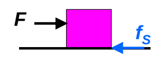
Review for Test 2

Test 2 will cover Ch. 5-10: Newton's Law with Friction, Circular Motion, ..., Rotational Kinematics, and basic concepts of Torques. There will be no questions on Static Equilibrium. The venue will be COM101: College of Communications Rm. 101, 640 Comm. Ave. It's in the short building just before the block where Starbucks and Warrens Tower is.)

Static Friction

Static friction (f_S) is the resistance force acting against an object to move from rest. If the force applied force (F) is less than the maximum static friction (f_S^{MAX}), which is a characteristic of the object and the surfaces in contact (discussed on next page),



$f_S = F$ if $F < f_S^{MAX}$.

With this, F is exactly balanced by f_S, so the object remains at rest. But when F exceeds f_S^{MAX}, we have

$f_S = f_S^{MAX}$ if $F > f_S^{MAX}$.

With this, there is a net force ($= F - f_S^{MAX}$) acting on the object and so the object will start to move.

Static Friction

It has been observed that the magnitude of f_S^{MAX} is proportional to the magnitude of the normal force, F_N, acting on the object, *i.e.*,

$$f_S^{MAX} = \mu_S F_N$$

where μ_S is a constant, known as the coefficient of static friction, which is characteristic of the contact between the object and the surface. Notice that μ_S has no unit, and f_S^{MAX} is independent of the area of contact between the object and the solid surface.

Kinetic (or Dynamic) Friction

Kinetic (or dynamic) friction (f_k) is the frictional force acting on an object when it is moving relative to a surface.

The magnitude of kinetic friction is also proportional to the magnitude of the normal force, F_N, acting on the object:

$$f_k = \mu_k F_N$$

Here, μ_k is the coefficient of kinetic friction. Kinetic friction is also independent of the area of contact between the object and the solid surface.

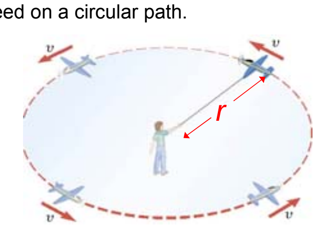
Newton's Second Law with Friction

Key concepts and skills you are assumed to know:

(1) how to draw a free-body-diagram
(2) how to apply the Newton's second law to a free-body-diagram (in both the x and y directions)
(3) how to express the normal force and hence the frictional forces in terms of the variables given in the equation, and have a good understanding of when to assume the friction to be the maximum static friction.
(4) how to solve the equations you obtained by combining (2) & (3), which would give you the acceleration of the system.
(5) Once you have the acceleration, you should know how to find the final position and velocity after a certain time.
(6) Graphical representation of the motion.

Definition of UNIFORM Circular Motion

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Centripetal acceleration, $a_c = v^2/r = r\omega^2$
Note that $v = r\omega$

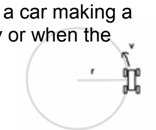
UNIFORM CIRCULAR MOTION

When to use $a_c = r\omega^2$?

We use it when ω is a constant (e.g. coins on a turntable, gravitron) or when the problem asks for ω or when the problem gives you its value.

When to use $a_c = v^2/r$?

We use it when v is a constant (e.g. a car making a turn) or when the problem asks for v or when the problem gives you its value.

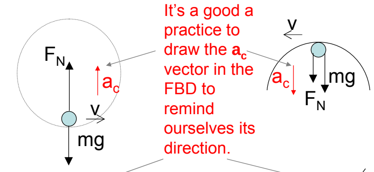


Vertical Circular Motion – Loop-the-loop

Here, centripetal acceleration, $a_c = v^2/r$ is still valid. However, v is **NOT** a constant.

At the bottom of the loop-the-loop:

At the top of the loop-the-loop:



It's a good a practice to draw the a_c vector in the FBD to remind ourselves its direction.

Choose "up" to be positive

Negative because a_c is pointing down.

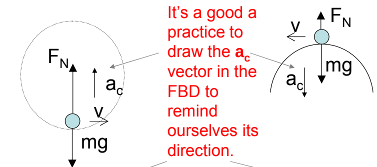
$F_N - mg = +mv^2/r$
$F_N = m(v^2/r + g)$

$-F_N - mg = -mv^2/r$
$F_N = m(-g + v^2/r)$

Vertical Circular Motion – Driving on a Hill

Driving at the bottom of a valley

Driving on top of a hill



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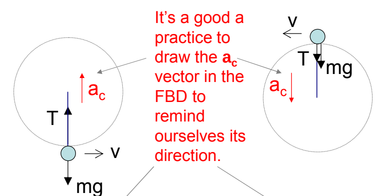
$F_N - mg = +mv^2/r$
$F_N = m(v^2/r + g)$

$F_N - mg = -mv^2/r$
$F_N = m(g - v^2/r)$

Vertical Circular Motion - Pendulum

At the bottom of a pendulum

At the top of a pendulum



It's a good a practice to draw the a_c vector in the FBD to remind ourselves its direction.

$T - mg = +mv^2/r$
$T = m(g + v^2/r)$

$-T - mg = -mv^2/r$
$T = m(v^2/r - g)$

Impulse

Definition: The impulse (J) of a force is the product of the average force and the time interval during which the force acts:

$$\mathbf{J} = \langle \mathbf{F} \rangle \Delta t$$

SI Unit is $kg \cdot m/s^2 \cdot s = kg \cdot m/s$

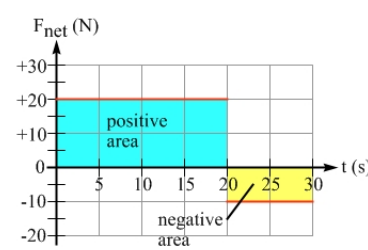
Given this definition, impulse is also the change in linear momentum.

$$\mathbf{J} = m\Delta v = \Delta p$$

Impulse = Change in momentum

The net force vs. time graph

The area under the net force vs. time graph gives the impulse or change in momentum.



Conservation of Linear Momentum

If no net external force (F_{net}) acts on a system (or if the system is isolated), the impulse ($=F_{net}\Delta t$) is zero. It follows that the total linear momentum of the system would be conserved.

$F_{net} = 0 \Rightarrow$ *P* <u>is a constant</u>

- This is **VERY** useful in analyzing collisions.

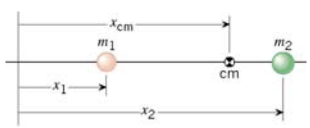
Center of Mass – in 1D

For a general system containing *N* masses,

$$P_{cm} = \frac{m_1 v_1 + m_2 v_2 + ... + m_N v_N}{m_1 + m_2 + ... + m_N} = \frac{\text{Total momentum}}{\text{Total mass}}$$

Center of Mass – in 1D

- The center of mass is a point that represents the average location for the total mass of a system.



If the system is consisted of only two masses,

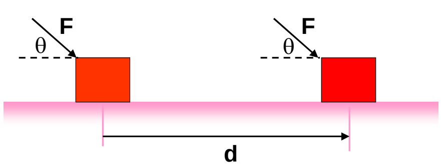
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Center of Mass – in 1D

For a general system containing *N* masses,

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + ... + m_N x_N}{m_1 + m_2 + ... + m_N}$$

Work Done by a Constant Force



In general, if the net force, **F**, makes an angle θ, with the displacement vector, **d**, the net work W by **F** is:

$W_{net} = F \cdot d\cos\theta$

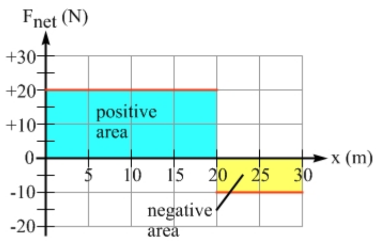
Kinetic Energy

The kinetic energy K of an object with mass *m* and velocity *v* is defined as:

$W_{net} = \Delta K = \Delta(\frac{1}{2} mv^2)$

The net force vs. position graph

The area under the net force vs. position graph represents the net work, W, which is also the change in kinetic energy, ΔK.



Gravitational Potential Energy

The potential energy U of an object with mass *m* situated at height *h* is:

U = mgh

In this equation, the absolute value of U depends on where zero height is chosen. However, in all problems we concern, only the change in height (and hence the change in U) matters so the choice of zero height is unimportant.

Non-Conservative Force & Work

In this course, the only kind of conservative force you encounter is gravitational force (mg). So, all forces other than mg are non-conservative. These include friction and forces applied by you, etc.

Non-conservative works (W_{nc}) are the works done by non-conservative works. They can be positive or negative. Typical negative non-conservative works arise from friction. Typical positive non-conservative works arise from external forces acted upon the object in its direction of motion.

Conservation of Mechanical Energy

$$W_{nc} = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i)$$

where W_{nc} is the work done by non-conservative forces (i.e., all forces except mg) on an object.

$$\Rightarrow \; U_i + K_i + W_{nc} = U_f + K_f$$

Five-term energy conservation equation, true in general

or $W_{nc} = E_f - E_i$ (where E = U + K is the mechanical energy.)

If the work done on an object by nonconservative forces is zero, its total mechanical energy **does not change**:

$$E_f = E_i$$

Conservation of mechanical energy, true only when $W_{nc} = 0$.

Collisions -- Conservation of Linear Momentum

All the collisions we encounter in this course involve isolated systems. Therefore, the law of conservation of linear momentum applies. That is,

P = constant, or

$m_{1i}v_{1i} + m_{2i}v_{2i} + \ldots = m_{1f}v_{1f} + m_{2f}v_{2f} + \ldots$

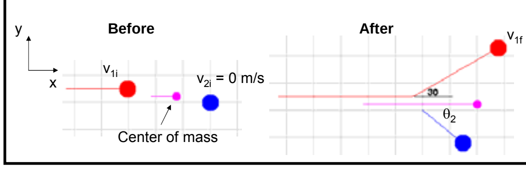
However, energy is not conserved in collisions in general.

Collisions in two dimensions

The Law of Conservation of Momentum applies in two and three dimensions, too. To apply it in 2-D, split the momentum into x and y components and keep them separate. Write out two conservation of momentum equations, one for the x direction and one for the y direction. That is,

$m_1v_{1,ix} + m_2v_{2,ix} + \ldots = m_1v_{1,fx} + m_2v_{2,fx} + \ldots$

$m_1v_{1,iy} + m_2v_{2,iy} + \ldots = m_1v_{1,fy} + m_2v_{2,fy} + \ldots$



Total Kinetic Energy

The total kinetic energy before collision is:

$K_i = (½)m_1v_{1,i}^2 + (½)m_2v_{2,i}^2 + …$

The total kinetic energy after collision is:

$K_f = (½)m_1v_{1,f}^2 + (½)m_2v_{2,f}^2 + …$

Elastic collision -- $K_f = K_i$

Super elastic collision – $K_f > K_i$

Inelastic collision – $K_f < K_f$

Completely inelastic collision – when the objects stick together after colliding. In that case, one often finds that $K_f << K_i$

Rotational variables

For rotational motion, we define a new set of variables that naturally fit the motion.

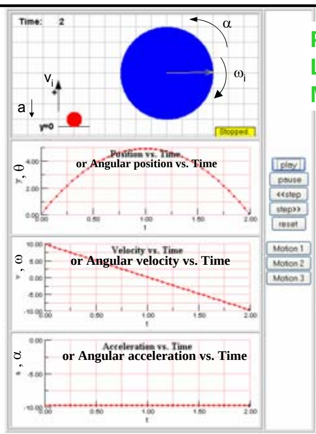
Angular position: θ , in units of radians. (π rad = 180°)

Angular displacement: $\Delta\vec{\theta}$

Angular velocity: $\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$, in units of rad/s.

For a direction, we often use clockwise or counterclockwise, but the direction is actually given by the right-hand rule.

Angular acceleration: $\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}$, in units of rad/s².



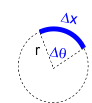
Parallel between Linear vs. Rotational Motions

It can not be distinguished which motion – the ball or the disk - the graphs in the simulation correspond to. This illustrates the parallel between linear motion under constant a and angular motion under constant α.

Analogy between 1D (tangential) and rotational motions

Below are several analogies between Linear motion variables and rotational motion variables.

Variable	Linear (tangential) motion	Rotational motion	Connec-tion
Displacement	Δx	$\Delta\theta$	$\Delta\theta = \frac{\Delta x}{r}$
Velocity	v	ω	$\omega = \frac{v_t}{r}$
Acceleration	a	α	$\alpha = \frac{a_t}{r}$



The subscript t stands for tangential.
Note that the variables above represent the magnitude of the respective vector quantity. Note also that θ is in rad, ω in rad/s and α in rad/s².

Constant acceleration equations

Straight-line motion equation	Rotational motion equation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(\Delta x)$	$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

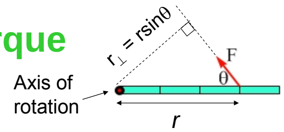
Don't forget to use the appropriate + and - signs!

Newton's Second Law for Rotation

Key concepts and skills you are assumed to know:

(1) Meaning of the equations of rotational motion and how apply them.
(2) Concept of "direction" in rotational motion
(3) Graphical representation of rotational motion.
(4) Draw the free body diagram of an object including a rod and disk involving torques.
(5) Determine the torque (both magnitude and direction) acting on an object from its free body diagram.

Torque



In short, **torque is a vector** with <u>magnitude</u> given by:

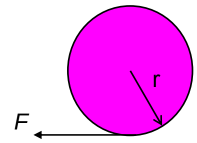
$$\tau = r\,F\sin\theta = Fr_\perp$$

where θ is the angle between ***r*** and ***F***.
Unit (SI): Nm

The <u>direction</u> of a torque (counterclockwise or clockwise) is determined by the direction of rotation the torque will cause an object to adopt from rest.

Example: Torque on a Disk

Assume the free-body-diagram shown at right. The torque acting on the disk is:



$\tau = Fr$

and it acts in the clockwise direction.