Fluids

PY105

The post-test is available on WebCT. Please complete it before Dec. 10. It's worth 0.5 point no matter what your score is as long as you put your best effort in it. Note that 0.5 point corresponds to about 1.4 point on a test.

Fluids

Fluids are materials that can flow.

Formally, they are defined as materials that have no shear rigidity. In other words, you can deform it with zero shear force. As a result, they do not maintain a well-defined shape.

Liquids and gases are fluids.

Buoyant Force

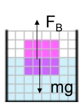
The buoyant force is an upward force exerted by a fluid on an object that is either fully or partly immersed in that fluid.

A Wooden Block

A wooden block with a weight of 100 N floats exactly 50% submerged in a particular liquid. The upward buoyant force exerted on the block by the liquid …

1. has a magnitude of 100 N
2. has a magnitude of 50 N
3. depends on the density of the liquid
4. depends on the density of the block
5. depends on both the density of the liquid and the density of the block

Ans. 100 N



Net force = 0
$\Rightarrow F_B = mg$

A Wooden Block, II

This example continues from example 1. Suppose a smaller wooden block with a weight of 50 N is added on top of the first block and the resulting stack (1 and 2) still floats on the liquid. The upward buoyant force exerted on the stack of blocks by the liquid …

1. has a magnitude of 50 N
2. has a magnitude of 100 N
3. has a magnitude of 150 N

Ans. 150 N, which is 50 N bigger than before.

Where does the extra buoyant force come from?

Note that the first block sinks more into the liquid after the smaller block is added. The extra buoyant force comes from an increase in the fraction of the first block immersed in the liquid.

Three beakers of liquids

The identical wooden block, with a weight of 100 N, floats in all three cases shown below, but a different percentage of the block is submerged in each case. In which case does the block experience the largest buoyant force?

25% submerged 80% submerged 50% submerged

Ans. The buoyant force is the same in all three cases.

What is the difference in the three liquids that give rise to the different submerged volumes?

The densities of the liquids are different. In the order of low to high density, 2 < 3 < 1.

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Mass Density

The mass density ρ is the mass m of a substance divided by its volume V:

$$\rho = \frac{m}{V}$$

SI Units of Mass Density: kg/m^3

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A table of mass densities

Material	Density (kg/m^3)
Interstellar space	10^{-20}
Air (20°C, 1 atm.)	1.21
Water (4°C, 1 atm.)	1000
Sun (average)	1400
Earth (the planet)	5500
Iron	8700
Mercury (the metal)	13600
Black hole	10^{19}

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Archimedes's Principle

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force F_B is:

$$F_B = W_{\text{fluid displaced}}$$

Where W_{fluid} is the weight of the fluid displaced by the portion of the object immersed in the fluid. If you know the mass density ρ_{fluid} of the fluid and the volume V_{immerse} of the object immersed in the fluid, you can find F_B:

$$F_B = \rho_{\text{fluid}} V_{\text{immerse}} g$$

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Archimedes's Principle

$$F_B = \rho_{\text{fluid}} V_{\text{immerse}} g$$

Since the maximum immersed volume V_{immerse} is the total volume of the object V_{tot}, the maximum buoyant force on an object by a fluid is:

$$F_{B,\max} = \rho_{\text{fluid}} V_{\text{tot}} g$$

For an object to float, $F_{B,\max} \geq mg = \rho_{\text{object}} V_{\text{tot}} g$. So the condition for an object to float on a fluid is:

$$\rho_{\text{fluid}} \geq \rho_{\text{object}}$$

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Fraction of Object Volume Immersed

At equilibrium, $F_B = mg$

$\rho_{\text{fluid}} V_{\text{immerse}} g$ $\qquad$ $\rho_{\text{object}} V_{\text{object}} g$

$$V_{\text{immerse}}/V_{\text{object}} = \rho_{\text{object}}/\rho_{\text{fluid}}$$

This is valid only if $\rho_{\text{fluid}} \geq \rho_{\text{object}}$ with which the fraction of volume immersed is less than 1.

If $\rho_{\text{fluid}} < \rho_{\text{object}}$, $F_B < mg$ even at 100% immersion and cannot support the object from sinking to the bottom of the water.

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Fraction of immersed volume, I

A rectangular block with mass density ρ_{object} is put into a fluid with a higher mass density of ρ_{fluid}. What fraction of the block's volume is immersed in the fluid?

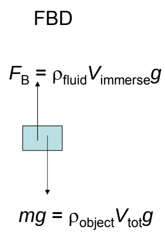
<u>Solution:</u>

Let V_{tot} be the total volume of the block.

Net force = 0

$\Rightarrow \rho_{object} V_{tot} g = \rho_{fluid} V_{immerse} g$

$$\Rightarrow \boxed{\frac{V_{immerse}}{V_{tot}} = \frac{\rho_{object}}{\rho_{fluid}}}$$



Fraction of immersed volume, II

For the same block and fluid in the last example, what fraction of the block's volume is immersed in the fluid if the block is partially supported by the tension *T* of a string?

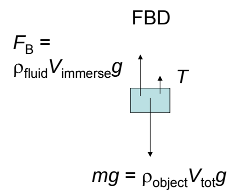
<u>Solution:</u>

Let V_{tot} be the total volume of the block.

Net force = 0

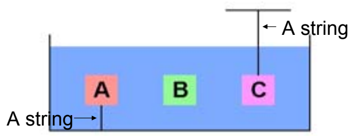
$\Rightarrow \rho_{object} V_{tot} g = \rho_{fluid} V_{immerse} g + T$

$$\Rightarrow \frac{V_{immerse}}{V_{tot}} = \frac{\rho_{object}}{\rho_{fluid}}\left(1 - \frac{T}{mg}\right)$$



m = mass of object = $\rho_{object} V_{tot}$

Three blocks, I

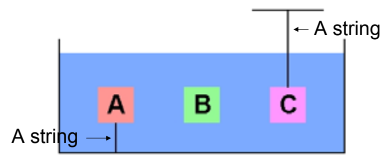


In the figure shown above, three blocks A, B and C with <u>identical volume</u> are totally submerged in the same fluid. It is given that the tensions in the string attached to block A and C are not zero. Which object has the largest buoyant force acting on it?

1. A 2. B 3. C
4. The cubes have equal buoyant force

Reason: $F_B = \rho_{fluid} V_{immerse} g$ is the same for all three blocks.

Three blocks, II



Continue from the previous example. Compare the density of the object with that of the fluid.

1. $\rho_{object} < \rho_{fluid}$ 2. $\rho_{object} = \rho_{fluid}$ 3. $\rho_{object} > \rho_{fluid}$

Three blocks, II

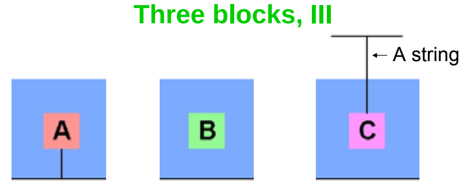
Ans. $\rho_A < \rho_B = \rho_{fluid} < \rho_C$

Reasoning:

Draw the FBD for each object. There, we can see that F_B = mg + T for A, F_B = mg for B and F_B = mg – T for C. Since F_B is the same for all three objects, $m_A < m_B < m_C$.

Moreover, because F_B = mg for B. So the density of B must be the same as the density of the fluid.

Three blocks, III



(choice 2) (choice 2) (choice 1)

Consider the same three objects as in the last example. Suppose we place them in three identical beakers of fluid, each with total weight, W, as shown in the figures above. Then we put a balance underneath each beaker. Which of the following statements is correct about the reading of the balance?

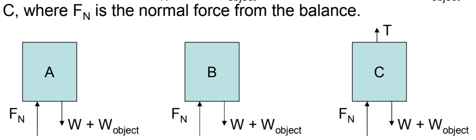
1. The reading is less than W plus the weight of the object
2. The reading is equal to W plus the weight of the object
3. The reading is bigger than W plus the weight of the object

Three blocks, III

Ans.: Balance reading $= W + W_{object}$ | Balance reading $= W + W_{object}$ | Balance reading $< W + W_{object}$

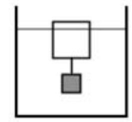
Reasoning:

Draw the FBD for the beaker+water+object system for each case. There, we can see that $F_N = W+W_{object}$ for A and B but is $W+W_{object}-T$ for C, where F_N is the normal force from the balance.



A Wooden Cube and a Metal Cube

A wooden cube measuring 20.0 cm on each side floats in water with 80.0% of its volume submerged. Suspended by a string below the wooden cube is a metal cube. The metal cube measures 10.0 cm on each side and has a density 5 times that of water.



(a) Which cube has a larger buoyant force acting on it?

[ ] the wooden cube [ ] the metal cube
[ ] neither, they're equal

(b) Taking the density of water to be 1000 kg/m³, what is the density of the wooden cube?

(c) What is the tension in the string between the cubes? Assume the string itself has negligible mass and volume.

(d) The pair of blocks is now placed in a different liquid. In this new liquid, the buoyant force acting on the wooden cube is exactly the same as the buoyant force acting on the metal cube. What is the density of this new liquid?

A Wooden Cube and a Metal Cube

(a) Which cube has a larger buoyant force acting on it?
[ ] the wooden cube [ ] the metal cube
[ ] neither, they're equal

Solution

The submersed volume of the wooden cube
$= (20\text{ cm})^3 \times 80\% = 6{,}400\text{ cm}^3$.

The submersed volume of the metal cube
$= (10\text{ cm})^3 = 1{,}000\text{ cm}^3$.

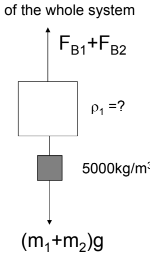
Since $F_{buoyant} \propto V_{immerse}$, the buoyant force acting on the wooden cube is bigger

A Wooden Cube and a Metal Cube

(b) Taking the density of water to be 1000 kg/m³, what is the density of the wooden cube?

Let V_1 and V_2 be the total volume of the wooden and metal cube, respectively:

FBD of the whole system



$F_{B1} + F_{B2} = (m_1+m_2)g = (\rho_1 V_1+\rho_2 V_2)g$ …(1)

$V_1 = (0.2)^3\text{m}^3 = 8(0.1)^3\text{m}^3 = 8V_2$

$V_{metal,immerse} = V_2 = (0.1)^3\text{m}^3$

$V_{wood,immerse} = (0.2)^3 \cdot 80\% = (0.2)^3(0.8)\text{m}^3 = 6.4V_2$

$(1) \Rightarrow (6.4V_2+V_2)\rho_{liq} g = (\rho_1 \cdot 8V_2+\rho_2 V_2)g$

$7.4 \cdot 1000V_2 = (\rho_1 \cdot 8V_2+5000 \cdot V_2)$

$7.4 \cdot 1000 - 5000 = \rho_1 \cdot 8$

$\rho_1 = 2400/8 = 300\text{ kg/m}^3$

A Wooden Cube and a Metal Cube

(c) What is the tension in the string between the cubes? Assume the string itself has negligible mass and volume.

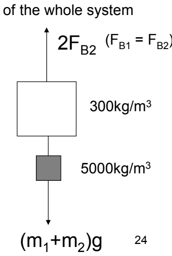
From part (b), $T = 4\rho_{water} V_2 g$
$= 4 \times 1000\text{ kg/m}^3 \times (0.1\text{ m})^3 \times 10\text{m/s}^2$
$= 40\text{ N}$

A Wooden Cube and a Metal Cube

(d) The pair of blocks is now placed in a different liquid. In this new liquid, the buoyant force acting on the wooden cube is exactly the same as the buoyant force acting on the metal cube. What is the density of this new liquid?

Solution

FBD of the whole system



$F_{B1} = F_{B2} \Rightarrow V_1=V_2=0.1^3$

$2F_{B2} = (m_1 + m_2)g$ ……. (1)

$m_1 = \rho_1(0.2)^3$

$m_2 = \rho_2(0.1)^3$

$(1) \Rightarrow 2\rho_{liq}(0.1)^3 = 300(0.2)^3+5000(0.1)^3$

$2\rho_{liq} = 2400 + 5000 = 7400\text{ kg/m}^3$

$\rho_{liq} = 3700\text{ kg/m}^3$