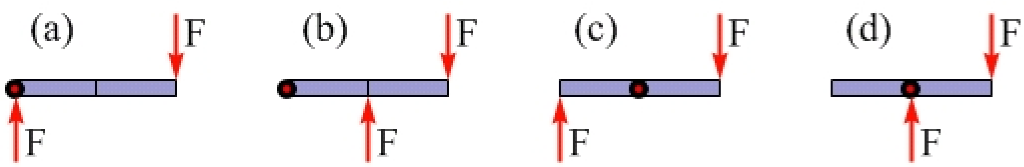
Solutions to Homework Set 9
Webassign
Physics 105

1) The figure below shows four different cases involving a uniform rod of length L and mass M is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. Rank these four situations based on the magnitude of the rod's angular acceleration, from largest to smallest. Use only > and/or = signs in your rankings, such as c>b=d>a.



Newton's second law for rotations states that the sum of the torques must equal the objects moment of inertia times the angular acceleration or,

$$\Sigma\tau = I\alpha$$

For the first two cases, the moment of inertia for a rod rotating about its end is given by

$$I_{end} = \frac{mL^2}{3}$$

Thus for part case (a) the acceleration is defined only by one torque equal to the force, F, acting at the full length of the rod, L.

$$\alpha_a = \frac{3F(L)}{mL^2} = \frac{3F}{mL}$$

For case (b), the acceleration is given by the sum of two torques, one acting at half the length and one at the full length.

$$\alpha_b = \frac{3(FL - \frac{FL}{2})}{mL^2} = \frac{3F}{2mL}$$

For the second two cases, the moment of inertia for a rod rotating about its center is given by

$$I_{center} = \frac{mL^2}{12}$$

The acceleration for case (c) is given by the sum of the two torques, both acting at half the length of the rod

$$\alpha_c = \frac{12(\frac{FL}{2} + \frac{FL}{2})}{mL^2} = \frac{12F}{mL}$$

Finally, the acceleration for case (d) is given by one force acting at half the length of the rod

$$\alpha_d = \frac{12(\frac{FL}{2})}{mL^2} = \frac{12F}{mL}$$

Hence, in order from largest to smallest: C>D>A>B

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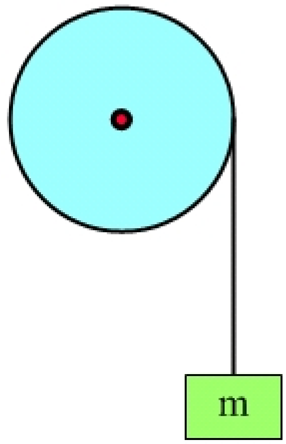
2)



The pulley shown above has a mass of 1.00 kg and radius R = 30.0 cm, and can be treated as a uniform solid disk that can rotate about its center. The block (which has a mass of m = 500 g) hanging from the string wrapped around the pulley is then released from rest. Use g = 10 m/s^2.
When the block has dropped through a distance of 3.00 m, what is the block’s speed?

NB: To distinctive methods exist for solving this problem. The first of which is by application of newtons second law. The second considers the energy of the system. Either are correct but this solution, for pedagogical reasons, will only only treat the former.

The goal of this approach is to find the acceleration of the block. Once found, the constant acceleration equations may be employed to find the final velocity after covering a distance of 'd' meters. It should be noted that the coordinate system where down and clockwise are positive is here adopted. So, in general, newton's second law for the block is:

$$\Sigma F = mg - T = ma$$

This acceleration must also be the tangential acceleration of the disk if the string is to unwind from the pully. Therefore the newton's second law for rotation may be applied to the pully:

$$\Sigma \tau = TRsin(\theta) = I\alpha$$

Where here, T is again the tension, R is the radius, I is the moment of inertia, and alpha is the angular acceleration. The force of tension acts tangent to the pulley and thus the angle between T and R is 90 degrees and the sin(90) is one.
The moment of inertia for a uniform disk is given by

$$I_{center} = \frac{MR^2}{2}$$

Hence the Tension force can be written as

$$T = \frac{MR^2\alpha}{2R} = \frac{MR\alpha}{2}$$

Re-writing this using the relation between the tangential accleration and the angular acceleration and plugging back into the equation for newtons second law for the block yields

$$mg - \frac{Ma}{2} = ma$$

Solving for the acceleration of the block results in

$$a = \frac{mg}{m + \frac{M}{2}}$$

Using the equation for constant acceleration where the initial velocity is zero:

The final velocity is

$$V_f = \sqrt{2a\Delta x}$$

$$V_f = \sqrt{\frac{2dmg}{m + \frac{M}{2}}}$$

or, for the values defined in this problem: V_f = sqrt(30)

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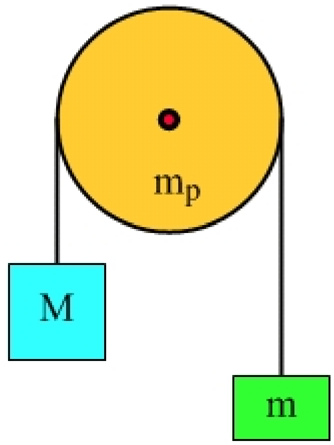
3)



Atwood's machine is a system consisting of two objects connected by a string that passes over a frictionless pulley, as shown in the figure above. Earlier in the course, we neglected the effect of the pulley, but now we know how to account for the pulley's impact on the system. The mass of the object on the left is M = 7.00 kg. The mass of the object on the right is m = 4.00 kg. The mass of the pulley is m_p = 2.00 kg. Use g = 10.0 m/s^2.

NB: For this problem, a coordinate system of down for M, up for m and counter clockwise for m_p is adopted to be positive.

(a) What is the magnitude of the acceleration of the system?

The acceleration of the system must be the acceleration of any one component of the system therefore, Newton's second law should be applied to each of the three bodies.

For M:

$$Mg - T_1 = Ma$$

For m:

$$T_2 - mg = ma$$

For m_p:

$$T_1R - T_2R = I\alpha = I\frac{a}{R}$$

This equation is certainly more complicated. The first term is the torque from M, the second term is the torque from m. The right hand side of the equation is found by using the relation between the angular and tangential acceleration. The moment of inertial for a disk is given by

$I = m_pR^2/2$

Now, taking the first two equations, solving for the corresponding tensions and plugging them into the third equation yields:

$$(Mg-Ma)R-(mg-ma)R=\frac{m_p R^2 a}{R}$$

Dividing through by R and solving for the acceleration gives:

$$a=\frac{g(M-m)}{M+m+\frac{m_p}{2}}$$

The exact value of this expression for the given masses above is a = 2 m/s^2

(b) What is the tension in the part of the string that is connected to the block of mass M?

Now that the acceleration is known, the tension for the string attached to block M is nothing more than

$$T_1=M(g-a)=Mg(1-\frac{M-m}{M+m+\frac{m_p}{2}})=7(8)=56N$$

(c) What is the tension in the part of the string that is connected to the block of mass m?

Similarly:

$$T_2=m(g+a)=mg(1+\frac{M-m}{M+m+\frac{m_p}{2}})=4(12)=24N$$

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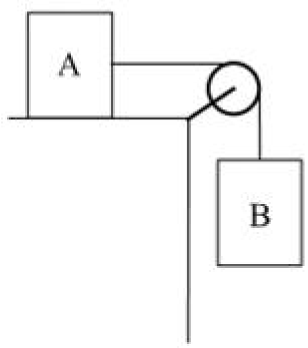
4)



As shown in the figure above, blocks A and B are connected by a massless string that passes over the outer edge of a pulley that is a uniform solid disk. Block A has a mass of 3.00 kg. Block B has a mass of 3.00 kg. The pulley has a mass of 4.00 kg. When the system is released from rest, it experiences a constant (and non-zero) acceleration. There is no friction between block A and the surface. Use g = 10.0 m/s^2.

NB: For this problem, a coordinate system of right for A, down for ***m*** *and clockwise for the pulley is adopted to be positive.*

(a) What is the acceleration of the system?

As with the previous problem, applying newtons second law to all three bodies is essential.

For A:

$$T_A=m_A a$$

For B:

$$m_B g-T_B=m_B a$$

For the Pulley:

$$T_B R - T_A R = I\alpha = \frac{m_p R^2 a}{2R} = \frac{m_p R a}{2}$$

Plugging in the tension forces from the first two equations into the third and dividing by R yields:

$$m_B(g-a) - m_A a = \frac{a m_p}{2}$$

Solving for the acceleration:

$$a = \frac{m_B g}{m_B + m_A + \frac{m_p}{2}}$$

or, for the specific question given above, a = 15/4 m/s^2

(b) What is the tension in the part of the string that is connected to block A?

From Newtons second law for A: T_A = (3)(15/4) = 45/4 N

(c) What is the tension in the part of the string that is connected to block B?

Again from Newtons second law: T_B = 75/4 N

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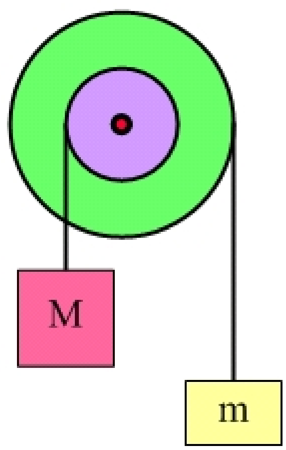
5)



A particular double pulley consists of a small pulley of radius 20 cm mounted on a large pulley of radius 50 cm, as shown in the figure. The pulleys rotate together, rather than independently. A block of mass m = 3.00 kg hangs from a string wrapped around the large pulley, while a second block of mass M = 8.00 kg hangs from the small pulley. Each pulley has a mass of 0.500 kg and is in the form of a uniform solid disk. Use g = 10 m/s^2.

NB: For this problem, a coordinate system of down for M, up for m and counter clockwise for both of the pulleys is adopted to be positive.

(a) What is the magnitude of the acceleration of the block attached to the large pulley?

First, it is important to find the total moment of inertia of the two pulleys. Since they rotate together, they should be treated as one body. $I_{total} = I_1 + I_2$. Since they are both solid disks, this total moment of inertia is found to be:

$$Itotal = \frac{m_1 R_1^2}{2} + \frac{m_2 R_2^2}{2} = \frac{1}{2}(m_1 R_1^2 + m_2 R_2^2)$$

Now, finding newtons second law for the two hanging masses and the combined pulley system:

$$Mg - T_1 = Ma_1$$
$$T_2 - mg = ma_2$$
$$T_1 R_1 - T_2 R_2 = I_{total}\alpha = \frac{(m_1 R_1^2 + m_2 R_2^2)\alpha}{2}$$

These equations are more complex than the previous cases because the accelerations are not the same for each of the hanging objects. But by writing the first two equations in terms of the angular accelerations and not the tangential ones, the third equation may then be used to solve for that quantity.

From the first two equations:

$$T_1 = M(g - \alpha R_1)$$
$$T_2 = m(g + \alpha R_2)$$

the third equation then becomes:

$$M(g - \alpha R_1)R_1 - m(g + \alpha R_2)R_2 = \frac{(m_1 R_1^2 + m_2 R_2^2)\alpha}{2}$$

Solving this equation for the angular acceleration yields:

$$\alpha = \frac{g(MR_1 - mR_2)}{R_2^2(m + \frac{m_2}{2}) + R_1^2(M + \frac{m_1}{2})} \approx 0.2432 rad/s$$

This expression is extremely useful in that the acceleration of the block on the big pulley is the radius of the big pulley times this value or 0.1216 m/s^2

(b) What is the magnitude of the acceleration of the block attached to the small pulley?

As with the previous problem, because we have the value of the angular acceleration, the tangential acceleration is easily computed as 0.0486 m/s^2