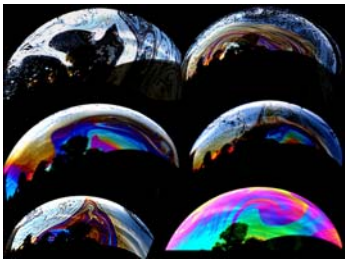
Thin Film Interference



Thin-film interference

Interference between light waves is the reason that thin films, such as soap bubbles, show colorful patterns.

Photo credit: Mila Zinkova, via Wikipedia

Thin-film interference

This is known as thin-film interference - interference between (1) light waves reflecting off the top surface of a film with (2) waves reflecting from the bottom surface. To obtain a nice colored pattern, the thickness of the film has to be comparable to the wavelength of light.

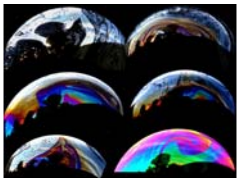
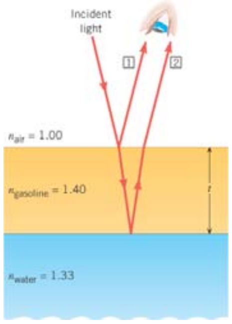


Photo credit: Mila Zinkova, via Wikipedia

Thin Film Interference



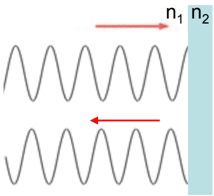
There will be constructive (or destructive) interference if the **effective path length difference (PLD)** between the no. 1 wave (reflection from top surface of the film) and the no. 2 wave (reflection from the bottom surface of the film) is an integer (or an integer plus a half) multiples of the wavelength λ_{film} of the in the film, where

$$\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n}$$

Effective PLD due to reflection at an interface

When light travels from a smaller refractive index medium towards a larger refractive index medium, reflection at the boundary occurs along with a phase change that is equivalent to an effective PLD equal to one-half of a wavelength in the film.

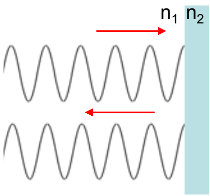
Case 1: $n_1 < n_2$



Effective PLD due to reflection at an interface

When light travels from a medium with a larger refractive index towards one with a smaller refractive index, there is no phase change and hence zero effective PLD upon reflection.

Case 2: $n_1 > n_2$



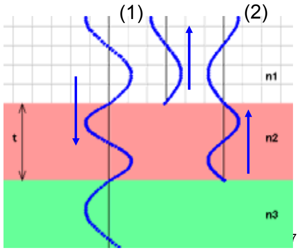
What kind of interference?

In the figure shown below, the film thickness is exactly one wavelength, so the wave that reflects off the bottom surface of the film travels a down-and-back extra distance of 2 wavelengths compared to the wave reflecting off the top surface.

What kind of interference do we get between the two reflected waves?

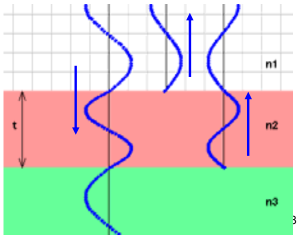
1. Constructive
2. Destructive

Simulation



What kind of interference?

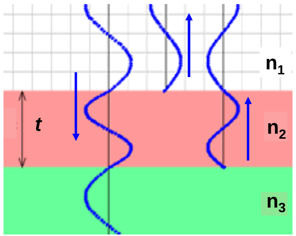
Even though the extra distance traveled is an integer number of wavelengths, we can see that the reflected waves interfere destructively. This is because the wave reflecting off the bottom surface is inverted, which is like an extra half-wavelength shift.



Thin films – a systematic approach

Let's use a five-step method to analyze thin films.

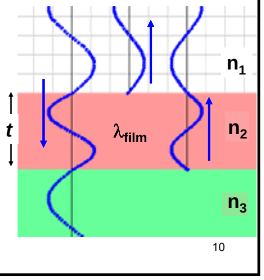
The basic idea is to determine the effective PLD between the wave reflecting from the top surface of the film and the wave reflecting from the bottom surface. The effective PLD accounts for (a) the extra distance of $2t$ traveled by the wave that reflects from the bottom surface, and any inversions upon reflection at the (b) top and (c) bottom surfaces of the film.



Thin films – a systematic approach

For a wave that gets inverted when it reflects, that is equivalent to a half-wavelength shift.

However, we have three media, and thus three different wavelengths! Because we're trying to match the wave that goes into the film with the wave bouncing off the top, it is the wavelength in the film, λ_{film}, that appears in the equations.



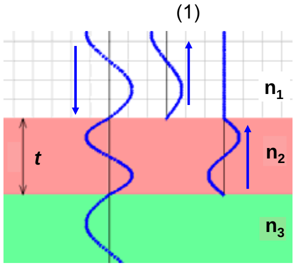
Thin films – the five-step method

Step 1 – Determine Δ_t , the shift for the no. 1 wave reflecting from the top surface of the film.

If $n_2 > n_1$, $\Delta_t = \frac{\lambda_{film}}{2}$

If $n_2 < n_1$, $\Delta_t = 0$

(This is what is shown in the drawing at right.)



Thin films – the five-step method

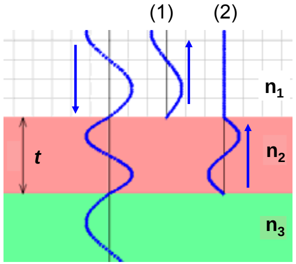
Step 2 – Determine the effective PLD Δ_b for the no. 2 wave reflecting from the bottom surface of the film. Then we should add this to $2t$ from the extra distance this wave traveled in comparison to the no. 1 wave.

If $n_3 > n_2$,

$\Delta_b = 2t + \frac{\lambda_{film}}{2}$

This is what is shown in the drawing at right.

If $n_3 < n_2$, $\Delta_b = 2t$



Thin films – the five-step method

Step 3 – Find the (total) effective path-length difference: $\Delta = \Delta_b - \Delta_t$

Step 4 – Bring in the appropriate interference condition, depending on the situation.

For constructive interference, $\Delta = m\lambda_{film}$

For destructive interference, $\Delta = (m+1/2)\lambda_{film}$

Step 5 – Solve the resulting equation. The equation generally connects the thickness of the film to the wavelength of the light in the film.

It is often useful to remember that $\lambda_{film} = \dfrac{\lambda_{vacuum}}{n_{film}}$

An example using the five-step method

White light in air shines on an oil film of thickness *t* that floats on water. The oil has an index of refraction of 1.50, while the refractive index of water is 1.33.

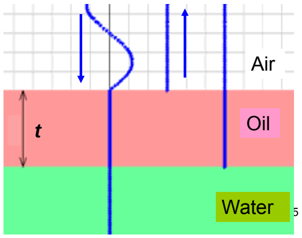
When looking straight down at the film, the reflected light looks orange, because the film thickness is just right to produce completely constructive interference for a wavelength, in air, of 600 nm.

What is the minimum possible thickness of the film?

Step 1

Step 1 – Determine Δ_t , the shift for the no. 1 wave reflecting from the top surface of the film. Which of the following is a suitable choice for Δ_t?

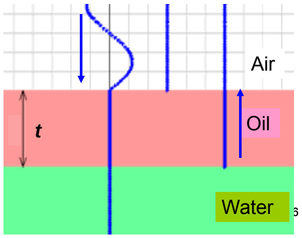
1. $\Delta_t = \dfrac{\lambda_{film}}{2}$
2. $\Delta_t = 0$



Step 2

Step 2 – Determine Δ_b, the shift for the no. 2 wave reflecting from the top surface of the film. Which of the following is suitable for Δ_b?

1. $\Delta_b = 2t + \dfrac{\lambda_{film}}{2}$
2. $\Delta_b = 2t$



Step 3

Step 3 – Determine Δ, the (total) effective path-length difference for the two reflected waves. Which of the following is a suitable choice for Δ?

1. $\Delta = 2t + \dfrac{\lambda_{film}}{2}$
2. $\Delta = 2t$
3. $\Delta = 2t - \dfrac{\lambda_{film}}{2}$

Step 4

Step 4 – Bring in the appropriate interference condition.

1. $2t - \dfrac{\lambda_{film}}{2} = m\lambda_{film}$
2. $2t - \dfrac{\lambda_{film}}{2} = (m+1/2)\lambda_{film}$

Step 4

In this situation, we were told that the film thickness was the minimum necessary to give constructive interference for a particular wavelength, so let's go with the first choice.

1. $2t - \frac{\lambda_{film}}{2} = m\lambda_{film}$

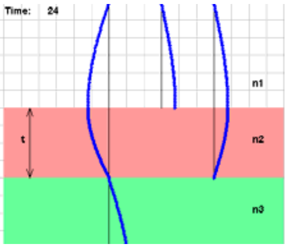
Re-arrange to get: $2t = (m + 1/2)\lambda_{film}$

This looks like destructive interference, but it is not!

Step 5

Find the unknown asked in the question.

$$\begin{aligned} 2t &= (m+1/2)\lambda_{film} \\ &= (m+1/2)\frac{\lambda_{vacuum}}{n_{film}} \\ &= (m+1/2)\frac{600 \text{ nm}}{1.5} \\ &= (m+1/2)(400 \text{ nm}) \end{aligned}$$

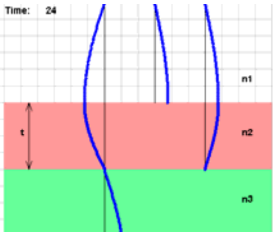


Step 5

$$2t = (m+1/2)(400 \text{ nm})$$

To find the minimum *t*, use the smallest *m* that makes sense, which in this case is

m = 0



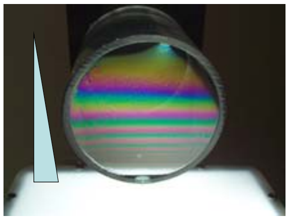
$$\begin{aligned} 2t_{\min} &= (0+1/2)(400 \text{ nm}) \\ &= 200 \text{ nm} \end{aligned}$$

$$t_{\min} = 100 \text{ nm}$$

A soap film

We make a soap film by dipping a loop into soap solution, and then hold the loop so it is vertical.

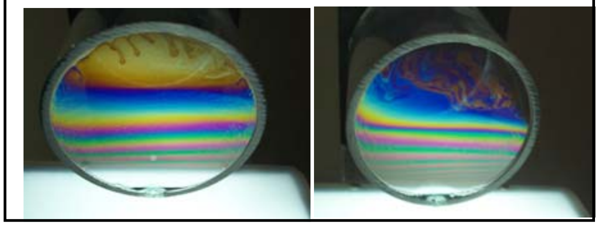
Why do we get horizontal bands on the soap film?



Gravity causes the film to be thicker at the bottom, with decreasing thickness as you move up. Different thicknesses correspond to different colors.

A soap film

As time goes by, the film gets increasingly thin, with the top of the film first going white/gold, and then black (non-reflective for all colors). Why does the film go black at the top before popping? (Use the five step method to answer this question.)



A soap film

Step 1 – Determine Δ_t, the shift for the wave reflecting from the top (or front) surface of the film.

$n_2 > n_1$, so $\Delta_t = \frac{\lambda_{film}}{2}$

Step 2 – Determine Δ_b, the shift for the wave reflecting from the bottom (or back) surface of the film.

$n_3 < n_2$, so $\Delta_b = 2t$

A soap film

Step 3 – Determine Δ , the effective path-length difference.

$$\Delta = \Delta_b - \Delta_t = 2t - \frac{\lambda_{film}}{2}$$

Steps 4-5 – What happens in the limit that the film thickness, t, approaches zero?

When $t \rightarrow 0$, the effective path-length difference, $\Delta \rightarrow 0 - \lambda_{film}/2 = -\lambda_{film}/2$, giving destructive interference. That's why the film appears black.