Interference of Light

1

Phase of a Wave

In interference phenomena, we often compare the phase difference between two waves arriving at the same point from different paths or sources. The phase of a wave at a point at distance L from the source and time t is the argument inside the wave function describing the wave.

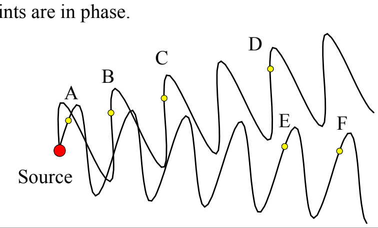
$$y = A\sin(\underbrace{kL \pm \omega t + \phi}_{\text{phase}}).$$

2

In Phase

The waves at two points are said to be in phase if their phase difference, $\Delta\Phi = 0, \pm 2\pi, \pm 4\pi, \ldots = 2m\pi$, where m is an integer.

For example, in the figure shown below, the phase of the waves at A, B, C, D, E, F is $\pi/4$, $\pi/4+2\pi$, $\pi/4+4\pi$, $\pi/4+8\pi$, $\pi/4+8\pi$, and $\pi/4+10\pi$, respectively. As a result, the phase difference between any two of them is a multiple of 2π, and the waves at all these points are in phase.

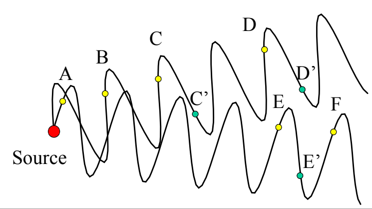


3

180º Out of Phase

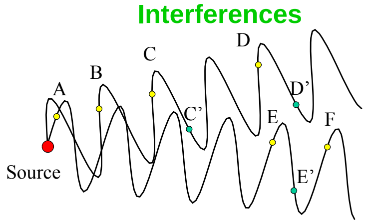
The waves at two points are said to be 180º out of phase if their phase difference is $(2m+1)\pi$ (or $\Delta\Phi = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$).

For example, in the figure shown below, the waves at points A, B, C, D, E and F are all 180º out of phase with the waves at points at C', D' or E'. Note however that the points C', D' and E' are in phase among themselves.



4

Constructive and Destructive Interferences



If we can manipulate or reflect the above two waves such that point A meets point C, for example, there will be **constructive** interference at the point where they meet at all times.

Similarly, if we can manipulate the above two waves such that point A meets point C', there will be **destructive** interference at that point at all times.

5

Coherent Sources

For constructive or destructive interference to be able to continue to occur at a point, the waves must come from **coherent sources**.

Two sources are coherent if the waves they emit have the same wavelength and maintain a constant phase difference, ϕ.

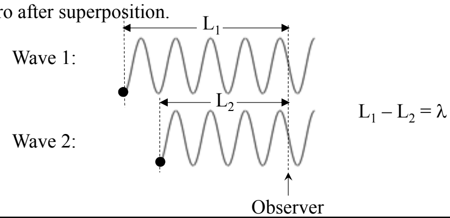
If the constant phase difference, ϕ, is zero, the two sources are said to be in phase.

If the constant phase difference, ϕ, is π, the two sources are said to be 180º out-of-phase.

6

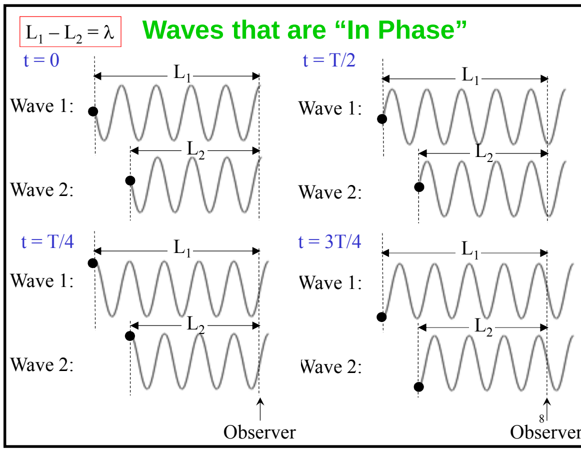
Waves that are "In Phase"

When two continuous waves with identical wavelength (λ) overlap at a point, their phase difference at that point is dictated by <u>the path length difference (PLD) as well as the phase difference between the two sources ϕ</u>. <u>That phase difference does not change with time</u> even though the waves are traveling. If the PLD is $\pm\lambda$, $\pm 2\lambda$, $\pm 3\lambda$, etc. and the sources are in phase, the two waves are **<u>in phase at ALL times (see example on next page)</u>**. Note that there can be constructive interference even if the net displacement of the waves is zero after superposition.



7

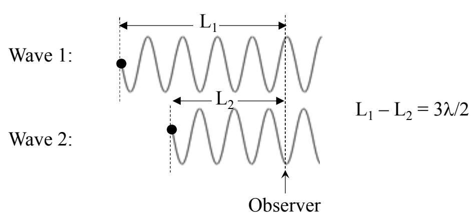
Waves that are "In Phase"



8

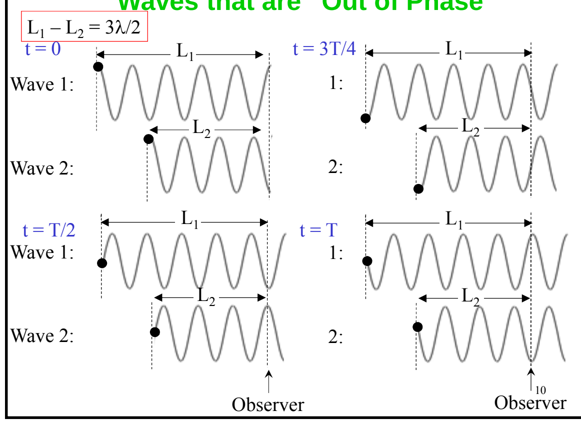
Waves that are "180º Out of Phase"

When the PLD is $\pm\lambda/2$, $\pm 3\lambda/2$, $\pm 5\lambda/2$, etc. and the sources are in phase, the two waves are **<u>180º out of phase at ALL times (see example below and on next page)</u>**.



9

Waves that are "Out of Phase"

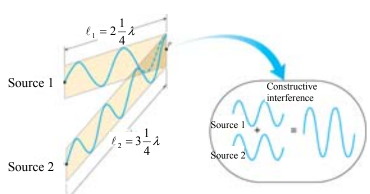


10

Conditions for constructive interference

When two or more light waves pass through a given point and arrive there in phase, **<u>constructive interference</u>** occurs. If the sources are in phase, the condition required for constructive interference is:

$$\text{PLD} = \ell_2 - \ell_1 = m\lambda \qquad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

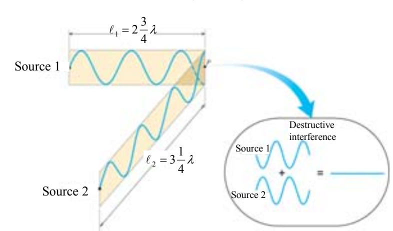


11

Conditions for destructive interference

When two or more light waves pass through a given point and arrive there out of phase, **<u>destructive interference</u>** occurs. If the sources are in phase, the condition required for destructive interference is:

$$\text{PLD} = \ell_2 - \ell_1 = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$



12

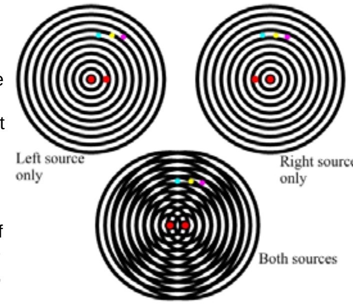
Conditions for constructive and destructive interference when the sources are "180° out-of-phase"

When the sources are 180° out-of-phase, there is an inherent phase difference of π between the two interfering waves even when the PLD is zero. This inherent phase difference of π is equivalent to an inherent PLD of $\lambda/2$. <u>So, the conditions for constructive and destructive interference for 180° out-of-phase sources are exactly reversed from those for sources that are in phase!</u>

13

Understanding interference pattern produced by two point sources

Consider circular waves spreading out from two coherent sources that are in phase and interfere. It is convenient to figure out the interference pattern by regarding the bright lines to be crests and the dark lines to be troughs. Therefore, they are out of phase, i.e., have a phase difference of π, 3π, 5π, 7π, etc.

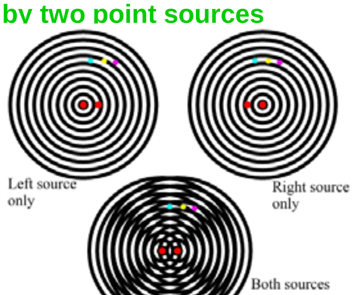


Simulation

14

Understanding interference pattern produced by two point sources

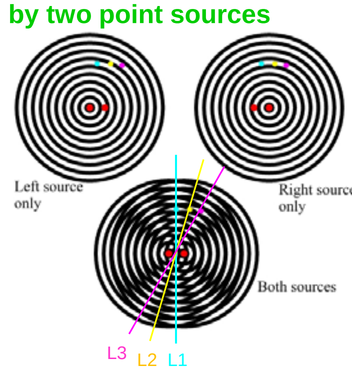
For the array of three points (blue on far left, yellow in the middle, magenta on far right) shown at right, the blue and magenta points will have constructive interference because they are where two bright lines (crests) overlap. But the yellow point will have destructive interference because it is where a dark line (trough) overlaps with a bright line (crest).



15

Understanding interference pattern produced by two point sources

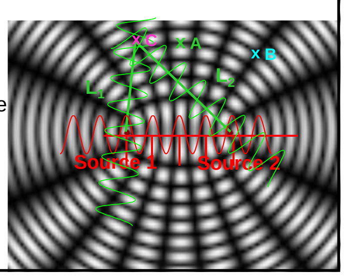
By using the above argument, we can deduces that all the other points lying on lines L1 and L3 should have constructive interference. Note that L1 is the perpendicular bisector of the line joining the two sources, so the PLD at any point on L1 should be zero. Similarly, all the points lying on line L2 have destructive interference.



16

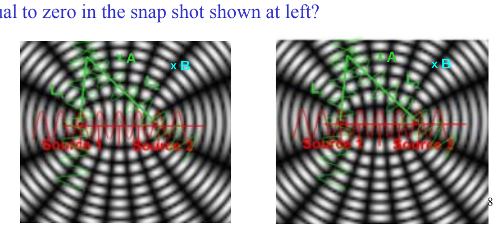
Interference from Two Sources

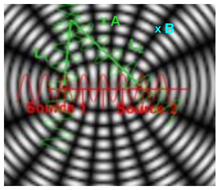
We have encountered interference from two sources for sound waves. There, we presented the interference pattern by an intensity map. Let's look at it again focusing on the directions along which there should be constructive and destructive interference. From the last slide, point C should have destructive interference. The green sine waves show that the PLD there is $3.5\lambda - 5\lambda = -1.5\lambda$. This account for the destructive interference. One can similarly show why there should be constructive interference at A and destructive Interference at B.



Understanding Interference from Two Sources

Below are two snap shots taken from the animated interference pattern in the last slide. From these snap shots, we see that the intensity at point A varies between zero and the maximum. But at point B, the intensity is zero at all times. Recall that there is completely constructive interference at A (PLD = 0), but completely destructive interference at B (PLD = $+5\lambda/2$). Why is the intensity at A equal to zero in the snap shot shown at left?



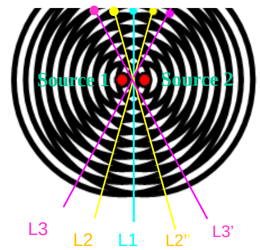


Why is the intensity at point A equal to zero in the snap shot shown on left?

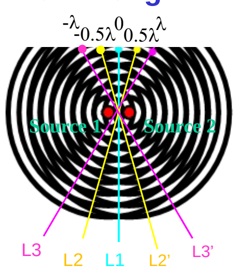
It is because the waves from both sources happen to have zero displacement at point A at that moment. By the principle of superposition, the resultant displacement is the sum of the displacement of individual waves. So, the resultant displacement at A in that snapshot is zero even though the two individual waves are in phase and interfere constructively.

Example: Path Length Difference

In the figure below, source 1 (left) and source 2 (right) are coherent and in phase. What is the path length different, $l_1 - l_2$, along L1, L2, L2', L3 and L3', respectively?



Example: Path Length Difference

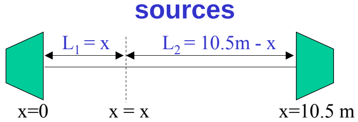


First, note that the PLD is positive for L1, L2 and L3, and negative for L1', L2' and L3'. Starting from the perpendicular bisector, the PLD must increase in the order 0, $\lambda/2$, λ as we move on to L2 then L3. Similarly, as we go from L1 to L2 to L3', the PLD must become increasingly negative in the order 0, $-\lambda/2$, and $-\lambda$.

Example: Standing wave between two sources

Two loudspeakers are arranged such that they face each other, with one located at x = 0 and the other at x = 10.5 m. Suppose they are coherent and emit sound waves with the same wavelength of 4 m. This sets up a standing wave in the space between the loudspeakers. Further suppose that the loudspeakers are in phase. Find all the positions for the antinodes and nodes.

Example: Standing wave between two sources



Consider a point at distance x from the first source. The path length from source 1 is $L_1 = x$, and that from source 2 is $L_2 = 10.5\text{ m} - x$.

There is constructive interference if $L_1 - L_2 = n\lambda$ $(n = 0, \pm1, \pm2, \ldots)$

$\Rightarrow x - (10.5 - x) = n\lambda$
$\Rightarrow 2x = 4n + 10.5$ (used $\lambda = 4$ m)
$\Rightarrow x = 5.25 + 2n$ $(n = 0, \pm1, \pm2, \ldots)$

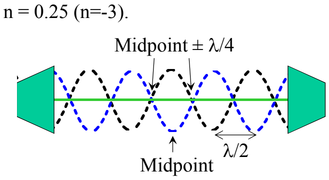
Since x must be between 0 and 10.5m, the allowed values of x are: 5.25m (n=0), 3.25m (n=-1), 5.25m (n=+1), 1.25m (n=-2) and 9.25m (n=+2). Note that these points are $\lambda/2$ apart. Can you explain why?

Example: Standing wave between two sources

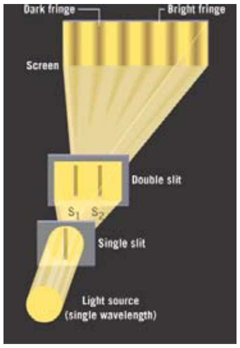
There is destructive interference if $L_1 - L_2 = (n+1/2)\lambda$ $(n = 0, \pm1, \pm2, \ldots)$

$\Rightarrow x - (10.5 - x) = (n+1/2)\lambda$
$\Rightarrow 2x = (n+1/2)(4) + 10.5$
$\Rightarrow x = 5.25 + (2n+1) = 6.25 + 2n$

Since x must be between 0 and 10.5m, the allowed values of x are: 6.25m (n=0), 4.25m (n=-1), 8.25m (n=+1), 2.25m (n=-2), 10.25m (n=+2), and n = 0.25 (n=-3).



Young's Double Slit Experiment



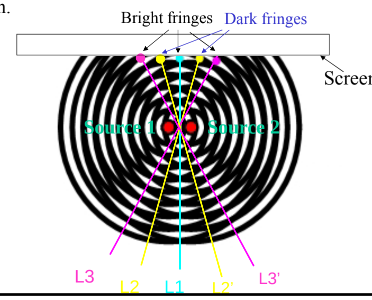
In Young's experiment, two slits acts as coherent sources of light.

Light waves from these slits interfere constructively and destructively on a distant screen.

25

Young's Double Slit Experiment

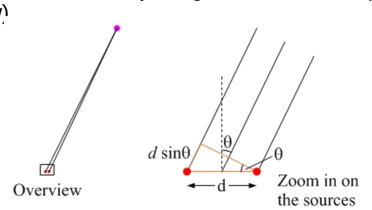
We can understand Young's Double Slit Experiment by considering the interference pattern we have been considering. Imagine if you put a screen at the position indicated below. You may expect alternating constructive and destructive interference fringes to form on the screen.



26

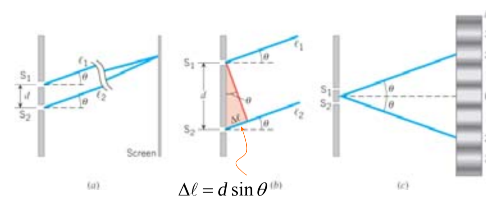
The double-source equation

For light, the two slits are usually several wavelengths apart, which is not that far. Let's call the distance between the sources *d*. For a point a long way from the sources, great simplification can be made. In particular, the path-length difference is approximately given by *d* sin*θ*, where *θ* is the angle measured from the direction perpendicular to the line joining the two sources (dashed line below)



27

The double-source equation



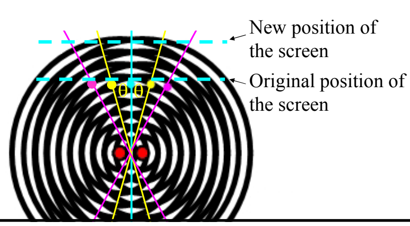
Bright fringes of a double-slit	$\sin\theta = m\dfrac{\lambda}{d}$	$m = 0,1,2,3,\ldots$
Dark fringes of a double-slit	$\sin\theta = \left(m+\tfrac{1}{2}\right)\dfrac{\lambda}{d}$	$m = 0,1,2,3,\ldots$

28

Increasing the distance between the source and the screen

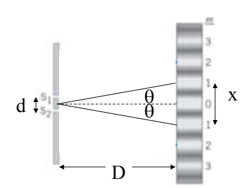
What happens to the pattern when you increase the distance between the source and the screen?

Ans. The widths of the fringes in the pattern increases. It is because the fringe separation is equal to the source-screen distance times tanθ.



Distance between two bright fringes

In a double slit experiment, the distance between the slits is d and that between the slits and the screen is D. Find an expression for the distance between the two first order maxima in terms of d, D and the wavelength of the light, λ. You may assume that λ/d << 1.

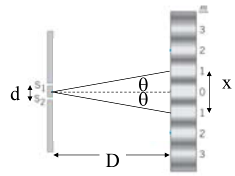


30

Distance between two bright fringes

Solution

The distance between the two first order maxima, x, is given by



(x/2)/D = tan θ ⇒ x = 2Dtanθ

$\sin\theta = m\dfrac{\lambda}{d}$, where m = 1 for first order maximum.

When λ << d, sinθ is << 1. As a result, sinθ ≈ tanθ. So, we have

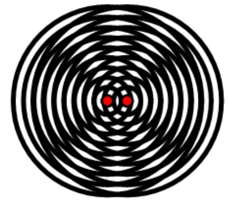
x = 2Dλ/d

31

Changing the wavelength

If the wavelength is decreased, what happens to the lines of constructive and destructive interference in the pattern?

1. They get farther apart
2. They get closer together
3. They stay the same

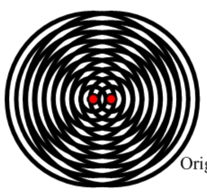


32

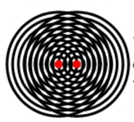
Changing the wavelength

$$\sin\theta = \frac{m\lambda}{d}$$

Decreasing the wavelength decreases the angles at which constructive and destructive interference happens, so the lines in the pattern get closer together.



Original



With decreased wavelength

33

Changing *d*

If the distance between the sources is decreased, what happens to the lines of constructive and destructive interference in the pattern?

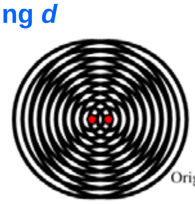
1. They get farther apart
2. They get closer together
3. They stay the same

34

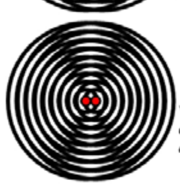
Changing *d*

$$\sin\theta = \frac{m\lambda}{d}$$

Decreasing *d* increases the angles at which constructive and destructive interference happens, so the lines in the pattern get farther apart.



Original



With decreased *d*

35