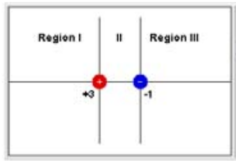
## Where is the field zero?

Two charges, $+3Q$ and $-Q$, are separated by 4 cm. Is there a point along the line passing through them (and at a finite distance from the charges) where the net electric field is zero? If so, where?

(Hint: First, think qualitatively.
Is there such a point to the left of the $+3Q$ charge?
Between the two charges?
To the right of the $-Q$ charge?)



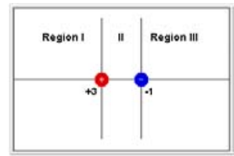

1

## Where is the net field equal to zero?

Is the net electric field equal to zero at some point in one of these three regions: to the left of both charges (Region I), in between both charges (Region II), and/or to the right of both charges (Region III)?

The field is zero at a point in:

1. Region I
2. Region II
3. Region III
4. two of the above
5. all of the above




2

## Where is the field zero? (cont'd)

In region I, the two fields point in opposite directions.
In region II, both fields are directed to the right, so they cannot cancel.
In region III, the two fields point in opposite directions.

Now think about the magnitude of the fields.
In region I, every point is closer to the +3Q charge than the -Q charge, so $E_{+3Q}$ is always > $E_{-Q}$ and their (vector) sum will not be zero.

In region III, we can strike a balance between the factor of 3 in the charges and the distances.

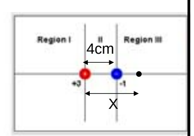
3

## Where is the field zero? (cont'd)

Let's calculate where the field is zero. Let that point be at a distance $x$ from the $+3Q$ charge. Then it is $(x - 4)$ cm away from the $-Q$ charge. Define right as positive, so:

$$\vec{E}_{net} = 0 \quad \Rightarrow \quad +\frac{3kQ}{x^2} - \frac{kQ}{(x-4\text{ cm})^2} = 0$$

$$\Rightarrow \quad \frac{3}{x^2} = \frac{1}{(x-4\text{ cm})^2}$$




Cross multiplying and expanding the brackets:

$$3\left[x^2 - (8\text{ cm})x + 16\text{ cm}^2\right] = x^2$$

4

## Where is the field zero? (cont'd)

$$3\left[x^2 - (8\text{ cm})x + 16\text{ cm}^2\right] = x^2$$

$$3x^2 - (24\text{ cm})x + 48\text{ cm}^2 = x^2$$

$$2x^2 - (24\text{ cm})x + 48\text{ cm}^2 = 0$$

$$x^2 - (12\text{ cm})x + 24\text{ cm}^2 = 0$$

Solve this using the quadratic formula to get two solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+12 \pm \sqrt{(-12\text{ cm})^2 - 4(1)(24\text{ cm}^2)}}{2}$$

The two solutions are x = 2.54 cm and x = 9.46 cm. Which one is correct?

5

## Two solutions

Which of the two solutions is the one we want?

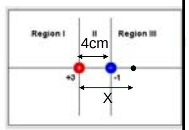
1. 2.54 cm
2. 9.46 cm
3. They are both valid solutions.

***Note:*** *even if you decide one solution is not valid, you should be able to explain what its physical significance is.*

6

## Where is the field zero? (cont'd)

The net electric field is zero at 9.46 cm to the right of the +3*Q* charge (and at 5.46 cm to the right of the –*Q* charge).




The other solution, x = 2.54 cm is between the two charges (region II), where the two fields point in the same direction. It arises from a variation of the force balance equation:

$$\vec{E}_{net}=0 \quad \Rightarrow \quad +\frac{3kQ}{x^2}-\frac{kQ}{(x-4\text{ cm})^2}=0$$

$$\Rightarrow \quad +\frac{3kQ}{x^2}-\frac{kQ}{(4\text{ cm}-x)^2}=0$$

This equivalent form of the force balance equation twists what we intend to assume for the force from the –Q charge.



## A test charge

A test charge has a small enough charge that it has a negligible impact on the local electric field.

Placing a positive test charge at a point can tell us the direction of the electric field at that point, and how strong the field is.

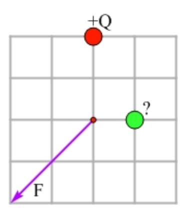
The force on a positive test charge is in the same direction as the electric field, because $\vec{F}=q\vec{E}$ .



## The net force on a test charge

The diagram shows the net force experienced by a positive test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge of +*Q* and one with an unknown charge. What is the sign and magnitude of the charge on the second ball?

1. +*Q*/4
2. +*Q*/2
3. +*Q*
4. +2*Q*
5. +4*Q*
6. none of these

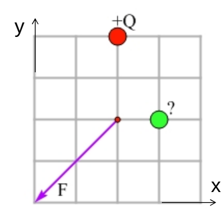





## The net force on a test charge (cont'd)

First, we note that the force from +Q must be along the y axis and that from the test charge along the x axis. That is, the two forces are perpendicular to each other.

The net force is at a 45° angle, so the two forces (or fields) must be identical. The +*Q* charge sets up a force (or field) directed down. The second ball must set up a force (or field) directed left, away from itself, so it must be _positive_.



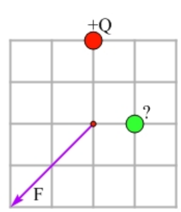



## The net force on a test charge (cont'd)

If the two forces (or fields) are the same, how does the magnitude of the charge on the second ball compare to *Q*?

It must be smaller than *Q*, because the second ball is closer to the test charge.

The first ball is twice as far away. Because distance is squared in the equation, the factor of 2 becomes a factor of 4. To offset this factor of 4, the second ball should have a charge of _+Q/4_.





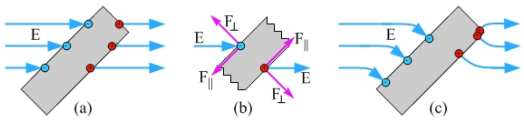

## Electric field near conductors, at equilibrium

A conductor is at electrostatic equilibrium when there is no net flow of charge. Equilibrium is reached in a very short time after being exposed to an external field. At equilibrium, the charge and electric field follow these guidelines:

- the electric field is zero in the material body of the conductor
- the electric field at the surface of the conductor is perpendicular to the surface
- any excess charge lies only at the surface of the conductor
- charge accumulates, and the field is strongest, on pointy parts of the conductor

## Electric field near conductors, at equilibrium (cont'd)

At equilibrium the field is zero inside a conductor and perpendicular to the surface of the conductor because the electrons in the conductor move around until this happens.




Excess charge, i.e., the net charge a conductor may possess, can only be found at the surface. If any was in the bulk, there would be a net field inside the conductor, making electrons move.

13

## A lightning rod

A van de Graaff generator acts like a thundercloud. We will place a large metal sphere near the van de Graaff and see how strong the sparks (lightning) we get. We will then replace the large metal sphere by a pointy piece of metal. In which case do we get more impressive sparks (lightning bolts)?

1. with the large sphere
2. with the pointy object
3. neither, the sparks are the same in the two cases

14

## A lightning rod

The big sparks we get with the sphere are dangerous, and in real life could set your house on fire.

With the pointy piece of metal, the charge (and field) builds up so quickly that it drains the charge out of the van de Graff slowly and continuously, avoiding the building up of charges in the van de Graff and so dangerous sparks. This is how lightning rod works.

The lightning rod was invented in the US by Benjamin Franklin .

15