Radioactivity

Activity, R and the Decay Constant, λ

It is impossible to predict when an individual nucleus will decay. However, radioactive decay is governed by statistics, so we can predict the rate of decay of a large number of radioactive nuclei.

The rate at which nuclei decay (also known as the **activity** of the nuclei) is proportional to *N*, the number of nuclei there are.

$$\boxed{R = -\frac{\Delta N}{\Delta t} = \lambda N}$$

where λ is the decay constant. The SI unit of activity, R, is the **becquerel (Bq)**; one Bq equals one disintegration per second. The SI unit of *the decay constant is* s^{-1}.

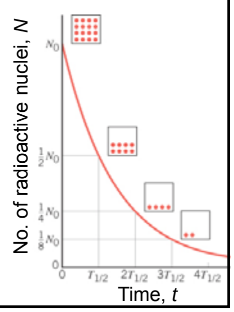
Number of Unstable Nuclides, *N* vs. time

As a result of the fact that the rate of a radioactive decay is proportional to the number of unstable nuclides that remain, the number of nuclides that haven't decayed decreases exponentially with *t*.

$$N = N_0 e^{-\lambda t}$$

Number remaining after a time t — *Initial number at t = 0*

n.b. The slope of the graph, d*N*/d*t* decreases with time because d*N*/d*t* is proportional to *N.*



Activity as a function of time

$$N = N_0 e^{-\lambda t}$$

$$R = \lambda N$$

$$= \underbrace{\lambda N_0}_{R_0} e^{-\lambda t}$$

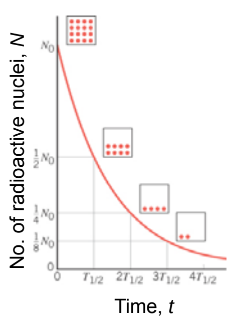
$$\boxed{R = R_0 e^{-\lambda t}}$$

Half-life, $T_{1/2}$ and the Decay Constant, λ

The decay constant is closely related to the half-life, the time for half the material to decay.

$$\boxed{T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}}$$

The activity of a sample or the no. of radioactive nuclei remaining after $t = nT_{1/2}$ is $1/2^n$ times the activity at *t* = 0.



Mass of a specimen

A specimen that undergoes β decay has an initial mass of 16 g and a half-life of 1 hr. What is its mass 3 hrs later? Choose the best answer from below.

1. 16 g
2. 8 g
3. 4 g
4. 2 g

Mass of a specimen

In β decays, a neutron is converted to a proton, which has a mass very close to that of a neutron. So, there is very little change (much less than a gram) in the mass from the decay.

7

Activity of radon, ^{222}Rn

Radon-222 undergoes alpha decay into ^{218}Po (Polonium 218). Suppose there are 256,000 radon atoms trapped in a basement. The $T_{1/2}$ of radon is 4 days. How many radon atoms remain after 16 days?

1. 256,000 · (1/2)
2. 256,000 · (1/4)
3. 256,000 · (1/8)
4. 256,000 · (1/16)
5. 256,000 · (1/32)

8

Activity of radon

How many radon atoms remain after 16 days?

Sixteen days = $4T_{1/2}$.

Hence, the no. of radon atoms remain

= $N_0/2^4$

= 256,000/16

=16,000

9

Activity of radon

What is the activity of the radon just after the basement is sealed?

1. 256,000 / (4 x 24 x 3,600) Bq
2. ln2 / (4 x 24 x 3,600) Bq
3. 256,000 x ln2 / (4 x 24 x 3,600) Bq
4. 1 / (4 x 24 x 3,600) Bq
5. Insufficient information to tell.

10

Activity of radon

What is the activity of the radon initially (just after the basement is sealed)?

Because the activity, $R = \lambda N$ and $\lambda = \ln 2/T_{1/2}$. Remember to convert $T_{1/2}$ to a value in units of s.

$R = N\ln 2/T_{1/2}$
= 256,000 x ln2 / (4d x 24hr/d x 3,600s/hr) Bq
= 0.5134 Bq

11

Activity of radon

What is the activity of the radon 16 days later?

The activity R (= λN) is proportional to N and N is reduced by a factor of $1/2^4$ 16 days later, so

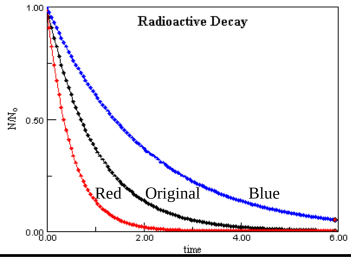
$R = R_0/2^4$
= 0.5134 Bq / 16
= 0.0321 Bq

12

Increasing the decay constant

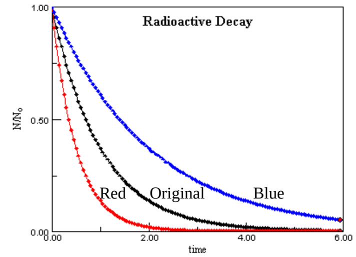
The black curve represents the decay curve for a particular radioactive sample. Which curve represents the decay of a second sample that has a larger decay constant?

1. the red curve
2. the blue curve



Increasing the decay constant

A larger decay constants means the decay happens more quickly. The red curve represents a larger decay constant, and a shorter half-life.



14

Radioactive dating

Radioactivity can be used to determine how old something is. When carbon 14 is used, the process is called radiocarbon dating, but radioactive dating can involve other radioactive nuclei. The trick is to use a nuclide with half-life of the order of, or slightly shorter than, the age of the object.

15

Radiocarbon dating

Carbon-14 is used because all living things take up carbon from the atmosphere. By natural exchange with the atomosphere and decay, C-14 maintains an equilibrium level of about 1 atom of C-14 for every 8.3 x 10^{11} atoms of carbon. When an organism dies, the carbon-14 continues to decay but the in-take from the atmosphere ceases to occur. So the level of C-14 in a specimen decreases with time, which can be used to deduce the time lapsed since the organism dies.

Carbon-14 has a half life of 5730 years, so it is useful for measuring ages of objects that are several hundred years, to several tens of thousands of years, old.

Applications of radiocarbon dating include dating of the shroud of Turin (to the 13^{th}-14^{th} centuries), and of Ötzi the Iceman (3300 BC), found in the Alps in 1991.

16

Example problem

A sample of wood has an activity of 0.22 Bq from carbon-14. An equivalent piece of wood cut from a growing tree would have an activity of 0.88 Bq from its carbon-14. How old is the sample?

17

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R_0 = 0.88 Bq , R = 0.22 Bq , $\lambda = \frac{0.693}{5730 \text{ y}} = 1.21\times10^{-4} \text{ y}^{-1}$

$0.22 = 0.88 e^{-\lambda t}$

Divide both sides by 0.88: $0.25 = e^{-\lambda t}$

Take the natural log of both sides: $\ln(0.25) = -\lambda t$

The age of the sample is:

$$t = -\frac{\ln(0.25)}{\lambda} = \frac{1.386}{1.21\times10^{-4} \text{ y}^{-1}} = 11460 \text{ years}$$

18

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Alternate (easier!) method: how many half-lives have passed?

The activity is ¼ of the original activity, so exactly two half-lives have gone by. Multiplying the half-life of C-14, which is 5730 y, by 2 gives 11460 years.

19