

## Review for Test 3

### Review for Test 3

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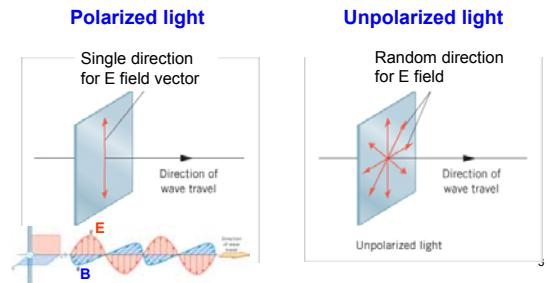
### Polarized light

No equation provided!

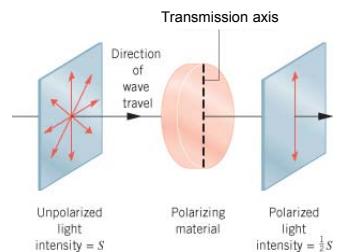
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### Polarized light

In linearly polarized light, the electric field vectors all lie in one single direction.



### Action of a Polarizer



Polarized light may be produced from unpolarized light with the aid of polarizing material.

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### Light Intensity after a Polarizer

1. If the incoming light is **unpolarized**: The light intensity after passing through a polarizer is **halved** irrespective of the transmission axis of the polarizer.

2. If the incoming light is **linearly polarized**: The light intensity after a polarizer is governed by **Malus' Law**.

$$I_1 = I_0 \cos^2 \theta_{01}$$

where  $\theta_{01}$  is the angle between the polarization direction of the incoming light and the transmission axis of the polarizer,  $I_0$  and  $I_1$  is the intensity of light before and after transmission through the polarizer. <sup>5</sup>

### Curved Mirrors

$$\text{Lenses and mirrors: } \frac{1}{f} = \frac{1}{d_s} + \frac{1}{d_i}$$

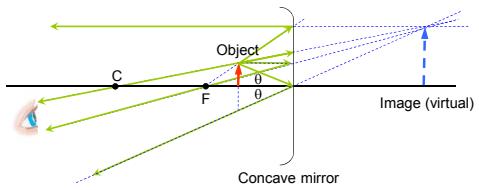
$$\text{Magnification, } m = \frac{h_i}{h_s} = -\frac{d_i}{d_s}$$

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# Review for Test 3

## Drawing Ray Diagrams for a Concave (Convergent) Mirror

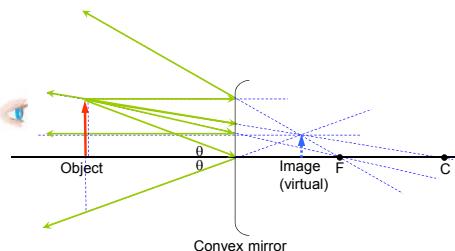
Draw the principal axis, the concave mirror in the middle, the object in front of the mirror, and **both F and C in front of the mirror**. There are four rays one typically draws to determine the location of the image.



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## Image Formation by Convex (Divergent) Mirrors

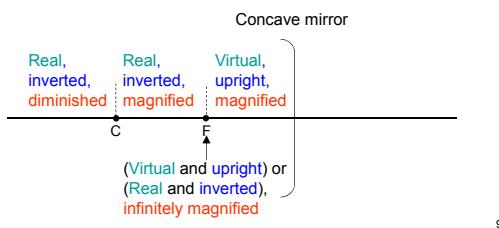
Draw the principal axis, the convex mirror in the middle, the object in front of the mirror, and **both F and C behind the mirror**. We can establish four similar ways as in the last slide to draw reflected rays for a convex mirror.



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## Properties of Images formed by a Concave (Convergent) Mirror

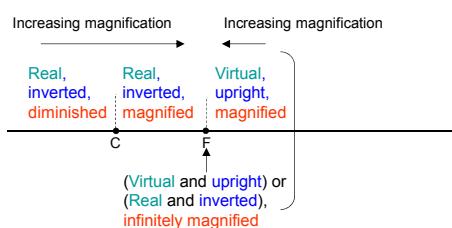
The properties depends on the position of the object relative to the mirror, F and C. Below is a summary of the image properties for different positions of the object.



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## Effect on the Image formed by a Concave Mirror by Moving the Object

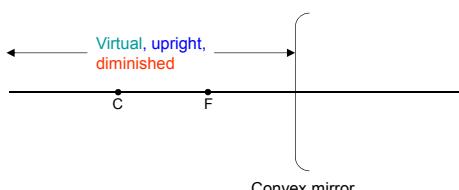
Both the magnification and image distance increase as the object moves toward the focal point from either side.



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## Properties of Images formed by a Convex (Divergent) Mirror

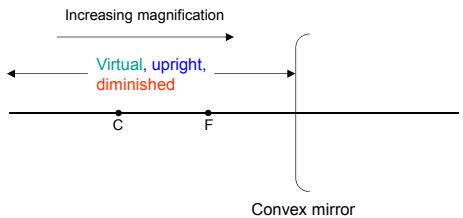
The images formed by a convex mirror are **always** virtual, upright and diminished. Below is an illustration of the image properties for different positions of the object.



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## Effect on the Image formed by a Convex Mirror by Moving the Object

Both the magnification and image distance increases as the object moves toward the mirror.



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# Review for Test 3

## The Mirror Equation

Drawing a ray diagram is a great way to get an idea about what's going on. But to find the precise distances of the object and/or image, we should use the mirror equation.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where  $d_o$  = object distance,  $d_i$  = image distance.

Magnification = (image height,  $h_i$ ) / (object size,  $h_o$ ) is another property of common and practical interest.

$$\text{Magnification, } m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

**Negative magnification means that the image is inverted!**

## Sign Convention of the Mirror Equation

The parameters,  $d_i$  and  $f$  can be positive or negative depending on the position of the image and the focal point, F, respectively, relative to the mirror. Below lists the sign convention.

	In front of the mirror	Behind the mirror
Image position	$d_i$ is +	
Focal point, F	$f$ is + (apply to concave or convergent mirrors)	$f$ is - (apply to convex or divergent mirrors)

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## Lenses

Make sure that you don't mix up the ray diagram of lenses with mirrors!

Lenses and mirrors:  $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

Magnification,  $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

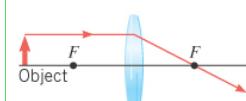
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## Convergent vs. Diverging Lenses

We have seen that convergent and Divergent **mirrors** converge and diverge light **reflected** upon them.

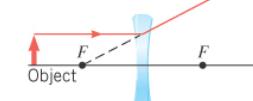
Convergent and Divergent **lenses** converge and diverge light **transmitted** through them.

### Convex (convergent) lenses



All parallel light rays converge to point F after transmission through the lens.

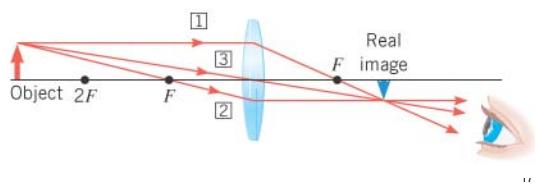
### Concave (divergent) lenses



Parallel light rays transmitted through the lens do not converge. But when they are extrapolated backward, they appear to.

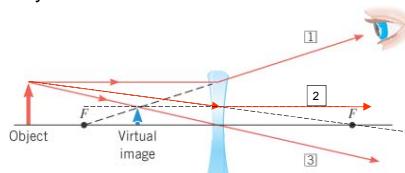
## Image formation with a convex (convergent) lens

Draw the principal axis, the convex lens in the middle, the object (on the left side of the lens), and the **focal point F on both sides of the lens**. Below shows the three rays one commonly draws in a ray diagram to determine the image formed by a convex lens.



## Image formation with a concave (divergent) lens

Draw the principal axis, the concave lens in the middle, the object (on the left side of the lens), and the **focal point F on both sides of the lens**. Below shows the three rays one commonly draws in a ray diagram to determine the image formed by a concave lens.

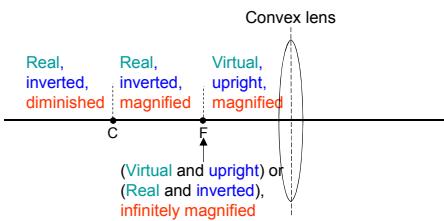


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# Review for Test 3

## Properties of Images formed by a Convex (Convergent) Lens

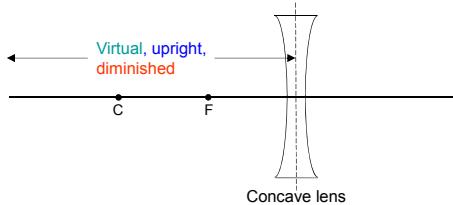
The properties depends on the position of the object relative to the mirror, F and C. Below is a summary of the image properties for different positions of the object. Note that it's the same as that for convergent mirrors.



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## Properties of Images formed by a Concave (Divergent) Lens

The images formed by a divergent lens are **always** virtual, upright and diminished. Below is an illustration of the image properties for different positions of the object. Note that it's same as that for divergent mirrors.



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## The Thin-lens Equation

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Conveniently, it's the same equation we used for mirrors!

$$\text{Magnification, } m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

Sign convention of m (same as before):  
Positive means that the image is upright.  
Negative means that the image is inverted.

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## Sign convention of the thin-lens equation

	Same side as the object (usually l.h.s. of the lens)	Opposite side to the object (usually r.h.s. of the lens)
Image position	$d_i$ is -	$d_i$ is +
Point where the transmitted rays converge or appear to converge	$f$ is -, applicable to divergent (concave) lens	$f$ is +, applicable to convergent (convex) lens

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## Refraction & Total Internal Reflection

Index of refraction,  $n = c/v = \lambda/\lambda'$

Snell's Law:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Critical angle,  $\theta_c = \sin^{-1}(n_2/n_1)$

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## Index of refraction

When an EM wave travels in a vacuum its speed is:  
 $c = 3.00 \times 10^8$  m/s.

In any other medium, light travels at a different speed,  $v$ , without changing the frequency. We define the index of refraction,  $n$ , of a material by:

$$n = \frac{c}{v} = \frac{f\lambda_0}{f\lambda} = \frac{\lambda_0}{\lambda}$$

Wavelength in vacuum      Wavelength in the medium

For naturally occurring materials,  $n > 1$  and so  $v < c$  and  $\lambda < \lambda_0$ .

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# Review for Test 3

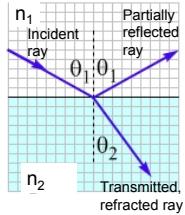
## Snell's Law

Due to the change in speed, light bends on entering a different medium at an oblique angle. This phenomenon is called **refraction**.

The angle of refraction  $\theta_2$  (the angle the ray in the second medium makes with the **normal** to the interface between the two media) is related to the angle of incidence  $\theta_1$  by

$$\text{Snell's Law: } n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where  $n_1$  and  $n_2$  is the refractive index of the first and second medium, respectively.



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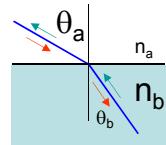
## Consequence of Snell's Law I

$$\text{Snell's Law: } n_1 \sin \theta_1 = n_2 \sin \theta_2 (= \text{constant})$$

Based on Snell's Law, we can see that if the refractive index of a medium is bigger, the angle made by the ray to the normal must be smaller to maintain the product of  $n$  and  $\sin \theta$  constant. This property is independent of whether the medium is the first (incoming) or the second (outgoing) one.

So, if light enters from a less dense to a denser medium, it bends toward the normal.

Conversely, if light enters from a denser to a less dense medium, it bends away from the normal.



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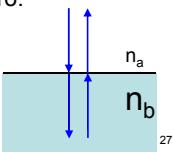
## Consequence of Snell's Law II

$$\text{Snell's Law: } n_1 \sin \theta_1 = n_2 \sin \theta_2 = \text{constant}$$

$$\text{If } \theta_1 = 0, n_1 \sin \theta_1 = n_2 \sin \theta_2 = 0 \\ = 0$$

Since  $n_2$  is not zero,  $\theta_2$  must be zero to make the product of  $n_2$  and  $\sin \theta_2$  equal to zero.

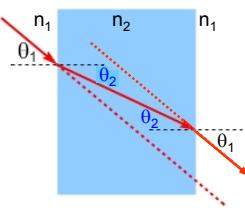
Light entering a medium perpendicularly is not bent.



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## Consequence of Snell's Law III

A ray entering a rectangular slab at angle  $\theta_1$  would emerge from the opposite edge of the slab at the same angle  $\theta_1$ , but is shifted laterally.



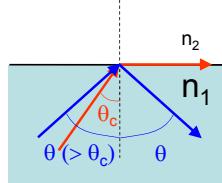
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## Consequence of Snell's Law IV

A ray going from a denser to a less dense medium suffers **total internal reflection** if the incident angle is bigger than the critical angle, which is the incident angle for which the refraction angle is  $90^\circ$ .

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\text{so } \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$



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## Interference Patterns

Constructive interference:  $\Delta L = m\lambda$

Destructive interference:  $\Delta L = (2m+1)\lambda/2$  or  $\Delta L = (m+1/2)\lambda$

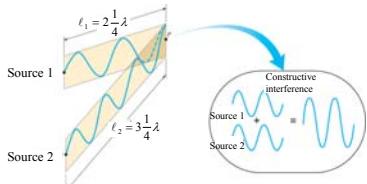
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# Review for Test 3

## Conditions for constructive interference

When two or more light waves pass through a given point and arrive there in phase, **constructive interference** occurs. If the sources are in phase, the condition required for constructive interference is:

$$PLD = \ell_2 - \ell_1 = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \dots$$

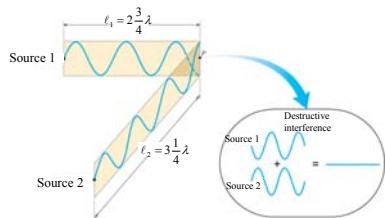


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## Conditions for destructive interference

When two or more light waves pass through a given point and arrive there out of phase, **destructive interference** occurs. If the sources are out of phase, the condition required for destructive interference is:

$$PLD = \ell_2 - \ell_1 = (m + \frac{1}{2})\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \dots$$

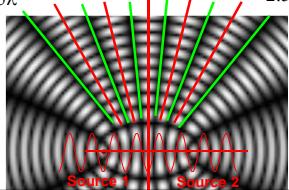


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## Interference from Two Sources

We can understand the pattern by recalling that PLD along the perpendicular bisector of the line joining the two sources is always zero. If we move systematically away from the perpendicular bisector to either side, the PLD increases in the order of  $\pm 0.5\lambda, \pm\lambda, \pm 1.5\lambda, \pm 2\lambda, \dots$  as shown below.

$$PLD = -2.5\lambda, -2\lambda, -1.5\lambda, -\lambda, -0.5\lambda, 0, 0.5\lambda, \lambda, 1.5\lambda, 2\lambda, 2.5\lambda$$



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## Double Slit

- Focus on the condition for constructive interference.

Interference and diffraction of light:

Constructive interference, two or more slits:  $d \sin\theta = m\lambda$

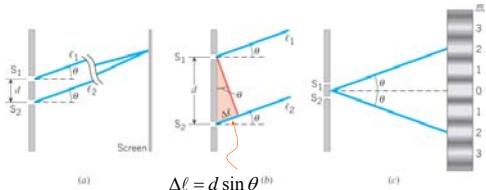
Destructive interference, double slit:  $d \sin\theta = (2m - 1)\lambda/2$  or  $d \sin\theta = (m + 1/2)\lambda$

Destructive interference, single slit:  $W \sin\theta = m\lambda$

- Destructive interferences occur right in the middle between two adjacent maxima.

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## The double-source equation



**Bright fringes  
of a double-slit**

$$\sin\theta = m \frac{\lambda}{d} \quad m = 0, \pm 1, \pm 2, \pm 3, \dots$$

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## Diffraction Grating

- We only consider the condition for constructive interference:

Interference and diffraction of light:

Constructive interference, two or more slits:  $d \sin\theta = m\lambda$

Destructive interference, double slit:  $d \sin\theta = (2m - 1)\lambda/2$  or  $d \sin\theta = (m + 1/2)\lambda$

Destructive interference, single slit:  $W \sin\theta = m\lambda$

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# Review for Test 3

## Diffraction Grating

Condition for constructive interference:

$$d \sin(\theta) = m\lambda, \quad (m = 0, \pm 1, \pm 2, \dots)$$

where  $d$  is the slit separation.

Below is the Intensity profile of the interference pattern produced by grating with different number of slits separated by the same distance,  $d$ .

By adding more slits, the interference maxima are much brighter, and a lot sharper.

## Diffraction by a Single Slit

- We only consider the condition for **destructive interference**:

Interference and diffraction of light:

Constructive interference, two or more slits:  $d \sin\theta = m\lambda$ .

Destructive interference, double slit:  $d \sin\theta = (2m - 1)\lambda/2$  or  $d \sin\theta = (m + 1/2)\lambda$

**Destructive interference, single slit:  $w \sin\theta = m\lambda$**

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## Diffraction (by a Single Slit)

**Interference pattern From diffraction:**

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## Dark Fringes in a Diffraction Pattern

There's no condition for constructive interference, except we know that there is constructive interference in the  $\theta=0$  direction, giving rise to the central maximum.

The condition for destructive interference is:

$$w \sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \dots$$

where  $w$  is the width of the slit. Note that  $m$  cannot be zero, at where the central maximum is.

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## Example: Width of the central maximum

Solution

The width of the central maximum of the interference pattern,  $x$ , is given by

$$(x/2)/D = \tan \theta \Rightarrow x = 2D \tan \theta \dots (1)$$

$$w \sin \theta = m\lambda \quad \text{or} \quad \sin \theta = m(\lambda/a), \dots (2)$$

where  $m = 1$  for the dark fringe right next to the central maximum.

When  $\lambda \ll w$ ,  $\sin \theta \approx \theta$ . As a result,  $\sin \theta \approx \tan \theta$ . By equations (1) and (2), we get

$$x = 2D\lambda/a$$

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## Realistic double-slit pattern

The realistic "double-slit" pattern is a combination of the **single slit pattern** and the **(ideal) double source pattern** we encountered at the beginning. The (ideal) double source pattern assumes the slits to be infinitely narrow.

Single-slit:  
Ideal double-source:  
Realistic double-slit:

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# Review for Test 3

## Wave-Particle Duality: Photoelectric Effect & De Broglie Wavelength

Values of some constants:

$e = 1.6 \times 10^{-19} \text{ C}$	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^3)$
$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$	$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$
$1 \text{ u} = 931.5 \text{ MeV/c}^2$	Planck's constant: $h = 6.63 \times 10^{-34} \text{ Js} = 4 \times 10^{-15} \text{ eV s}$
$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$	$hc = 1200 \text{ eV nm}$

Waves:  $v = f\lambda$

Photon energy:  $E_{\text{ph}} = hf$

Photo-electric effect:  $hf = W_0 + K_{\text{max}}$

Particle waves:  $\lambda = h / p$

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## The photoelectric effect

In the photoelectric effect, electrons are given out by a metal when it is shone by optical or UV light with high enough frequency,  $f$ , to overcome the work function,  $W_0$ , of the metal. Given the particle model, the energy conservation for the photoelectric effect is:

$$hf = W_0 + K_{\text{max}}$$

where  $K_{\text{max}}$  is the maximum kinetic energy of the photoelectrons.

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## The electron volt

$$\begin{aligned} 1 \text{ eV} &= 1.60 \times 10^{-19} \text{ J} \\ 1 \text{ J} &= 1 / (1.6 \times 10^{-19}) \text{ eV} \end{aligned}$$

Use  $c = f\lambda$  and  $E = hf$ , we can determine the frequency ( $f = c/\lambda$ ) and energy for the photons ( $E = hc/\lambda$  (in J) =  $hc/e\lambda$  (in eV)) in the visible spectrum (i.e.,  $400\text{nm} < \lambda < 700\text{nm}$ ).

Wavelength	Frequency	Energy
400 nm (violet)	$7.5 \times 10^{14} \text{ Hz}$	3.1 eV
700 nm (red)	$4.3 \times 10^{14} \text{ Hz}$	1.8 eV

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## The photoelectric effect – a graph

Pay attention to what is being plotted against what! Example: How should a graph of  $K_{\text{max}}$  vs. photon energy ( $hf$ ) look like? Consider cases where  $hf < W_0$  and  $hf > W_0$ .

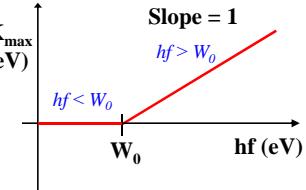
(i) When  $hf \leq W_0$ ,

$$K_{\text{max}} = 0$$

(ii) When  $hf > W_0$ ,

$$hf = W_0 + K_{\text{max}}$$

$$K_{\text{max}} = hf - W_0$$



What would the slope be if the plot is  $K_{\text{max}}$  vs.  $f$ ?

Coefficient of term  $hf = 1 \Rightarrow \text{Slope} = 1$

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## The de Broglie wavelength

The momentum of a photon is given by:

$$p = \frac{h}{\lambda}$$

In 1923, Louis de Broglie predicted that objects we generally think of as particles (such as electrons and neutrons) should also exhibit a wave-like nature, with a wavelength of:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

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## Nuclear Reactions

Values of some constants:

$e = 1.6 \times 10^{-19} \text{ C}$	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^3)$
$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$	$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$
$1 \text{ u} = 931.5 \text{ MeV/c}^2$	Planck's constant: $h = 6.63 \times 10^{-34} \text{ Js} = 4 \times 10^{-15} \text{ eV s}$
$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$	$hc = 1200 \text{ eV nm}$

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# Review for Test 3

## The nucleus

A nucleus consists of protons and neutrons; these are known as **nucleons**. Each nucleus is characterized by two numbers:  $A$ , the atomic mass number (the total number of nucleons); and  $Z$ , the atomic number (the number of protons). The number of neutrons is  $N$ , so  $A = N + Z$ .

Any nucleus can be written in a form like this:



The  $X$  stands for the chemical symbol. On the right is a particular isotope of aluminum, aluminum-27.

**Isotopes** of an element have the same  $Z$ , but different number of neutrons.

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## The atomic mass unit

$$\begin{aligned} 1 \text{ u} &= (\text{Mass of } {}^{12}\text{C})/12 \\ &= 1.660540 \times 10^{-27} \text{ kg} \\ &= 931.5 \text{ MeV}/c^2 \end{aligned}$$

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## The mass defect

Each carbon 12 atom is made up of 6 neutrons, 6 protons, and 6 electrons, which separately have a mass of:

six neutrons:  $6 \times 1.008664 \text{ u} = 6.051984 \text{ u}$

six protons:  $6 \times 1.007276 \text{ u} = 6.043656 \text{ u}$

six electrons:  $6 \times 0.00054858 \text{ u} = 0.00329148 \text{ u}$

Sum = 12.098931 u

When these are combined into a carbon-12 atom, the atom has a mass of precisely 12.000000 u. The missing 0.098931 u worth of mass is the **mass defect**.

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## The most famous equation in physics

$$E = mc^2$$

The missing mass accounts for the **binding energy** of the atom (almost all of which is in the nucleus).

At the rate of 931.5 MeV/c<sup>2</sup> per u, the mass defect of 0.098931 u corresponds to 92.15 MeV worth of binding energy in the carbon-12 atom.

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## General requirements for a permissible radioactive decay

(1) The total mass of the daughter nuclides must be smaller than that of the parents. This is effectively saying that the daughter nuclides are on average more stable than the parent nuclides

(2) The sum of the atomic numbers and mass numbers of all the parent nuclides/particles and daughter nuclides/particles before and after the decay must agree. In checking if this requirement is met, take for neutrons ( ${}^1_0 n$ ),  $A = 1$  and  $Z = 0$ ; for protons ( ${}^1_1 p$ ),  $A = 1$  and  $Z = 1$ ; and for electrons ( ${}^0_{-1} e$ ),  $A = 0$  and  $Z = -1$ .

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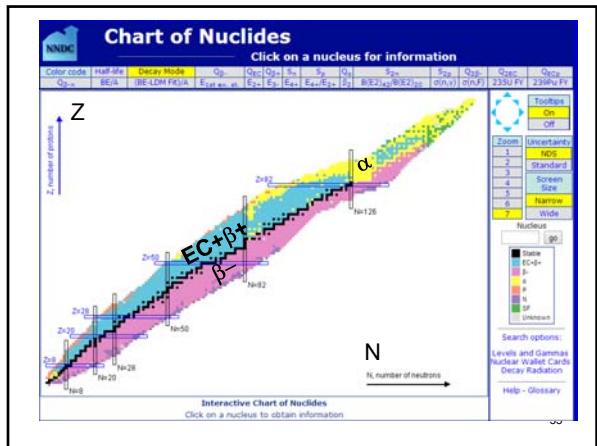
## Chart of the nuclides

The chart (next page) shows the existing nuclides and their lifetime, where a darker shade represents nuclides with a longer half-life. In particular, the nuclides shaded in black have half-life  $> 30$  million years and are thus regarded as stable.

With increased resolution (found in <http://www-nds.iaea.org/relnsd/vchart/index.html>), one can see that for atomic number  $0 \leq Z \leq 7$ , the stable nuclides always have the neutron number,  $N = Z$  and/or  $Z+1$ , except for hydrogen which has  ${}^1_1 H$  and  ${}^2_1 H$  as the only stable nuclides. For  $Z > 7$ , the stable nuclides have  $N$  increasingly bigger than  $Z$  as  $Z$  increases.

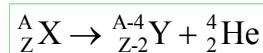
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# Review for Test 3



## Alpha decay

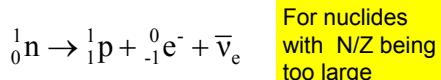
If a heavy unstable nuclide  ${}^A_Z X$  is located near a lighter nuclide with atomic number  $Z-2$  and mass number  $A-4$  and a longer half-life, it will undergo alpha decay.



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## Beta-minus ( $\beta^-$ ) decay

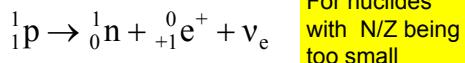
In a beta-minus decay, a neutron is turned into a proton by giving up an electron, an electron (a  $\beta^-$  particle) and an anti electron neutrino.



Generally,  ${}^A_Z X \rightarrow {}^{A-1}_{Z+1} Y + {}^0_{-1} e^- + \bar{\nu}_e$

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## Beta-plus ( $\beta^+$ ) decay



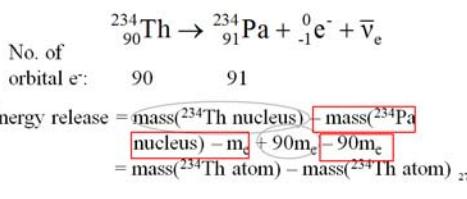
In beta-plus decay, a proton turns into a neutron, positron, and an electron neutrino. In general, we get:



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## Finding the mass defect for $\beta^-$ decay from atomic masses

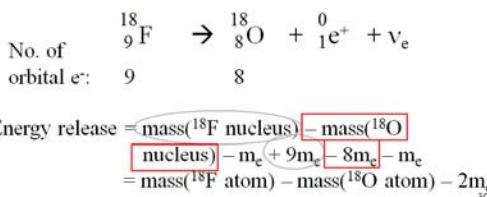
The nuclear reaction  ${}^A_Z X \rightarrow {}^{A-1}_{Z+1} Y + e^- + \bar{\nu}_e$  only shows the nuclei involved in the reaction. In calculating the mass defect from the atomic masses (not the nuclear masses), one needs to keep track of the number of electrons contained in each reactant atom. Take an example,



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## Finding the mass defect for $\beta^+$ decay from atomic masses

Again, the nuclear reaction  ${}^A_Z X \rightarrow {}^{A-1}_{Z-1} Y + e^+ + \nu_e$  only shows the nuclei involved in the reaction. In calculating the mass defect from the atomic, one needs to keep track of the number of electrons contained in each reactant atom. Take an example,



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# Review for Test 3

## Radioactivity

$$\text{Radioactivity: } R = -\frac{\Delta N}{\Delta t} = \lambda N$$

$$N = N_0 e^{-\lambda t}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.7}{\lambda}$$

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## Activity, $R$ and the Decay Constant, $\lambda$

The rate at which nuclei decay (also known as the **activity** of the nuclei) is proportional to  $N$ , the number of nuclei there are.

$$R = -\frac{\Delta N}{\Delta t} = \lambda N$$

where  $\lambda$  is the **decay constant**. The SI unit of activity is the **becquerel (Bq)**; one Bq equals one disintegration per second. The SI unit of the decay constant is  $s^{-1}$ .

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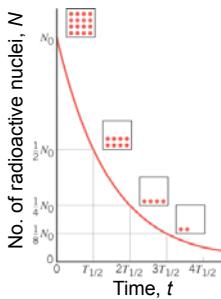
## Number of Unstable Nuclides, $N$ vs. time

As a result of the fact that the rate of a radioactive decay is proportional to the number of unstable nuclides that remain, the number of nuclides that haven't decayed decreases exponentially with  $t$ .

$$N = N_0 e^{-\lambda t}$$

*Number remaining after a time  $t$*       *Initial number at  $t = 0$*

n.b. The slope of the graph,  $dN/dt$  decreases with time because  $dN/dt$  is proportional to  $N$ .



## Activity as a function of time

$$N = N_0 e^{-\lambda t}$$

$$R = \lambda N$$

$$= \lambda N_0 e^{-\lambda t}$$

$\underbrace{N_0}_{R_0}$

$$R = R_0 e^{-\lambda t}$$

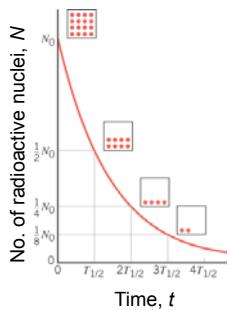
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## Half-life, $T_{1/2}$ and the Decay Constant, $\lambda$

The decay constant is closely related to the **half-life**, the time for half the material to decay.

$$T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

The activity of a sample or the no. of radioactive nuclei remaining after  $t = nT_{1/2}$  is  $1/2^n$  times the activity at  $t = 0$ .



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Review examples given in the class notes.

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