Rotational Kinematics

1

Rotational variables

For rotational motion, we define a new set of variables that naturally fit the motion.

Angular position: θ , in units of radians. (π rad = 180°)

Angular displacement: $\Delta\vec{\theta}$

Angular velocity: $\vec{\omega} = \dfrac{\Delta\vec{\theta}}{\Delta t}$, in units of rad/s.

For a direction, we often use clockwise or counterclockwise, but the direction is actually given by the right-hand rule.

Angular acceleration: $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t}$, in units of rad/s².

2

Right-hand rule for the conventional direction of the angular velocity vector, $\vec{\omega}$

Counterclockwise: +z

Clockwise: -z

The angular acceleration vector, $\vec{\alpha}$ and angular displacement vector, $\vec{\theta}$ follows this same convention.

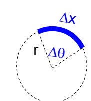
Note that this is a convention, not the rule. If a problem tells you to adopt clockwise to be positive, you should do as you are told.

3

Analogy between 1D (tangential) and rotational motions

Below are several analogies between Linear motion variables and rotational motion variables.

Variable	Linear (tangential) motion	Rotational motion	Connec-tion
Displacement	Δx	$\Delta\theta$	$\Delta\theta = \dfrac{\Delta x}{r}$
Velocity	v	ω	$\omega = \dfrac{v_t}{r}$
Acceleration	a	α	$\alpha = \dfrac{a_t}{r}$



The subscript t stands for tangential.
Note that the variables above represent the magnitude of the respective vector quantity. Note also that θ is in rad, ω in rad/s and α in rad/s².

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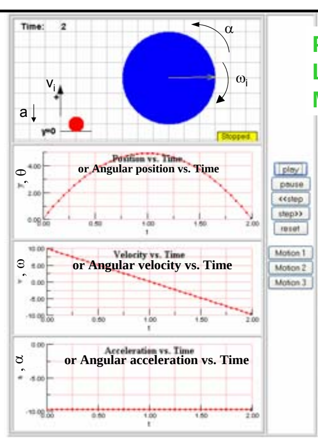

Parallel between Linear vs. Rotational Motions

It can not be distinguished which motion – the ball or the disk - the graphs in the simulation correspond to. This illustrates the parallel between linear motion under constant a and angular motion under constant α.

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Constant acceleration equations

Straight-line motion equation	Rotational motion equation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(\Delta x)$	$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

Don't forget to use the appropriate + and - signs!

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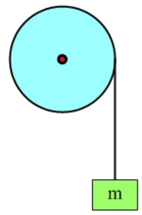
Rotation of a pulley

A large block is tied to a string wrapped around the outside of a large pulley that has a radius of 2.0 m. When the system is released from rest, the block falls with a constant acceleration of 0.5 m/s^2, directed downward.

What is the angular speed of the disk after 4.0 s?

What angle (in rad) does the disk rotate in 4.0 s?



Rotation of a pulley

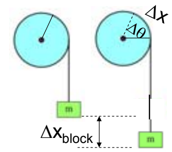
(a) What is the angular speed of the disk after 4.0 s?

The important thing to notice is that because the pulley and the block are connected by a string, the angular velocity of the pulley, ω, and the velocity of the block, v, are related by ω = v/r. Similarly, $\Delta\theta = \Delta x_{block}/r$. To find ω, we first find v:

a = 0.5 m/s^2

So v = 0 + at = (0.5 m/s^2)(4 s) = 2 m/s.

ω = v/r = 1 rad/s



(b) What angle does the pulley rotate in 4 s?

To find $\Delta\theta$, we first find Δx_{block}:

Obviously, $\Delta x_{block} = \Delta x$.
So, $\Delta\theta = \Delta x_{block}/r$

$\Delta x_{block} = v_0 t + at^2/2$
= 0 + (0.5 m/s^2)(4 s)^2/2
= 4 m.

$\Delta\theta = \Delta x_{block}/r$ = 2 rad

Ferris wheel

You are on a ferris wheel that is rotating at the rate of 1/(2π) revolution every second. The operator of the ferris wheel decides to bring it to a stop and so puts on the brake. The brake produces a constant acceleration of -0.1 radians/s^2.

(a) If your seat on the ferris wheel is 4 m from the center of the wheel, what is your speed when the wheel is turning at a constant rate, before the brake is applied? (Ans. 4 m/s)

(b) How long does it take before the ferris wheel comes to a stop? (Ans. 10 s)

(c) How many revolutions does the wheel make while it is slowing down? (Ans. 0.8 rev)

(d) How far do you travel while the wheel is slowing down? (Ans. 20 m)

Ferris wheel

Organization of the information:
- Radius, *r* = 4 m
- Pick the positive direction: the direction of motion
-Use a consistent set of units: The problem provides the value of the initial angular speed of the Ferris wheel in revolution per second. We need to convert into rad/s:

1/(2π) rev. per second = [1/(2π) rev/s] x [2π rad/rev] = 1 rad/s.

$\Delta\theta$	?	← Part (b)
ω_0	1 rad/s	
ω	0	
α	-0.1 rad/s^2	
t	?	← Part (c)

Ferris wheel

(a) The question asks about the tangential motion, use v = rω:

$$v_0 = r\omega_0 = 4\,\text{m} \times 1\,\text{rad/s} = 4\,\text{m/s}$$

(b) Use the equation: $\omega = \omega_0 + \alpha t$

Substitute the values of the variables organized in the table.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{(0-1)\,\text{rad/s}}{0.1\,\text{rad/s}^2} = 10\,\text{s}$$

(c) Use the equation: $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

Substitute the values of the variables organized in the table.

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(0\,\text{rad/s})^2 - (1\,\text{rad/s})^2}{2\times(-0.1\,\text{rad/s}^2)} = 5\,\text{rad} = \frac{5\,\text{rad}}{2\pi\,\text{rad/rev}} = 0.8\,\text{rev}$$

(d) It's the distance you travel along the circular arc. The arc length. Let the arc length be s:

$$s = r(\Delta\theta) = 4\,\text{m} \times 5\,\text{rad} = 20\,\text{m}$$

Front wheel of a bike.

While fixing the chain on your bike, you have the bike upside down. Your friend comes along and gives the front wheel, which has a radius of 30 cm, a spin. You observe that the wheel has an initial angular velocity of 2.0 rad/s, then comes to rest after 50 s.

Assume that the wheel has a constant angular acceleration. Determine how many revolutions the wheel makes.

Front wheel of a bike.

Question: Determine how many revolutions the wheel makes.

$\Delta\theta$	?
ω_0	2.0 rad/s
ω	0
α	Don't know
t	50 s

$\alpha = \dfrac{\omega - \omega_o}{t}$ = (-2.0 rad/s)/50s = -0.04 rad/s^2

$\Delta\theta = \omega_o t - \dfrac{1}{2}\alpha t^2$ = (0.5)(-0.04 rad/s^2)(2500s^2)

= 50 rad = $50 \text{ rad} \times \dfrac{1 \text{ rev}}{2\pi \text{ rad}} = \dfrac{25}{\pi}$ rev

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Front wheel of a bike.

An alternative method is to use the fact that the average angular velocity = $(\omega_o + \omega)/2$ = 1 rad/s is related to the average angular displacement $<\Delta\theta>$ by:

$$<\Delta\theta> = <\omega>t.$$

Over a time of 50 s, the wheel makes an angular displacement of 1.0 rad/s multiplied by 50 s, or 50 rad. The corresponding number of revolutions is:

$$50 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = \frac{25}{\pi} \text{ rev}$$

This is the same answer as before.

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