Class 34

Diffraction Grating

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Adding sources

What happens to the interference pattern when we add more sources? Let's start by switching from two sources *d* apart to three sources *d* apart.

Do we still get maxima at the same angles where we got maxima for two sources?

1. Yes
2. No

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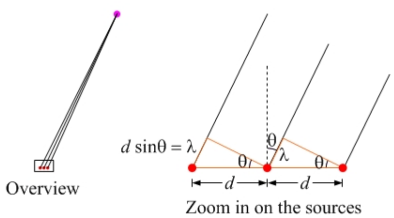
Adding sources

Yes, the constructive interference equation ***applies for any number of sources separated by a distance d*** : $d \sin(\theta) = m\lambda$. Now we're simply adding three waves in phase instead of two, and for *N* sources we'd add *N* waves in phase.



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Adding sources, II

What happens to the interference pattern when we add more sources? Let's start by switching from two sources *d* apart to three sources *d* apart.

Do we still get minima at the same angles where we got minima for two sources?

1. Yes
2. No

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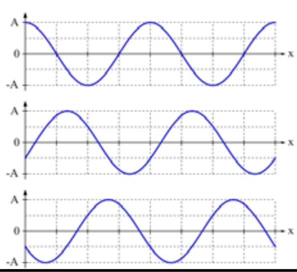
Adding sources, II

No! We don't got destructive interference when the path-length difference is half a wavelength because while the first two waves canceled, there is nothing to cancel the third wave.

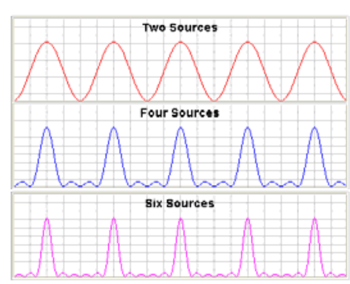
To get three waves to cancel, the path-length difference is one-third or two-thirds of a wavelength, not half a wavelength.



Adding slits

For light, adding sources really means adding slits (openings) for the laser light to pass through – each slit acts as a source.

By adding more slits, the interference maxima are much brighter, and a lot sharper.



A diffraction grating

Taking this adding slits business to an extreme, a diffraction grating consists of a large number of long slits on a glass or plastic slide. Diffraction gratings are commonly used in spectroscopy to separate light into its various colors, often to determine the chemical composition of the source of the light.

Because of the way they are made, CD's act like diffraction gratings.

Image credit: CSIRO, Australia

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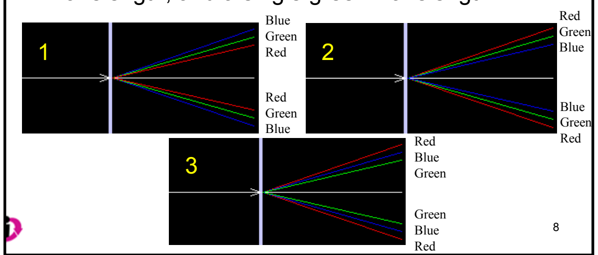
Diffraction grating, first order

Which picture shows correctly the first-order spectrum (m = 1) for a beam of light consisting of a single red wavelength, a single blue wavelength, and a single green wavelength?



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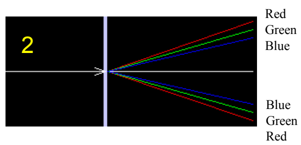
Diffraction grating, first order

For the diffraction grating, $d \sin(\theta) = m\lambda$.

Ranking the colors by increasing wavelength, we have blue, green, red. The longer the wavelength, the larger the angle.

Is this the same as what happens with a prism?



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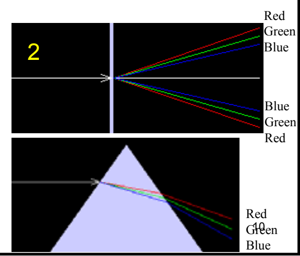
Diffraction grating, first order

For the diffraction grating, $d \sin(\theta) = m\lambda$.

Ranking the colors by increasing wavelength, we have blue, green, red. The longer the wavelength, the larger the angle.

Is this the same as what happens with a prism?

This is opposite to what happens with a prism.



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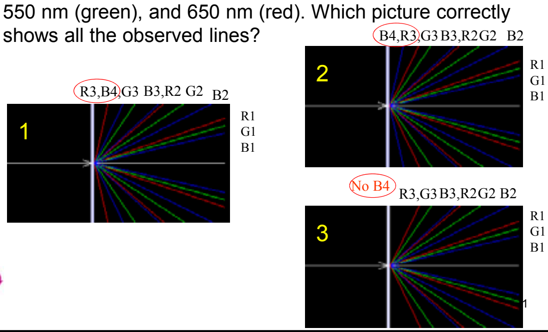
Diffraction grating, higher orders

A diffraction grating has a grating spacing of d = 2000 nm. The wavelengths in the beam of light are 450 nm (blue), 550 nm (green), and 650 nm (red). Which picture correctly shows all the observed lines?



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Diffraction grating, higher orders

For the diffraction grating, $\sin\theta = \frac{m\lambda}{d}$

Order	Blue (450 nm)	Green (550 nm)	Red (650 nm)
m = 1			
m = 2			
m = 3			
m = 4			

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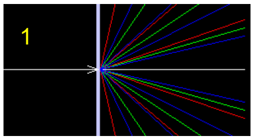
Diffraction grating, higher orders



Order	Blue (450 nm)	Green (550 nm)	Red (650 nm)
m = 1	sinθ = 450/2000	sinθ = 550/2000	sinθ = 650/2000
m = 2	sinθ = 900/2000	sinθ = 1100/2000	sinθ = 1300/2000
m = 3	sinθ = 1350/2000	sinθ = 1650/2000	sinθ = 1950/2000
m = 4	sinθ = 1800/2000	~~sinθ = 2200/2000~~	~~sinθ = 2600/2000~~

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Diffraction of Light

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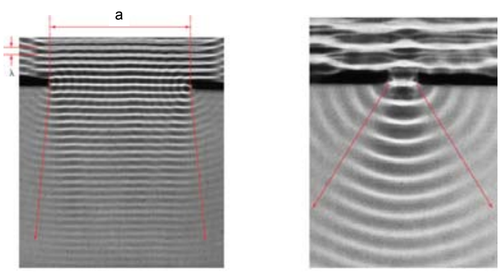
Diffraction



(a) Smaller value for λ/a, less diffraction.

(b) Larger value for λ/a, more diffraction.

The extent of the diffraction increases as the ratio of the wavelength to the width of the opening increases.

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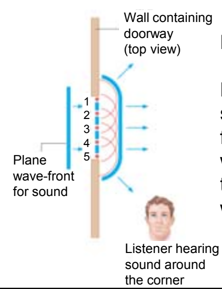
Diffraction – due to a single opening or object

Diffraction is the spreading out of a wave when it encounters a single object or opening.

Simulation



Huygens' principle

Every point on a wave front acts as a source of tiny wavelets that move forward with the same speed as the wave; the wave front at a latter instant is the surface that is tangent to the wavelets.

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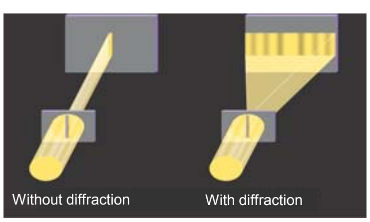
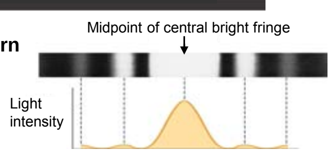
Observation of Diffraction



Interference pattern From diffraction:



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Diffraction vs. Double-Slit Patterns

Double-slit pattern:

Single-slit (or diffraction) pattern:

For double slits, we had derived expressions for the θ positions of the bright and dark fringes.

For single-slits, it turns out it would be straightforward only if we attempt to calculate the positions of the dark fringes.

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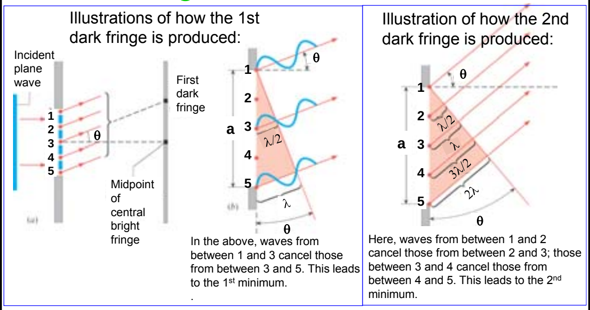
Dark Fringes in a Diffraction Pattern



The condition for destructive interference is:

$a \sin\theta = m\lambda$ or $\sin\theta = m(\lambda/a)$, $m = \pm1, \pm2, \pm3, \ldots$

Note that *m* cannot be zero, at where the central maximum is.

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Example: Width of central maximum

A slit with width w is used to observe diffraction from a light beam with wavelength λ. A screen is placed at a distance D from the slit to project the interference pattern. Find an expression for the width of the central maximum of the interference pattern in terms of w, D and λ. You may assume that $\lambda/d \ll 1$.

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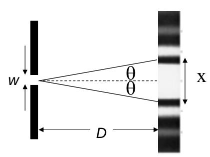
Example: Width of central maximum

Solution

The width of the central maximum of the interference pattern, x, is given by



$(x/2)/D = \tan\theta \Rightarrow x = 2D\tan\theta$

$w\sin\theta = m\lambda$ or $\sin\theta = m(\lambda/a)$, where $m = 1$ for the central maximum.

When $\lambda \ll d$, $\sin\theta$ is $\ll 1$. As a result, $\sin\theta \approx \tan\theta$. So, we have

$x = 2D\lambda/a$

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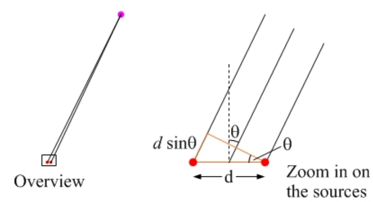
Recall the double-source equation

For two sources at a distance *d* apart, constructive interference occurs when

$d \sin(\theta) = m\lambda$

$m = 0, \pm1, \pm2, \ldots$

Question: How will diffraction modify the double source interference pattern?



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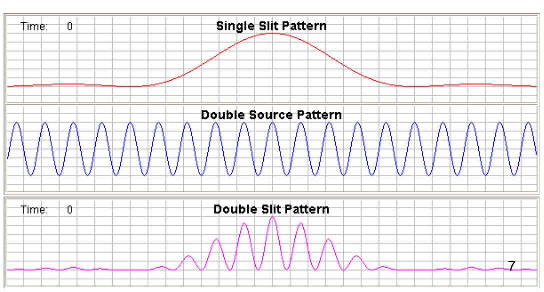
Realistic double-slit pattern

The realistic "double-slit" pattern is a combination of the single slit pattern and the double source pattern – formed by enclosing the double source pattern under an envelop of the single slit pattern.



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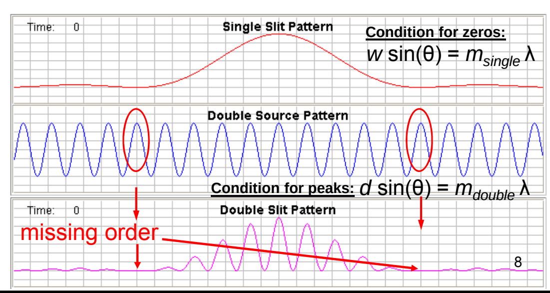
Realistic double slit pattern

The "Double Slit" pattern shows missing orders. The peaks predicted by the double-source equation are not present, because they coincide with the zeros in the single slit pattern.



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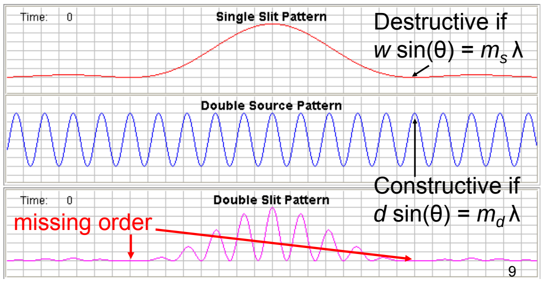
The double slit

What is the ratio of *d* to *a* in this double slit? (Hint: It's an integer less than 10.)



The double slit

$$\frac{d\sin\theta}{w\sin\theta}=\frac{m_d\lambda}{m_s\lambda} \Rightarrow \frac{d}{w}=\frac{m_d}{m_s}=\frac{5}{1}$$