Rotational Kinetic Energy

Energy associated with rotation is given by an equation analogous to that for straight-line motion.

For an object that is moving but not rotating: \( K = \frac{1}{2}mv^2 \)

For an object that is rotating only: \( K = \frac{1}{2}I\omega^2 \)

For an object that is rolling, i.e., translating and rotating simultaneously, the total kinetic energy of such an object is:

\[
K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2
\]

Racing Shapes - Revisited

We have three objects, a solid disk, a ring, and a solid sphere, all with the same mass, \( M \) and radius, \( R \). If we release them from rest at the top of an incline, which object will win the race? Assume the objects roll down the ramp without slipping.

1. The sphere
2. The ring
3. The disk
4. It’s a three-way tie
5. Can’t tell - it depends on mass and/or radius.

What does this tell us?

\[
Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2
\]

Because the object rolls without slipping, we can use \( \omega = \frac{v}{R} \). Next, substitute \( I = cMR^2 \), where \( c = \frac{1}{2} \) for the disc, \( \frac{2}{5} \) for the sphere and \( \frac{1}{2} \) for the ring.

\[
Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(cMR^2)\left(\frac{v}{R}\right)^2
\]

Both the mass and the radius cancel out!

\[
g\frac{h}{2} = \frac{1}{2}v^2 + \frac{1}{2}cv^2
\]

Solving for the speed at the bottom:

\[
v = \sqrt{\frac{2gh}{1+c}}
\]
A spinning figure skater is an excellent example of angular momentum conservation. The skater starts spinning with her arms outstretched, and has a rotational inertia of $I_i$ and an initial angular velocity of $\omega_i$. When she moves her arms close to her body, she spins faster. Her moment of inertia decreases, so her angular velocity must increase to keep the angular momentum constant.

Conserving angular momentum:

\[ L_i = L_f \]
\[ I_i \omega_i = I_f \omega_f \]

**Question:** In this process, what happens to the skater’s kinetic energy?

The terms in brackets are the same, so the final kinetic energy is larger than the initial kinetic energy, because $\omega_i < \omega_f$.

Where does the extra kinetic energy come from?
The skater does work on her arms in bringing them closer to her body, and that work shows up as an increase in kinetic energy.

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**A ball rolling down a ramp**

**Solution:**

Conservation of mechanical energy gives $E_i + W_{nc} = E_f$

Initial mechanical energy,
\[ E_i = K_i + U_i = 0 + Mgh = Mgh \]

Final mechanical energy,
\[ E_f = K_f + U_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 + \frac{1}{2}(c/2)Mv^2 = (1+c)Mv^2/2 \]

To find $v_f$, we use
\[ v_f^2 = v_i^2 + 2as = 0 + 2[gsinθ/(1+c)][-h/sinθ] = 2gh(1+c) \]

\[ E_i + W_{nc} = Mgh - W_{nc} \]
\[ E_f = (1+c)Mv_f^2/2 = Mgh \]
\[ E_i + W_{nc} = E_f \] gives $W_{nc} = 0$

This shows that $f_s$ actually does no work on the ball!