Electric Potential Energy (Cont'd)

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Example: where is the potential zero?

Region I | Region II | Region III

4 cm

+3Q   -Q
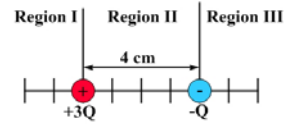
Two charges, $+3Q$ and $-Q$, are separated by 4 cm. Is there a point along the line passing through them (and a finite distance from the charges) where the net electric potential is zero? If so, where?

First, think qualitatively.
Is there such a point in Region I?
Region II?
Region III?

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Example: where is the potential zero?

Unlike electric field, where we had to worry about two vectors being equal and opposite, we just have to worry about two numbers having the same magnitude but opposite sign.
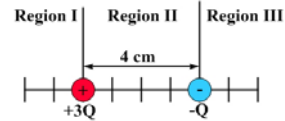
One charge has three times the magnitude of the other. Thus, we're looking for points that are three times farther from the $+3Q$ charge than the $-Q$ charge. (Recall that $V = kq / r$)

In which region(s) can we find such points?

Ans. Region II and Region III.

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Example: where is the potential zero?

Region I | Region II | Region III

4 cm

+3Q   -Q

The two charges are separated by 4 cm.
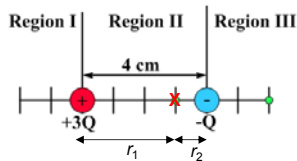At what location between the charges is the net electric potential equal to zero?

Ans. 1 cm from the –Q charge, and 3 cm from the +3Q charge.

At what location to the right of the –Q charge is the net electric potential equal to zero?

Ans. 2 cm from the –Q charge, and 6 cm from the +3Q charge.

Let's also set up and solve the equations …

Region I | Region II | Region III

4 cm

+3Q   -Q

r_1 r_2

Case I: Location lying in Region II

For this case, we have the following relations:
$r_1 + r_2 = 4$ cm   (1)
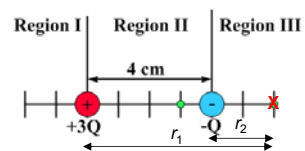$r_1 = 3r_2$ (2)
where r_1 is the distance of the location from the +3Q charge and r_2 is the distance of the location from the –Q charge.

Substitute (2) into (1), one finds: $3r_2 + r_2 = 4$ cm $\Rightarrow r_2 = 1$ cm

Hence the location with zero potential is 1 cm to the left of the –Q charge.

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Let's also set up and solve the equations …

Region I | Region II | Region III

4 cm

+3Q   -Q r_2

r_1

Case II: Location lying in Region III

For this case, we have the following relations:
$r_1 - r_2 = 4$ cm   (1)
$r_1 = 3r_2$ (2)
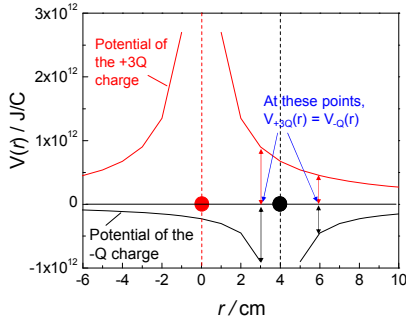
Substitute (2) into (1), one finds: $3r_2 - r_2 = 4$ cm $\Rightarrow r_2 = 2$ cm

Hence, the location with zero potential is 2 cm to the right of the –Q charge.

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Let's look at V(r) …

Plotting the potentials due to the +3Q and –Q charges may also help one visualize where V(r) = 0.

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Capacitors

A capacitor is a device for storing charge. The simplest type of capacitor is made up of two parallel conducting plates separated by either empty space or by an insulating material known as the dielectric. For a capacitor storing charge Q, one conductor has a charge of $+Q$ and the other has a charge of $-Q$.

The amount of charge a capacitor can store per applied potential difference is given by its capacitance, C. It is determined by the capacitor geometry and the dielectric material used.
The unit of capacitance is farad (F).

For a capacitor with a charge of $+Q$ on one plate, $-Q$ on the other and voltage ΔV across the two plates:

$$Q = C\,\Delta V$$

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A parallel-plate capacitor

A parallel-plate capacitor is a pair of identical conducting plates, each of area A, placed parallel to one another and separated by a distance d. With nothing between the plates, the capacitance is:

$$C = \frac{\varepsilon_0 A}{d}$$

ε_0 is known as the permittivity of free space.

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85\times 10^{-12}\ \mathrm{C^2/(N\ m^2)}$$

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Playing with a capacitor I – While the capacitor is connected to the power supply, vary the gap between the plates

Take a parallel-plate capacitor and connect it to a power supply. The power supply sets the potential difference between the plates of the capacitor.

While the capacitor is still connected to the power supply, the distance between the plates is increased. When this occurs, what happens to C, Q, and ΔV?

1. C decreases, Q decreases, and ΔV stays the same
2. C decreases, Q increases, and ΔV increases
3. C decreases, Q stays the same, and ΔV increases
4. All three decrease
5. None of the above

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Playing with a capacitor I – While the capacitor is connected to the power supply, vary the gap between the plates

Does anything stay the same?

While the capacitor is still connected to the power supply, the potential difference ΔV cannot change.

Moving the plates further apart decreases the capacitance, C because: $C = \frac{\varepsilon_0 A}{d}$

To see what happens to the charge, use $Q = C\Delta V$, which shows that decreasing C decreases the charge, Q stored on the capacitor.

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Playing with a capacitor, II – with the power supply disconnected, vary the gap

Take a parallel-plate capacitor and connect it to a power supply. Then disconnect the capacitor from the power supply. After this, the distance between the plates is increased. When this occurs, what happens to C, Q, and ΔV?

1. C decreases, Q decreases, and ΔV stays the same
2. C decreases, Q increases, and ΔV increases
3. C decreases, Q stays the same, and ΔV increases
4. All three decrease
5. None of the above

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Playing with a capacitor, II – with the power supply disconnected, vary the gap

Does anything stay the same?

When the power supply is disconnected, the charge is stranded on the capacitor plates. So, the charge cannot change.

Moving the plates further apart decreases the capacitance, because: $C = \dfrac{\varepsilon_0 A}{d}$

To see what happens to the potential difference, look at $Q = C\Delta V$, which shows that decreasing C while keeping the charge the same, the potential difference increases.

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Change in capacitance – effect of plate area

Our basic capacitor equations are: $Q = C\Delta V$

and, for a parallel-plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$

Increase the area of each plate. The capacitance ...

1. Increases
2. Decreases
3. Stays the same

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Change in capacitance – effect of plate area

Capacitance is proportional to area, so increasing area increases capacitance.

$$C = \frac{\varepsilon_0 A}{d}$$

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Change in capacitance – effect of varying the charge

Double the charge on each plate. The capacitance ...

1. Increases
2. Decreases
3. Stays the same

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Change in capacitance – effect of varying the charge

Recall: $C = \dfrac{\varepsilon_0 A}{d}$

Increasing Q does not change the capacitance at all. The capacitance is a constant because it is determined by how the capacitor looks like. With this, $Q = C\Delta V$ tells us that <u>the potential difference across the capacitor doubles when the charge on each plate doubles.</u>

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Energy in a capacitor

When we move a single charge q through a potential difference ΔV, its potential energy changes by $q\,\Delta V$.

Charging a capacitor involves moving a large number of charges from one capacitor plate to another. If ΔV is the final potential difference on the capacitor, and Q is the magnitude of the final charge on each plate, the energy stored in the capacitor is:

$$U = \frac{1}{2}Q\Delta V$$

The factor of 1/2 is because, on average, the charges were moved through a potential difference of ½ ΔV. (Note that the potential difference changed from zero to ΔV in the process. So the average value is ½ ΔV)

Using Q = CΔV, the energy stored in a capacitor can be written as:

$$U = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

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Example: Energy stored in a capacitor

How much energy is stored in a 8 μF capacitor that has a potential difference of 4000 V?

Solution:

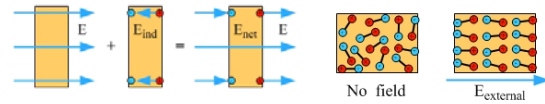$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(8 \times 10^{-6} \text{ F})(4000 \text{ V})^2$$

The factor of 10^{-6} in the capacitance cancels the factor of 1000^2, so we get:

$$U = \frac{1}{2}(8 \text{ F})(4 \text{ V})^2 = 64 \text{ J}$$

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Dielectrics

When a material (generally an insulator) is inserted into a capacitor, we call the material a dielectric. Adding a dielectric allows the capacitor to store more charge for a given potential difference.

When a dielectric is inserted into a charged capacitor, the dielectric is polarized by the field. The electric field from the dielectric will partially cancel the electric field from the charge on the capacitor plates. If the capacitor is connected to a battery at the time, the battery is able to store more charge in the capacitor, bringing the field back to its original value.

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Dielectrics (cont'd)

Thus, when a dielectric is inserted in a charged capacitor (not connected to a power supply), the electric field would be decreased and so would the voltage (= Ed). Since $C = Q/V$, this means that C must be bigger when a dielectric is inserted.

For a parallel-plate capacitor containing a dielectric, the capacitance is:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

where κ is the dielectric constant.

In general, adding a dielectric to a capacitor increases the capacitance by a factor of κ.

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The dielectric constant

Every material has a dielectric constant κ, which as discussed above tells you how effective the dielectric is at increasing the amount of charge a capacitor can store for given voltage difference applied across it. Alternatively, we can think of the dielectric constant as telling you how effectively the dielectric is in reducing the electric field of a charged capacitor not connected to a power supply.

$$\kappa = \frac{E_0}{E_{net}} > 1$$

E_0 is the field without the dielectric.
E_{net} is the field with the dielectric.

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The dielectric constant of a conductor

$$\kappa = \frac{E_0}{E_{net}}$$

What is the dielectric constant of a conductor?

1. Zero
2. Infinity
3. This question makes no sense – a dielectric is an insulator, so a conductor does not have a dielectric constant.

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