Electric Potential Energy

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Electric Potential

Electric potential (V) at a point is defined as the work done (U) required to bring a charge (q) from infinity to that point divided by the charge:

$$\boxed{V = U/q.}$$

With this definition, $V = 0$ at infinity.

Important: electric potential is a scalar.

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Electric Potential from point charges

If the electric potential arises from a single point charge, Q,

$$\boxed{V(r) = \frac{kQ}{L}}$$

where L is the distance from Q to the point of interest.

For electric potential arising from more than one charge, say, Q_1, Q_2, …, one can use superposition.

$$\boxed{V(r) = \frac{kQ_1}{L_1} + \frac{kQ_2}{L_2} + \cdots}$$

Scalar sums

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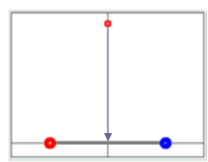
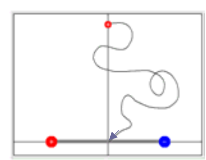
A charge and a dipole

A dipole is placed on the x-axis with its center on the origin. A positive point charge is brought from very far away on the y-axis to the origin. In Case 1 (below left), it is moved straight down the y-axis to the origin. In Case 2 (below right), it follows a complicated path, but with the same start and end points.





Which case takes more net work?

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[Simulation]

A charge and a dipole

Which case takes more net work?

1. Case 1
2. Case 2
3. The net work done is the same in both cases

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A charge and a dipole

Like gravity, the electrostatic force is conservative. When the only forces acting are conservative it doesn't matter how an object gets from A to B, the work done is always the same.

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How much work?

How much work is required to bring the charge from very far away to the center of the dipole?

1. Zero
2. The work done is positive
3. The work done is negative

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How much work?

Let's first find the potential, V, at the center of the dipole due to the two charges. Because electric potential is a scalar, we can add the contributions from the two charges (+Q and -Q) algebraically. Let the distance from either charge to the center of the dipole be r:

$V = k(+Q)/r + k(-Q)/r = 0.$

Note that unlike electric fields, we do put the sign of the source charge in the formula to find the electric potential the charge produces.

The potential energy there is: $U = qV = 0$

So the net work done we would have to do to bring the charge from very far away against the field, which is U, is 0.

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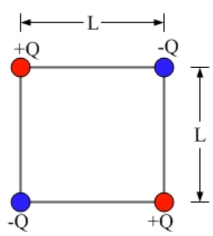
Four charges in a square

Four charges of equal magnitude are placed at the corners of a square that measures L on each side. There are two positive charges $+Q$ diagonally across from one another, and two negative charges $-Q$ at the other two corners. How much potential energy is associated with this configuration of charges?

1. Zero
2. Some positive value
3. Some negative value



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Four charges in a square

The potential energy of a given charge (or set of charges) is the same as the work done spent in bringing it (them) from infinity to its (their) current position(s). It is because electric forces are conservative, all the work done is converted to potential energy in the process.

$$U_f + \cancel{K_f} = W \Rightarrow U_f = W$$

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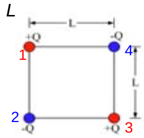
Four charges in a square

Label the four charges as 1, 2, 3, 4 as below. Suppose you start with no charge in the square and (1) you bring charge 1 in. This should cause no energy since there is no charge and so the electric potential is zero. (2) Then you bring charge 2 in. The work required is $(-Q)V_{12}$. where V_{12} is the potential produced by charge 1 at where charge 2 is placed. (3) Next you bring charge 3 in. The work required is $(+Q)[V_{13} + V_{23}]$. (4) Finally, you bring charge 4 in. The work required is $(-Q)[V_{14} + V_{24} + V_{34}]$.

Among these, U_{12}, U_{23}, U_{34}, and U_{14} look like: $-\frac{kQ^2}{L}$

And U_{13} and U_{24} look like: $+\frac{kQ^2}{\sqrt{2}L}$



When we add them up, we get an overall negative value.

Convince yourself that the answer is independent of the order by which we bring the charges in.

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Electric potential Energy

We have focused on electric potential, V, which is related to potential energy, U, in a way that is similar to how electric field is related to force.

$$\vec{E} = \frac{\vec{F}}{q} \qquad V = \frac{U}{q}$$

Electric potential has units of J/C or Volts. Like field, electric potential is a way to visualize how a charged object, or a set of charged objects, affects the region around it electrically.

A positive test charge placed in the region always tend to move towards another point with a lower potential. Conversely, a negative test charge tends to move towards another point with a higher potential. The change in potential energy in either case (= $Q\Delta V$) is negative, which is what drives the movement.

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Visualizing electric potential

We often draw equipotentials (lines of constant electric potential) on a picture involving charges and/or fields. An equipotential is analogous to contour lines on a map, such as this map of the summit of Mt. Rainier. The contour lines in this map are lines of constant altitude. With equipotentials, we can draw electric field lines easily. Electric field lines are perpendicular to the equipotentials and points towards equipotentials with lower potential. (Why?)

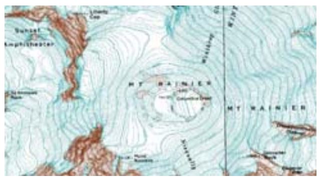

Photo credit: NASA/USGS

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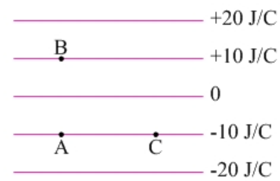
Equipotentials in a uniform field

Here is a picture of equipotentials in a uniform electric field. A uniform electric field means that the density of the electric field lines is the same and they point in the same direction everywhere. Can you tell why these equipotentials correspond to a uniform electric field?

With the equipotentials shown, which direction is the electric field?

Down – field points in the direction of decreasing potential.



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Finding the electric fields from the equipotentials

The direction of the electric field, ***E***, at a point can be readily marked by drawing an arrow that is perpendicular to the equipotential at that point and points towards another equipotential with a lower potential.

To find the magnitude of ***E***, recall that ***E*** is the force per unit charge (***F***/*q*) experienced by a positive test charge (*q*) placed at that point. Consider a positive charge *q* going from one point to a neighboring point with a lower potential. The change in potential energy of the charge, $\Delta U = (U_f - U_i) = q\Delta V = q(V_f - V_i)$. The work performed by the field in moving the charge is $W = Fd = qEd$, where *d* is the displacement of the charge parallel to *F*. (This work is converted into the kinetic energy gain of the charge in the end.) Set $U_i = U_f + W$, we have $-\Delta U = qEd$

$$\boxed{\Rightarrow \quad E = -\Delta V/d}$$

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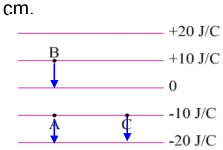
Equipotentials in a uniform field

What is the magnitude of the electric field in the uniform field region shown? Assume that the distance between neighboring equipotential lines is 1 cm.



Solution: Because the field is uniform, we can just find the electric field at any one point and that should represent the field in the whole region.
Consider the electric field at point A. Imagine a positive test charge moving from point A, under the influence of the electric field at A, to the nearby, lower -20 V equipotential line. Apply the equation, $E = -\Delta V/d$ to find E. We get
$E = -[-20 \text{ V} - (-10 \text{ V})] / (0.01 \text{ m}) = 1000 \text{ V/m}$

Verify that the fields at points B and C are also 1000 V/m.

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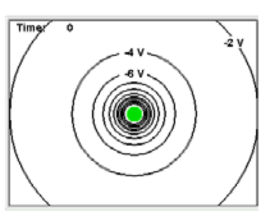
Which way is the field?

The figure shows the equipotentials for a non-uniform field, specifically the field from a point charge. In which direction is the field?

1. Clockwise
2. Counter-clockwise
3. Toward the center
4. Away from the center
5. There is not enough information to say



Simulation

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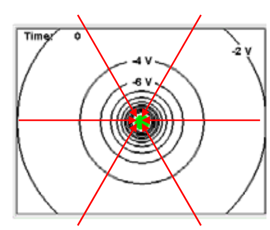
Which way is the field? (cont'd)

Field points perpendicular to the equipotentials and in the direction of decreasing potential. In this case, that is radially toward the center.



You may recognize that such pattern of field lines is characteristic of those produced by a negative point charge.

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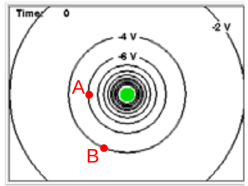
Which has a high field?



Which point, A or B has a high electric field?

Ans. Point A has a higher electric field.

This is because $|E| = |\Delta V|/d$. The two equipotential lines differing by ± 2 V from the equipotential line where point A sits are closer together than those at point B. That is, d is on average bigger at point B than at point A. So point A should have a higher electric field.

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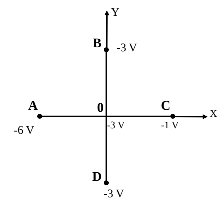
A question from an old test

The drawing shows the equipotential lines at five points on the X and Y axes. Each of the four outer points is 1 mm from the point at the origin. The electric field is **uniform** in the X > 0 and X < 0 regions separately.

(a) On the diagram, draw equipotential lines for V = -6V, -5V, -4V, -2V and -1V. Note that the Y-axis is the V = -3 equipotential line.



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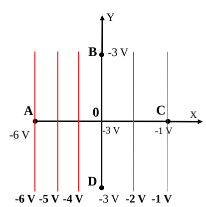
A question from an old test

Solution:

Because the field is given to be uniform in the X < 0 region, the equipotential lines in that region should all be parallel, straight lines. Since the V = -3 V line (i.e., the Y-axis) is vertical, all the other lines should also be vertical. In particular, the V = -6 V line should be vertical, passing through point A.

Also, because the field is uniform, the V = -5 and -4 V equipotential lines should be equally spaced between the V = -3 V and -6 V lines.

Similarly, we may figure that the V = -1 V line is the vertical line passing through point C. The V = -2 V line is the vertical line mid-way between the V = -3 V line (Y-axis) and the V = -1V line.



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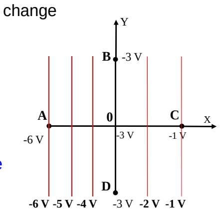
A question from an old test

Choose from the following for all the parts (b) to (f) below:
[ ] decreases [ ] increases [ ] does not change
[ ] cannot be decided

(b) If a positive charge moves straight from point A to the origin, its electric potential energy …

increases -- because the potential at the origin (= -3V) is higher than that at point A (= -6V), the electric potential energy of a positive charge increases in going from point A to the origin.



(c) If a positive charge moves from point A to the origin through point C, its electric potential energy …

also increases – the change in potential energy is independent of the path taken because electrostatic force is conservative. The answer should be the same as that in (b).

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A question from an old test

(d) If a negative charge moves from point A to the origin, its electric potential energy …

decreases – because for negative charges, the sign change in potential energy is opposite to that for positive charges. So the answer should be opposite to that of (b).

(e) If a negative charge moves from point A to the origin through point C, its electric potential energy …

also decreases – again, the change in potential energy is independent of the path taken. So the answer should be the same as that in (d).

(f) If a negative charge moves from point A to point D and back to point A, its electric potential energy …

makes no change -- since the charge returns to the starting point, there should not be any change in its electric potential energy.

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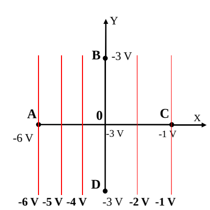
A question from an old test

(g) Find the magnitude and direction of the field in the x > 0 region.

$|E| = |\Delta V|/d = |-1V-(-3V)| / 1 \text{ mm} = 2 \text{ V} / 0.001 \text{ m} = 2000$ V/m.

The direction is perpendicular to the equipotential line and directed from high to low equipotentials. So E is pointing left.



(h) Find the magnitude and direction of the field in the x < 0 region.

$|E| = |\Delta V|/d = |-3V-(-6V)| / 1\text{mm} = 3000$ V/m.

The direction of E is still pointing left by the same reasoning used for (g).

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