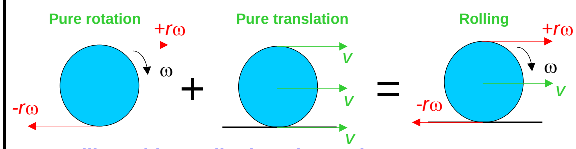
Rolling

Rolling

Rolling simulation

We can view rolling motion as a superposition of pure rotation and pure translation.



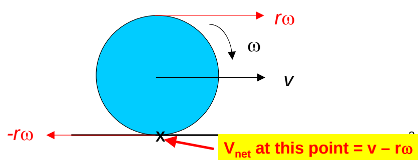
For rolling without slipping, the net instantaneous velocity at the bottom of the wheel is zero. To achieve this condition, $0 = v_{net}$ = translational velocity + tangential velocity due to rotation. In other wards, $v - r\omega = 0$.

When $v = r\omega$ (i.e., rolling without slipping applies), the tangential velocity at the top of the wheel is twice the translational velocity of the wheel ($= v + r\omega = 2v$).

Condition for Rolling Without Slipping

When a disc is rolling without slipping, the bottom of the wheel is always at rest instantaneously. This leads to $\omega = v/r$ and $\alpha = a/r$

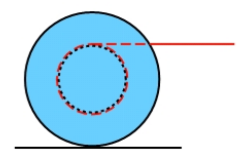
where v is the translational velocity and a is acceleration of the center of mass of the disc.



Big yo-yo

A large yo-yo stands on a table. A rope wrapped around the yo-yo's axle, which has a radius that's half that of the yo-yo, is pulled horizontally to the right, with the rope coming off the yo-yo above the axle. In which direction does the yo-yo move? There is friction between the table and the yo-yo. Suppose the yo-yo is pulled slowly enough that the yo-yo does not slip on the table as it rolls.

1. to the right
2. to the left
3. it won't move



Big yo-yo

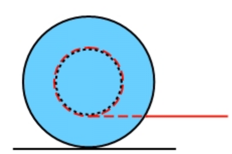
Since the yo-yo rolls without slipping, the center of mass velocity of the yo-yo must satisfy, $v_{cm} = r\omega$. With this, the direction of v_{cm} (= direction of motion) is determined entirely by whether the yo-yo rolls clockwise or counter-clockwise.

In this example, the tension on the rope produces a clockwise torque, which would cause the yo-yo to roll clockwise. So the yo-yo will move to the right.

Big yo-yo, again

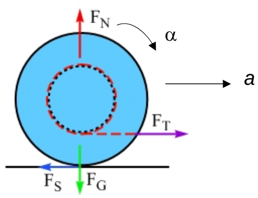
The situation is repeated but with the rope coming off the yo-yo below the axle. If the rope is pulled to the right, which way will the yo-yo move now? Again, suppose that there is no slip when the yo-yo rolls.

1. to the right
2. to the left
3. it won't move



Analyzing the yo-yo

The key to determining which way the yo-yo moves is to look at the torque due to the tension about the point of contact. All the other forces acting at the point of contact will contribute no torque about the point of contact.

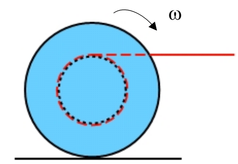


To realize this motion, the net torque about the center of the yo-yo (= the torque due to static friction, τ_{Fs}, plus the torque due to F_T, τ_{FT}) must be clockwise. Since τ_{FT} is counterclockwise, this means that τ_{Fs} must be clockwise and bigger than τ_{FT}. This requires F_S to be pointing left and bigger than $F_T/2$ (so that $F_S R > F_T R/2$). At the same time, F_T must be bigger than F_S in order to produce an acceleration that's pointing right. Altogether, $F_T > F_S > F_T/2$.

Big yo-yo

Given the axle of a yo-yo to be half the radius of the yo-yo, and the yo-yo moves a distance *L* to the right when the rope is pulled from above the axle, how far does the end of the rope move? Again, the yo-yo rolls without slipping.

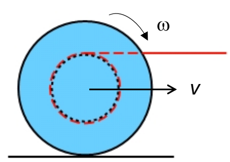
1. it doesn't move at all
2. *L*/3
3. *L*/2
4. 2*L*/3
5. *L*
6. 4*L*/3
7. 3*L*/2
8. 5*L*/3
9. 2*L*



The distance moved by the rope

Given the axle to be half the yo-yo's radius, a point on the outer edge of the axle has a rotational (tangential) speed equal to half the yo-yo's translational speed. Let's call the translational speed *v*. Above the axle, where the rope is unwinding, the net velocity is 1.5 *v* (= $v + \frac{1}{2} r\omega = v + v/2$).

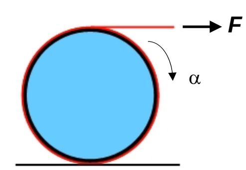
If the yo-yo moves a distance *L*, the end of the rope would move a distance 1.5 *L*.



An accelerating cylinder

A cylinder of mass *M* and radius *R* has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force *F*, what is the acceleration of the cylinder if the cylinder rolls without slipping? (Hint: Consider the direction of the frictional force and whether it will assist or resist *F*.)

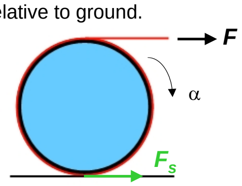
1. $a = \frac{F}{m}$
2. $a < \frac{F}{m}$
3. $a > \frac{F}{m}$



An accelerating cylinder

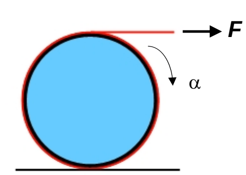
We would expect the frictional force to be pointing forward since the tensional force would produce a torque that rotates the cylinder clockwise, which produces a tendency for the bottom of the cylinder to move backward relative to the ground. Hence, the net force would be the sum of the frictional force and the tensional force (bigger than the tensional force along). Notice that the friction is static friction since there's no slip, which would mean that the bottom of the wheel is momentarily at rest relative to ground.

Simulation



An accelerating cylinder – Finding a and F_s

A cylinder of mass *M* and radius *R* has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force *F*, what is the acceleration of the cylinder if the cylinder rolls without slipping? What is the frictional force acting on the cylinder?



An accelerating cylinder – Finding a and Fs

Take positive to be right, and clockwise positive for torque.

The normal force cancels *Mg* vertically. Apply Newton's Second Law for horizontal forces, and for torques:

Forces	Torques
$\sum \vec{F}_x = M\vec{a}$	$\sum \vec{\tau} = I\alpha$
$+F + F_S = Ma$	$+RF - RF_S = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$
	$F - F_S = \frac{1}{2}Ma$

* Only for rolling without slipping can we use $\alpha = \frac{a}{R}$

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An accelerating cylinder – Finding a and Fs

Forces	Torques
$+F + F_S = Ma$	$F - F_S = \frac{1}{2}Ma$

Adding these two equations gives $2F = \frac{3}{2}Ma$,

which leads to the surprising result, $a = \frac{4F}{3M}$ (> F/m).

We can make sense of this by solving for the force of static friction. $F_S = +\frac{1}{4}Ma = +\frac{1}{3}F$

The positive sign means that our initial guess for the direction of the friction force being in the same direction as ***F*** is correct.

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Racing Shapes

We have three objects, a solid disk, a ring, and a solid sphere, all with the same mass, *M* and radius, *R*. If we release them from rest at the top of an incline, which object will win the race? Assume the objects roll down the ramp without slipping.

1. The sphere
2. The ring
3. The disk
4. It's a three-way tie
5. Can't tell - it depends on mass and/or radius.

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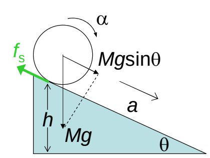
Racing Shapes

Question: For the situation considered in the previous question, find the frictional force for various shapes. Use your result to explain why the sphere should win the race.



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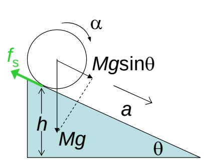
Racing Shapes

Forces	Torques
$Mg\sin\theta - f_s = Ma$	$f_sR = cMR^2\underbrace{(a/R)}_{\alpha}$
(2): $g\sin\theta - f_s/M = a$	(3): $f_s/cM = a$



(2) and (3) $\Rightarrow g\sin\theta - f_s/M = f_s/cM$

$\Rightarrow Mg\sin\theta = f_s(1/c+1)$

$\Rightarrow f_s = Mg\sin\theta/(1/c+1)$

Note that f_s, which produces the rotational motion of the objects, reduces the linear acceleration. From eqn. (3), the shape with a bigger moment of inertia requires a bigger f_s to produce the same angular and hence linear acceleration. So the sphere, with the smallest moment of inertia, accelerates the fastest down the incline.

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A race

If we take the winner of the rolling race (the sphere) and race it against a frictionless block, which object wins the race? Assume the sphere rolls without slipping.

1. The sphere
2. The block
3. It's a tie
4. Can't tell

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