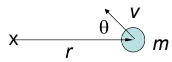
Angular momentum

The angular momentum of a spinning object is represented by L.



1. $\vec{L} = I\vec{\omega}$.

If the object is a point mass, $I = mr^2$ and $L = mr^2\omega = mrv\sin\theta$, where θ is the angle between v and r so that $v\sin\theta$ is the tangential velocity.

2. Angular momentum is a vector, pointing in the direction of the angular velocity.

3. If there is no net torque acting on a system, the system's angular momentum is conserved.

4. A net torque produces a change in angular momentum that is equal to the torque multiplied by the time interval during which the torque was applied.

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A Figure Skater

A spinning figure skater is an excellent example of angular momentum conservation. The skater starts spinning with her arms outstretched, and has a rotational inertia of I_i and an initial angular velocity of ω_i. When she moves her arms close to her body, she spins faster. Her moment of inertia decreases, so her angular velocity must increase to keep the angular momentum constant.

Conserving angular momentum:

$$\vec{L}_i = \vec{L}_f$$

$$I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

http://www.youtube.com/watch?v=9921LChDlBc&feature=related at 0:28, 1:04
http://www.youtube.com/watch?v=O9B4CBcFlYw at 2:17

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A Bicycle Wheel

Question: A person standing on a turntable while holding a bicycle wheel is an excellent place to observe angular momentum conservation in action. Initially, the bicycle wheel is rotating about a horizontal axis, and the person is at rest. Can you predict what happens when the person flips the wheel to bring the rotational axis vertical?

Solution: The initial angular momentum about a horizontal axis contributes no angular momentum to the turntable, which can rotate about a vertical axis only. If the person re-positions the bicycle wheel so its rotation axis becomes vertical, the person must spin (on the turntable) in the opposite direction to maintain the total angular momentum (about the turntable's rotational axis) zero at all times.

Flipping the bike wheel over makes the person spin in the opposite direction.

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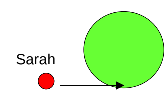
Jumping on a Merry-go-around

This is an example involving a rotational collision:

Question: Sarah, with mass m and velocity v, runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah and the merry-go-round (mass M, radius R, and $I = cMR^2$) then spin together with a constant angular velocity ω_f. If Sarah's initial velocity is tangent to the circular merry-go-round, what is ω_f?

Simulation



Solution:

What concept should we use to attack this problem?

Conservation of angular momentum.

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Jumping on a Merry-go-around

The system clearly has angular momentum after the completely inelastic collision, but where is the angular momentum beforehand?

It's with Sarah. Sarah's linear momentum can be converted to an angular momentum relative to an axis through the center of the merry-go-around. Here, we can treat Sarah as a point mass with mass m and initial velocity v_i.

$L_i = rmv_i\sin\theta$, where θ is the angle between r and v_i.

In this case, $\theta = 90^o$ and the angular momentum is directed counter-clockwise.

$L_i = Rmv_i\sin(90^o) = Rmv_i$

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Jumping on a Merry-go-around

Conserving angular momentum: $\vec{L}_i = \vec{L}_f$

Let's define counterclockwise to be positive.

$$+Rmv_i = +I_{tot}\omega_f$$

$$+Rmv_i = +(cMR^2 + mR^2)\omega_f$$

Solving for the final angular speed: $\omega_f = \dfrac{mv}{cMR + mR}$

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