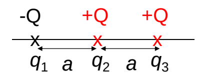
Force and Fields

A 1-dimensional example

Three charges are equally spaced along a line. The distance between neighboring charges is *a*. From left to right, the charges are:

$q_1 = -Q$
$q_2 = +Q$
$q_3 = +Q$



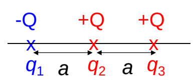
What is the magnitude of the force experienced by q_2, the charge in the center?

Simulation

A 1-dimensional example

Let's define positive to be right. The net force on q_2 is the vector sum of the forces from q_1 and q_3.

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32} = -\frac{kQ^2}{a^2} - \frac{kQ^2}{a^2} = -\frac{2kQ^2}{a^2}$$



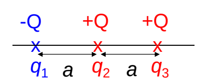
The force has a magnitude of $\frac{2kQ^2}{a^2}$ and points to the left.

Handling the signs correctly is critical. The negative signs come from the direction of each of the forces (both to the left), not from the signs of the charges.

Ranking based on net force

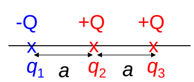
Rank the charges according to the magnitude of the net force they experience, from largest to smallest.

1. 1 = 2 > 3
2. 1 > 2 > 3
3. 2 > 1 = 3
4. 2 > 1 > 3
5. None of the above.



Ranking based on net force

Both charges 1 and 3 have 1 close neighbor and 1 far neighbor. Will they experience forces of the same magnitude?



No, because both forces acting on charge 1 are in the same direction, while the two forces acting on charge 3 are in opposite directions. Thus, 1 > 3.

Both charges 1 and 2 have two forces acting in the same direction. Will they experience forces of the same magnitude?

No, because one force acting on charge 1 is the same magnitude as one acing on charge 2, while the second force acting on charge 1 is smaller – it comes from a charge farther away. Thus, 2 > 1.

Ranking based on net force (cont'd)

We can calculate the net force, too.

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} = +\frac{kQ^2}{a^2} + \frac{kQ^2}{(2a)^2} = +\frac{5kQ^2}{4a^2}$$

$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32} = -\frac{kQ^2}{a^2} - \frac{kQ^2}{a^2} = -\frac{2kQ^2}{a^2} = -\frac{8kQ^2}{4a^2}$$

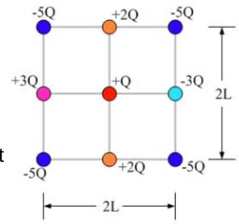
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = -\frac{kQ^2}{(2a)^2} + \frac{kQ^2}{a^2} = +\frac{3kQ^2}{4a^2}$$

If we add these forces up, what do we get? Is that a fluke?

Ans. $(+5-8+3)*kQ^2/(4a^2) = 0$. This is a result of Newton's 3^{rd} Law. Why?

Nine charges on a grid

Nine different charged balls, which we treat as point charges, are arranged in a highly symmetric pattern around a square.

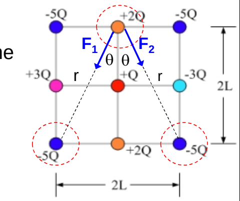


(a) What is the magnitude of the net force experienced by the +2Q ball at the top due to the two -5Q balls at the two lower corners of the square? Express your answer in terms of k, Q, and L.

Nine charges on a grid

Solution

First, notice that the distance r between the +2Q ball with either of the -5Q charges is the same equal to $(L^2+(2L)^2)^{1/2} = 5^{1/2}L$.

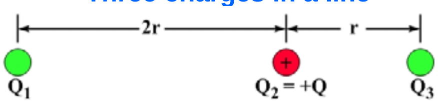


Secondly, the x-components of the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ from the two -5Q charges cancel.

So to find the net force, we only need to add the y-components of $\mathbf{F_1}$ and $\mathbf{F_2}$. Let up be positive:

$F_{net} = [-k(2Q)(5Q)/(5L^2) \times \cos\theta] \times 2$
$= -8kQ^2/(5^{1/2}L^2)$ (n.b. $\cos\theta = 2/5^{1/2}$)
$= -3.58kQ^2/L^2$

Three charges in a line



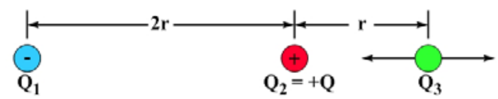
Ball 1 has an unknown charge and sign.
Ball 2 is positive, with a charge of +*Q*.
Ball 3 has an unknown non-zero charge and sign.

Ball 3 is at equilibrium - it feels no net electrostatic force due to the other two balls.

What is the sign of the charge on ball 1?

1. Positive
2. Negative
3. We can't tell unless we know the sign of the charge on ball 3.

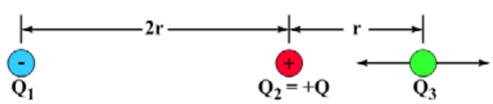
Three charges in a line (cont'd)



Ball 3 is at equilibrium means that it experiences equal-and-opposite forces from the other two balls. So ball 1 must have a negative charge. Flipping the sign of the charge on ball 3 reverses the sign of both these forces, so they still cancel.

So, we don't need to know the sign of the charge on ball 3 to tell the sign of the charge on ball 1. How about the magnitude of the charge on ball 1?

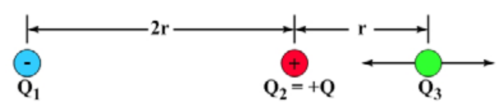
Three charges in a line (cont'd)



What is the magnitude of the charge on ball 1? Can we tell even if we don't know what Q_3 is?

Yes, we can! For the two forces to be equal-and-opposite, with ball 1 three times as far from ball 3 as ball 2 is, and the distance being squared in the force equation, the charge on ball 1 must have a magnitude of 9*Q*.

Three charges in a line (Cont'd)



Let's do the math. Since it doesn't matter what the sign of Q3 is, we'll assume it to be positive and define to the right as positive.

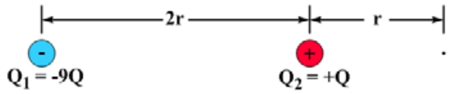
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = 0$$

$$-\frac{k|Q_1|\cancel{Q_3}}{(3r)^2} + \frac{kQ\cancel{Q_3}}{r^2} = 0$$

$$\frac{|Q_1|}{9} = Q$$

$$|Q_1| = 9Q$$

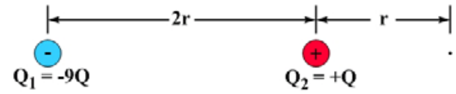
Two charges in a line (cont'd)



The neat thing here is that we don't need to know anything about ball 3. We can put whatever charge we like at the location of ball 3 and it will feel no net force because of balls 1 and 2.

Ball 3 isn't special - it's the location that's special. So, let's get rid of ball 3 from the picture and think about how the two charged balls influence the point where ball 3 was.

Two charges in a line (cont'd)



Ball 2's effect on ball 3 is given by Coulomb's Law:

$$|\vec{F}_{23}| = \frac{kQQ_3}{r^2}$$

Ball 2's effect on the point where ball 3 was is given by the Electric Field, E_2, as follows: $|\vec{E}_2| = \frac{kQ}{r^2}$

The electric field from ball 1 and the electric field from ball 2 cancel out at the location where ball 3 was. We will discuss the direction of ***E*** in relation to the electric force ***F*** shortly.

Electric field

A field is something that has a magnitude and a direction at every point in space. A field at a point determines the force that will be experienced by a massive object placed at that point. For gravity, the gravitational field is *g* and the gravitational force on an object with mass *m* is the product of *m* and *g*: $\vec{F}_g = m\vec{g}$. Note that *g* has a dual role. It is also the acceleration due to gravity:

$$\vec{F}_g = m\vec{g} = m\vec{a} \quad \Rightarrow \quad \vec{a} = \vec{g}$$

Similarly, for a charged object acted on by an electric field ***E***, the electric force $\boldsymbol{F}_E$ is the product of the charge on the object, *q* and ***E***:

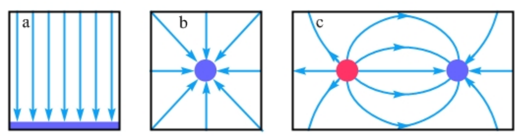
$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_E = q\vec{E} = m\vec{a} \quad \Rightarrow \quad \vec{a} = \frac{q\vec{E}}{m}$$

Simulation

Electric field lines

Field line diagrams show the direction of the field, as well as a qualitative view of the magnitude of the field at a point – the magnitude of the field is reflected from the density of the field lines there. The field is stronger where the lines are closer together.



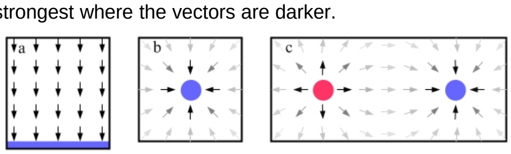
a – a uniform electric field directed down

b – the field near a negative point charge

c – field lines start on positive charges and end on negative charges. This is an electric dipole – two charges of opposite sign and equal magnitude separated by some distance.

Electric field vectors

Field vectors give an alternate representation, and reinforce the idea that there is an electric field everywhere. The field is strongest where the vectors are darker.



a – a uniform electric field directed down

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c – field lines start on positive charges and end on negative charges. This is an electric dipole – two charges of opposite sign and equal magnitude separated by some distance.

Force on a test charge in an electric field

Recall that the electric force, ***F*** (a vector) acting on a charge, *q*, is *q**E*** (note that ***E*** is also a vector). Mathematically,

$$\boldsymbol{F} = q\boldsymbol{E}$$

This equation shows that if *q* is positive, ***F*** and ***E*** point in the same direction. Therefore, the direction of the field line at a point indicates the direction of force a positive test charge will experience.

Getting quantitative about field

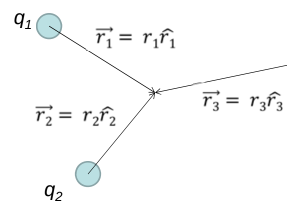
The field line and field vector diagrams are nice, but when we want to know the magnitude of the electric field at a particular point those diagrams are not very useful.

If the electric field comes from individual charges that are highly localized (which we call "point charges"), we can use superposition to quantify the field. Specifically, the net electric field at a particular point is the vector sum of the individual electric fields from the point charges at that point.

For any point charge q_i at a distance r_i from the point of interest, the magnitude of the electric field is: $E_i = \frac{kq_i}{r_i^2}\hat{r}_i$

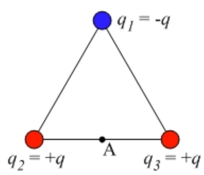
If q_i is +, the field $\boldsymbol{E}_i$ points away from q_i. If q_i is -, the field points towards q_i. The net field is given by the vector sum of all the $\boldsymbol{E}_i$'s.

Meaning of $\hat{r}_i$



A triangle of point charges

Three point charges, having charges of equal magnitude, are placed at the corners of an equilateral triangle. The charge at the top vertex is negative, while the other two are positive.



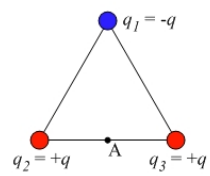
In what direction is the net electric field at point A, halfway between the positive charges?

We could ask the same question in terms of force.
In what direction is the net electric force on a positive charge located at point A?

Net electric field at point A

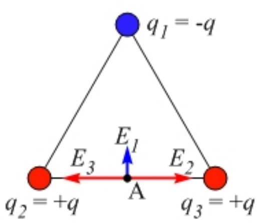
In what direction is the net electric field at point A, halfway between the positive charges?

1. up
2. down
3. left
4. right
5. other



Net electric field at point A

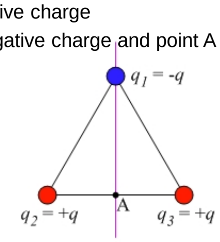
The fields from the two positive charges cancel one another at point A.
The net field at A is due only to the negative charge, which points toward the negative charge (up).



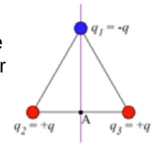
Net electric field equals zero?

Are there any locations, a finite distance from the charges, on the straight line passing through point A and the negative charge at which the net electric field due to the charges equals zero? If so, where is the field zero?

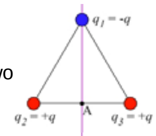
1. At some point above the negative charge
2. At some point between the negative charge and point A
3. At some point below point A
4. Both 1 and 3
5. Both 2 and 3
6. None of the above



Above the triangle, the field from the negative charge is directed down. The net field from the two positive charges is directed up. The former dominates at positions near q1, but the latter dominates at positions far away. In between there should be a compromise.



Inside the triangle, the field from the negative charge is directed up. The net field from the two positive charges is directed up.



Below the triangle, the field from the negative charge is directed up. The net field from the two positive charges is directed down. At point A, the total field doesn't have contribution from q_2 and q_3. But far below, it is dominated by q_2 and q_3. In between, there should be a compromise.