Coulomb's Law

The force between electric charges

Two charged objects, of charge *q* and *Q*, separated by a distance *r*, exert a force on one another. The magnitude of this force is given by:

Coulomb's Law: $F = \frac{kqQ}{r^2}$

where k is a constant:

$$k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \approx 9 \times 10^9 \text{ N m}^2/\text{C}^2$$

The direction of the force is toward the second object if the objects have opposite signs, and away from the second object if the signs are the same.

What does Coulomb's Law remind us of? Gravity

Comparing gravity and the interaction between charges

- In general, the force of gravity is much weaker than electrostatic interactions.
- Gravity is always attractive, while the force between charges can be attractive or repulsive.
- The equations have similar forms, giving rise to similar behavior.

$F = \frac{GmM}{r^2}$ $\qquad$ $F = \frac{kqQ}{r^2}$

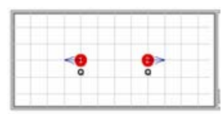
G = 6.67e-11 $\text{N}^2\text{m}^2/\text{kg}^2$ $\qquad$ k = 8.99e9 $\text{N}^2\text{m}^2/\text{C}^2$

The most abundant charged particles are protons and electrons, with proton heavier. For protons, *q* ~ 1e-19 C, *m* ~ 1e-27 kg. So F_G/Fe ~ 1e(-11-2x27)/1e(9-2x19) = 1e-36. This ratio is even smaller for electrons.

Forces between two charges

Two equal charges *Q* are placed a certain distance apart. They exert equal-and-opposite forces *F* on one another. Now one of the charges is doubled in magnitude to 2*Q*. What happens to the magnitude of the force each charge experiences?

1. Both charges experience forces of magnitude 2*F*.
2. The *Q* charge experiences a force of 2*F*; the 2*Q* charge experiences a force *F*.
3. The *Q* charge experiences a force of *F*; the 2*Q* charge experiences a force 2*F*.
4. None of the above.



Forces between two charges

Let's examine this question from two perspectives.

1. Newton's Third Law – can one object experience a larger force than the other?

   No – the objects experience equal-and-opposite forces.

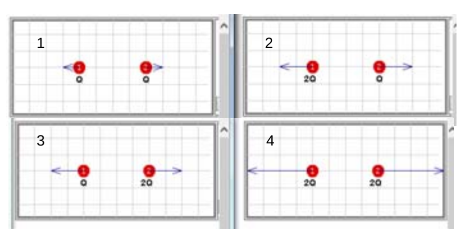
2. Coulomb's Law: $F = \frac{k(Q)(Q)}{r^2}$.

   If we double one charge, the force will be:

   $\frac{k(Q)(2Q)}{r^2} = 2F$

Forces between two charges

Figures 2 and 3 show what happens to the forces when one of the charges is doubled in magnitude to 2*Q*. Figure 4 shows what happens when both charges are doubled in magnitude.



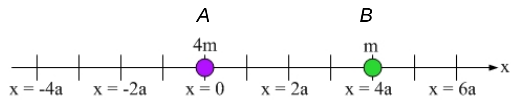
Superposition

If an object experiences multiple forces, we can use:

The principle of superposition - the net force acting on an object is the vector sum of the individual forces acting on that object.

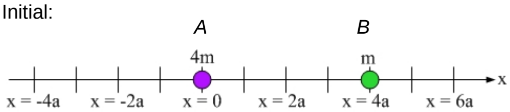
A one-dimensional situation with masses

Ball *A*, with a mass $4m$, is placed on the x-axis at $x = 0$. Ball *B*, which has a mass m, is placed on the x-axis at $x = +4a$. Where would you place ball *C*, which also has a mass m, so that ball *A* feels no net force because of the other balls? Is this even possible?

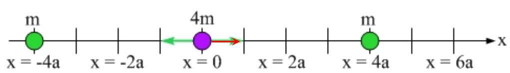


A one-dimensional situation with masses (cont'd)

Initial:

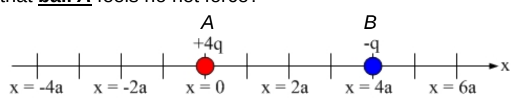


By drawing the force acting on *A* due to *B* (red arrow below), one may see that we should put ball *C* to the left of ball *A*, at the same distance ball A keeps from ball B.

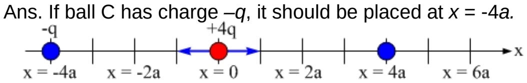


A similar one-dimensional situation, but with charges (I)

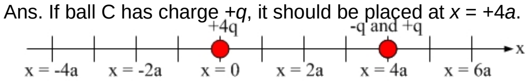
Ball *A*, with charge $+4q$, is placed at $x = 0$. Ball *B*, which has a charge $-q$, is placed at $x = +4a$. Where would you place Ball *C*, which has a charge of magnitude q, and could be + or -, so that **ball *A*** feels no net force?



Ans. If ball C has charge $-q$, it should be placed at $x = -4a$.

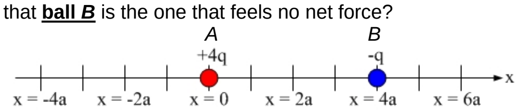


Ans. If ball C has charge $+q$, it should be placed at $x = +4a$.

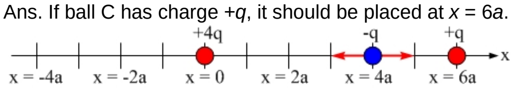


A similar one-dimensional situation, but with charges (II)

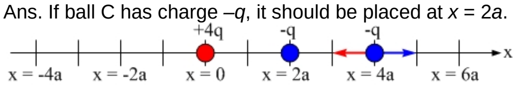
Ball *A*, with charge $+4q$, is placed at $x = 0$. Ball *B*, which has a charge $-q$, is placed at $x = +4a$. Where would you place Ball *C*, which has a charge of magnitude q, and could be + or -, so that **ball *B*** is the one that feels no net force?



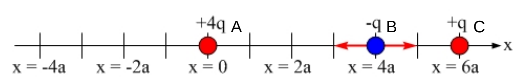
Ans. If ball C has charge $+q$, it should be placed at $x = 6a$.



Ans. If ball C has charge $-q$, it should be placed at $x = 2a$.



Case 1: Suppose ball *C* has charge $+q$ (and so must be on the right side of *B*):



Let r be the distance of ball *C* from ball *B*, and choose the direction pointing at right to be positive.

Total force acting on *B*, $F = (-k)(4q)(q)/(4a)^2 + (+k)(q)(q)/r^2$

The first term is the force acting on ball *B* due to ball *A*. We assign this term negative because the figure shows that this force is pointing to the left. By assigning the sign of the force, we have taken care of whether the charges are like or unlike so we should not include the sign of the charges in the equation anymore. Otherwise we double-count the effect!

From the above, the total force acting on *B*,

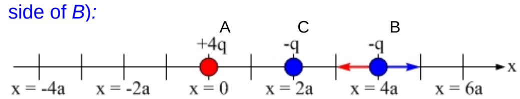
$F = (-k)(4q)(q)/(4a)^2 + (+k)(q)(-q)/r^2$

The second term – *i.e.*, the force acting on *B* due to *C* is similarly obtained. Solve the equation for *r* and obtain:

$F = 0 \Rightarrow 4/(4a)^2 = 1/r^2 \Rightarrow r^2 = 4a^2 \Rightarrow r = \pm 2a.$

Since ball *C* must be on the right side of *B*, we should choose $r = +2a$. That gives the position of ball *C* to be $x = 4a + 2a = +6a$.

Case 2: Ball *C* has charge $-q$ (and so must be on the left side of *B*):



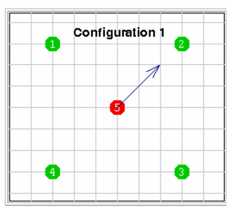
Total force acting on *B*, $F = (-k)(4q)(q)/(4a)^2 + (+k)(q)(q)/r^2$

By setting $F = 0$, we obtain the same equation as the one we wrote before for case 1, so it should have the same solution, i.e., $r = \pm 2a$. Since ball *C* in this case must be on the left side of *B*, we should choose $r = -2a$. That gives the position of ball *C* to be $x = 4a + (-2a) = +2a$.

A two-dimensional situation 1

Simulation

Case 1: There is an object with a charge of $+Q$ at the center of a square. Can you place a charged object at each corner of the square so the net force acting on the charge in the center is directed toward the top right corner of the square? Each charge has a magnitude of Q, but you get to choose whether it is + or – .

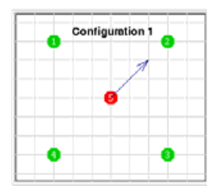


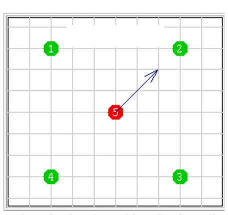
Case 1 – let me count the ways.

There is an object with a charge of $+Q$ at the center of a square. Can you place a charged object at each corner of the square so the net force acting on the charge in the center is directed toward the top right corner of the square? Each charge has a magnitude of Q, but you get to choose whether it is + or – .

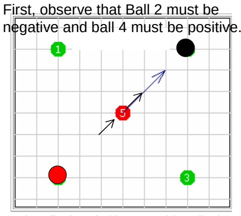
How many possible configurations can you come up with that will produce the required force?

1. 1
2. 2
3. 3
4. 4
5. either 0 or more than 4

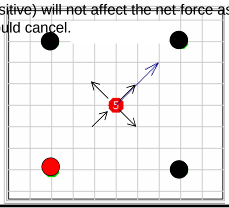


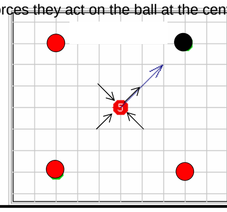


First, observe that Ball 2 must be negative and ball 4 must be positive.



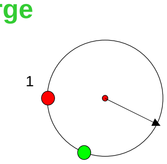
With that, by having either both Balls 1&3 negative (below left) or positive (below positive) will not affect the net force as the forces they act on the ball at the center would cancel.



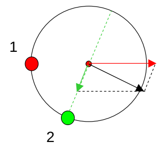


An unknown charge

Two charges 1 and 2 are placed on the circumference of a circle. Charge 1 is positive while the sign of charge 2 is unknown. Suppose the arrow indicates the direction of the net force due to the two charges on a positive test charge at the center of the circle. What is the sign of the unknown charge? How is its magnitude compared to that of charge 1?



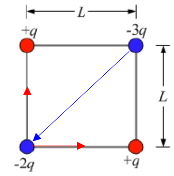
An unknown charge



The force due to charge 1 must be pointing right so as to be away from charge 1 as shown by the red arrow. The force due to charge 2 must lie along the line joining charge 2 and the test charge (i.e., the green dash line). The net force is a vector sum of the forces from charge 1 and 2 on the positive test charge. To have the net force points in the direction given, the force due to charge 2 must point towards charge 2. That means the unknown charge 2 is negative. Since the net force is closer to the force vector due to charge 1 than it is to that due to charge 2, the contribution from the former must be bigger. (The three force vectors in the actual situation are shown in the diagram above.) So, the magnitude of charge 1 must be larger than the unknown charge. Note that we can ignore the distance in our consideration of the forces because the distance between the test charge and charges 1 and 2 are the same.

Direction of the net force

What is the direction of the net force acting on the -2q charge in the square shown at right?



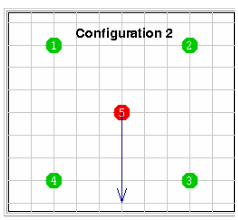
Solution

We first consider the forces due to the two +q charges. They should produce a net force along the diagonal towards the -3q charge. On the other hand, the −3q charge produces a force along the diagonal away from itself (see the blue arrow). The magnitude of the vector sum of the forces due to the two +q charge is: $[2(2kq^2/L^2)^2]^{1/2} = 2\sqrt{2}kq^2/L^2$. The magnitude of the force due to the -3q charge is $3kq^2/L^2$ (Note: The distance between the -3q charge and -2q charge is $\sqrt{(2L)}$). Since this is bigger, the net force on the -2q charge points away from the -3q charge.

A two-dimensional situation 2

Simulation

Case 2: The net force on the positive center charge is directed straight down. What are the signs of the equal-magnitude charges occupying each corner?
How many possible configurations can you come up with that will produce the desired force?

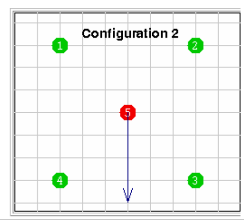


Case 2 – let me count the ways.

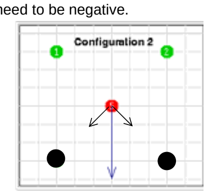
There is an object with a charge of +*Q* at the center of a square. Can you place a charged object at each corner of the square so the net force acting on the charge in the center is directed straight down? Each charge has a magnitude of *Q*, but you get to choose whether it is + or – .

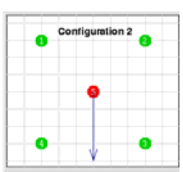
How many possible configurations can you come up with that will produce the required force?

1. 1
2. 2
3. 3
4. 4
5. either 0 or more than 4

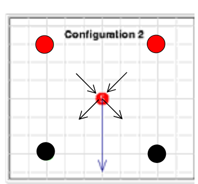


First, observe that both charges 3 & 4 need to be negative.





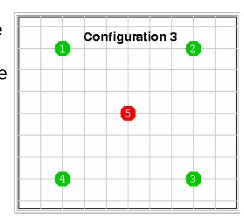
With that, both charges 1 & 2 must be positive to maintain the direction of the net force (see figure at left.)



A two-dimensional situation

Simulation

Case 3: There is no net force on the positive charge in the center. What are the signs of the equal-magnitude charges occupying each corner?
How many possible configurations can you come up with that will produce no net force?

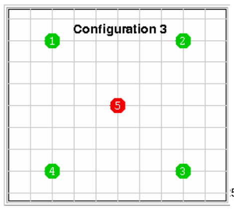


Case 3 – let me count the ways.

There is an object with a charge of +*Q* at the center of a square. Can you place a charged object at each corner of the square so there is no net force acting on the charge in the center? Each charge has a magnitude of *Q*, but you get to choose whether it is + or – .

How many possible configurations can you come up with that will produce no net force?

1. 1
2. 2
3. 3
4. 4
5. either 0 or more than 4



Case 3 – let me count the ways.

So long as the two charges along each diagonal have the same sign, the forces acting on the charge at the center would be canceled. So, there are 2 x 2 = 4 ways.

2 signs   2 diagonals