pull the dielectric out of the capacitor, the energy in the capacitor energy goes back up again. Where can it come from?

Ans. The side of the dielectric closest to the positive capacitor plate is negatively charged and the side closest to the negative plate is positively charged. So, the dielectric is attracted to the capacitor. The capacitor does work pulling the dielectric in, and you do work pulling it out.

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A question from an old test

A parallel-plate capacitor, with air between the plates (dielectric constant = 1) is charged by connecting it to a battery that has a voltage of V_0. Then, a series of steps is carried out, as described below. For each step, fill in the table with the potential difference across the capacitor, in terms of V_0; the capacitance, in terms of the initial capacitance, C_0; the charge stored in the capacitor, in terms of the initial charge Q_0; the magnitude of the uniform electric field in the capacitor, in terms of the initial value E_0; and the stored electric energy, in terms of the initial energy U_0.

Step 1: With the wires still connecting the battery to the capacitor, the distance between the plates is halved.
Step 2: After completing step 1, with the wires still connecting the battery to the capacitor, a dielectric, with a dielectric constant of 3 is placed in the capacitor, completely filling the space between the plates.

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A question from an old test

Step 1: With the wires still connecting the battery to the capacitor, the distance between the plates is halved.
Step 2: After completing step 1, with the wires still connecting the battery to the capacitor, a dielectric, with a dielectric constant of 3 is placed in the capacitor, completely filling the space between the plates.

	Potential difference	Capacitance	Charge	Field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
1. Wires still connected, distance between plates halved	V_0	$2C_0$	$2Q_0$	$2E_0$	$2U_0$
2. Wires still connected, dielectric inserted	V_0	$6C_0$	$6Q_0$	$2E_0$	$6U_0$

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A question from an old test

1. Potential difference

As long as the capacitor is connected to the battery, the potential difference remains the same as the electromotive force (EMF) of the battery. So, $V = V_0$ in both steps.

2. Capacitance

Use $C = \kappa\varepsilon_0 A/d$.
In step 1, d is halved. So, $C = 2C_0$.
In step 2, a dielectric constant with $\kappa = 3$ is inserted, so the capacitance is thrice the value before or $C = 3 \times (2C_0) = 6C_0$.

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A question from an old test

3. Charge

In step 1, the capacitor is still connected to the battery, so $V = V_0$. Use $Q = CV$ and $C = 2C_0$, we have $Q = 2C_0V_0$. This is twice the value of the initial charge, $Q_0 = C_0V_0$.
In step 2, V is still V_0, but C is increased to $6C_0$, so $Q = 6C_0V_0 = 6Q_0$.

4. Field

Use $E = V/d$
In step 1, $V = V_0$ and d is halved. So, $E = 2V_0/d_0 = 2E_0$.
In step 2, a dielectric constant is inserted. But this doesn't change V since the capacitor is wired to the battery. With d still halved from step 1, E is still equal to $2E_0$.

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A question from an old test

5. Energy

You can use either equation, $U = CV^2/2$ or $U = Q^2/(2C)$. Just be sure that you use the corresponding value of the parameters in the same row in the table.

In step 1, $U = (2C_0)V_0^2/2 = 2(C_0V_0^2/2) = 2U_0$.
In step 2, $U = (6C_0)V_0^2/2 = 6V_0^2/2 = 6U_0$.

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Minimizing the charge

You have a parallel plate capacitor whose gap distance can be adjusted, a battery and some dielectric that can fill the capacitor gap. Find the order of steps from below that will leave the capacitor with the minimum possible charge in the end.

Step A: The battery to the capacitor, the distance between the plates is halved.
Step B: a dielectric, with a dielectric constant of 3 is placed in the capacitor, completely filling the space between the plates.
Step C: Disconnect the battery from the capacitor.
Step D: Connect the battery to the capacitor.

Ans. Considering that $Q = CV$, the capacitor is minimally charged when it has the minimum capacitance when connected to the battery. Recognize that the capacitor has the minimum capacitance before the its plate separation is halved or the dielectric is inserted. So we should carry out step D first, then step C. After these, no matter what we do to the capacitor, its charge won't change. So the order of steps is: D, C, A, B or D, C, B, A.

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