PY105 (C1)

1. Assignment 6 has been posted on WebCT.
2. Lecture notes can be downloaded from the following URL:
   http://physics.bu.edu/~okctsui/PY105.html

Energy Conservation

Work-Energy Theorem

$$W_{nc} + W_c = \Delta\text{KE}$$

$$W_c = \text{PE}_o - \text{PE}_f = -\Delta\text{PE}$$

$$\Rightarrow W_{nc} = \Delta\text{KE} + \Delta\text{PE}$$

$$= \Delta\text{E} \quad \text{(where E = KE + PE)}$$

So, $\boxed{W_{nc} = \text{E}_f - \text{E}_o}$

Or, $\boxed{\text{E}_f = \text{E}_o \text{ if } W_{nc} = 0}$

This means that if $W_{nc} = 0$, the system's ***mechanical energy*** (E = KE + PE) before and after a motion are the same. In physics, we say that the mechanical energy is conserved.

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

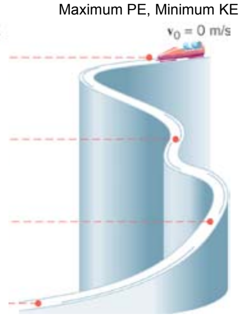
The ***total mechanical energy*** (E = KE + PE) of an object remains constant as the object moves, provided that the net work done by external nononservative forces is zero.

Example 1: A car moving down a hill.

If friction and air resistance can be ignored, $W_{nc} \approx 0$. This causes $E_f = E_o$ or $\Delta KE = -\Delta PE$

Maximum PE, Minimum KE

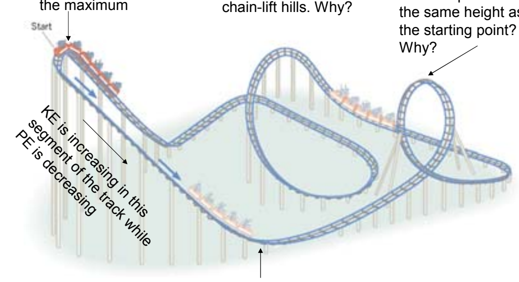
KE	PE	E = KE + PE
0 J	700 000 J	700 000 J
200 000 J	500 000 J	700 000 J
400 000 J	300 000 J	700 000 J
600 000 J	100 000 J	700 000 J



Minimum PE, Maximum KE

Example 2: A roller-coaster where the cars are not powered.

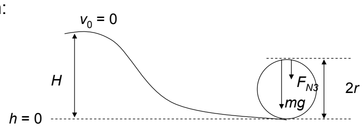
If $W_{nc} \approx 0$, $\Delta KE = -\Delta PE$



Example 3: Designing a loop for a roller coaster

Question: A roller coaster contains a loop with a radius of 10 m. Neglect friction, what is the minimum height of the starting point from the bottom of the loop if there the cars are not powered?

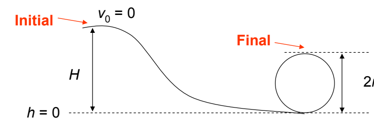
Solution:



For the cars to remain in contact with the track throughout the motion in the loop, the normal force at the peak position, F_{N3}, can be no less than zero. The condition, $F_{N3} = 0$ sets the minimum speed, v, required at the peak for the motion around to loop to complete (Please see the lecture notes on vertical circular motion.)

$F_{N3} = 0 \Rightarrow v_f^2/r = g \Rightarrow v_f = (rg)^{1/2}$

Example 3: Designing a loop for a roller coaster (cont'd)



Since there is no friction and the cars are not powered, $W_{nc} = 0$. Notice that the normal force does no work since it always makes an angle 90° with the displacement vector of the cars and so its work done is $F_N s\cos 90° = 0$.

$W_{nc} = 0 \Rightarrow KE_f + PE_f = KE_o + PE_o$.

We must now identify the initial and final position for the equation. Since the question concerns (1) the height H of the starting point of the rail and the condition for the circular motion to be successful (which concerns (2) the speed at the peak of the circular path), we choose the initial position to be the starting point of the rail and the final point to be the peak of the circular path.

Example 3: Designing a loop for a roller coaster (cont'd)

	Initial	Final
Speed	$v_0 = 0$	$v_f = (rg)^{1/2}$
Height	$h_0 = H$	$h_f = 2r$

$KE_f - KE_0 = (m/2)(v_f^2 - v_0^2)$ (m = mass of the whole train of cars)

$PE_f - PE_0 = mg(h_f - h_0)$

$KE_f + PE_f - (KE_o + PE_o) = 0 \quad (0 = W_{nc} = \Delta KE + \Delta PE)$

$\Rightarrow m/2(v_f^2 - v_0^2) + mg(h_f - h_0) = 0$

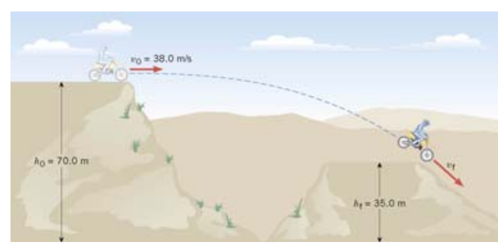
$\Rightarrow v_f^2 - v_0^2 = 2g(h_0 - h_f)$

$\Rightarrow rg/2 = g(H - 2r)$

$\Rightarrow H = 2.5rg = 2.5(10\text{m}) = 25$ m.

Example 4: A Daredevil Motorcyclist

A motorcyclist tries to leap across the canyon by driving horizontally off a cliff at a speed of 38.0 m/s. Ignore air resistance, find the speed with which the cycle strikes the ground on the other side.



Example 4: A Daredevil Motorcyclist (cont'd)

Solution:

	Initial	Final
Speed	v_0=38 m/s	v_f = ?
Height	h_0 = 70 m	h_f = 35 m

$\Delta KE = KE_f - KE_0 = m/2(v_f^2 - v_0^2)$

$\Delta PE = PE_f - PE_0 = mg(h_f - h_0)$

Ignoring air resistance, $W_{nc} = 0$ and so $\Delta KE + \Delta PE = 0$

$\Rightarrow m/2(v_f^2 - v_0^2) + mg(h_f - h_0) = 0$

$\Rightarrow v_f^2 = v_0^2 + 2g(h_0 - h_f)$

$\Rightarrow v_f^2 = (38\text{m/s})^2 + 2(9.8\text{m/s}^2)(70\text{m} - 35\text{m})$

$\Rightarrow v_f = 46.2$ m/s

Conceptual example 5: Sliding a block on a rough surface.

Question: A block with initial speed, v_0, slides on a rough surface until it comes to rest. What is the work done by friction?

Ans.

$$W_{nc} + E_o = E_f$$

The final KE is 0 because the block comes to rest in the end. So the non-conservative work done by friction must be minus the initial KE.

Conceptual example 5: Pushing a block on a rough surface.

Question: A block is pushed by a force, F, to move on a rough surface, where the kinetic frictional force between the block and the surface is f_k. What is the change in KE of the block if it has been pushed over a distance of s?

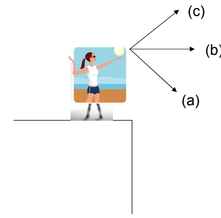
Ans.

The net force acting on the block during the displacement is $F - f_k$. We notice that this force is non-conservative. Therefore, $\boldsymbol{W_{nc} = (F - f_k)s}$.

There is no change in PE in this motion.

So $\boldsymbol{\Delta KE = W_{nc} = (F - f_k)s}$.

Conceptual example 6: Throwing a ball in three different ways down a cliff.



Question: Mary throws three balls down a cliff. Each time she imparts the same speed to the ball but directs it at a different angle relative to horizontal as shown above. When the ball hits the bottom of the cliff, which case will have the biggest final speed for the ball?

Choices:
1. (a)
2. (b)
3. (c)
4. The speeds are the same in all cases.

Other Forms of Energies

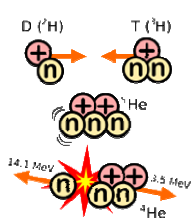
So far we have discussed only a very small sub-group of energies (including (1) kinetic energy and (2) work done due to nonconservative force and (3) potential energy). There are a vast number of other forms of energies such as heat, chemical energy (stored in our food), electrical energy, sound energy, light energy (that provides the energy required in photosynthesis – a process that green plant leaves synthesis glucose from CO_2 and water), energy due to the mass of a matter according to Einstein's equation, $E = mc^2$), *etc*. Together, they warrant that the total energy of the universe in any process is conserved.

The Principle of Conservation of Energy

Energy can neither be created not destroyed, but can only be converted from one form to another.

Example 7: Other Forms of Energy: Energy Release in a Fusion Reaction.

$^2H + ^3H \rightarrow n + 4He$



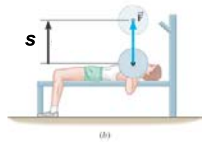
The total mass on the L.H.S. is not the same as the total mass on the R.H.S. The energy release in this reaction is Δmc^2. Since $c = 3\text{x}10^8$m/s. The energy release can be very large even though Δm may only be ~1% of a neutron mass. Assuming this mass loss per reaction, the corresponding energy release would be ~240 TJ/kg (Tera means 10^{12})

A Word about Nonconservative Work Done

Note that the word "non-conservative" carries no implication about whether the non-conservative work done is conserved or not. In general, one should never assume the colloquial meaning of a word that is used for a technical term in physics.

Conceptual example 7 Energies in weight lifting

The weight-lifter does positive work to the weight during the “up” cycle. But why is the speed of the weight equals to zero in the end of the up cycle?



The weight-lifter does negative work to the weight during the “down” cycle. This means that the weight’s total energy is reduced in the end. What is the major kind of energy reduced here?