Assignment 4 Solutions
PY 105

1. For this problem it is important to remember that the force of friction always opposes motion, and that the force of static friction applies in situations where the object is not moving, while the force of kinetic friction applies in situations where the object is moving.

   (a) **The truck is moving at constant velocity to the right**: In this situation, since there's no acceleration, the object experiences no horizontal forces, so it acts just as it would just sitting on a table—that means that there will be no friction force applied, and the answer is $\boxed{\text{FBD 3}}$

   (b) **The truck is moving at a constant velocity to the left**: This is the same situation as (a), and because there are no horizontal forces, the same free-body-diagram is used, $\boxed{\text{FBD 3}}$

   (c) **The truck, while moving right, is speeding up**: We now have some acceleration! Intuitively, we all know from having ridden in cars all our lives that the object will feel a force to the left. Just remember, when a car accelerates, you feel like you're being pushed backwards into the seat. Because the box is predisposed to slipping to the left, the friction will act against that attempt to move by acting toward the right. Since the box isn't actually moving, we are dealing with static friction, so here the answer is $\boxed{\text{FBD 4}}$

   (d) **The truck, while moving right, is slowing down**: Again, remember that when a car brakes, you are pushed forward. Similarly, the block wants to move to the right, so the force of friction will oppose that motion and be oriented to the left. Since the block isn't moving, we are again dealing with static friction, so our answer is $\boxed{\text{FBD 1}}$

   (e) **The truck, while moving right, is stopping so quickly that the box slides over the floor of the truck**: We know that the box will scoot off to the right, due to inertia—it wants to keep going the way it had been going. Since it's moving, we know we're dealing with kinetic friction now. Since friction opposes motion, it's going to be directed towards the left, so our answer is $\boxed{\text{FBD 2}}$

   (f) **The truck, while moving right, is speeding up so rapidly that the box slides over the floor of the truck**: Again, inertia tells us that the box wants to keep going at the constant velocity it was going at before the truck started speeding up. That means it's going to shoot to the left, and the opposing kinetic friction will be in the right direction, so we use $\boxed{\text{FBD 5}}$

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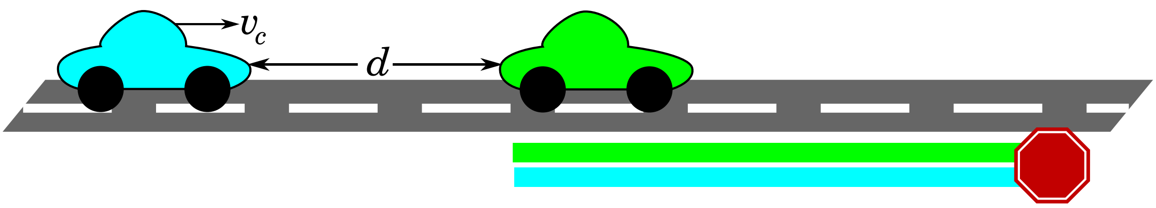
2. (a) Here we assume that the cars both brake in the same manner—that is, it takes them the same distance to come to a complete halt. That means that the follower has to apply his brakes at the same location that the leader started to brake at so that their bumpers are just kissing at the very end.



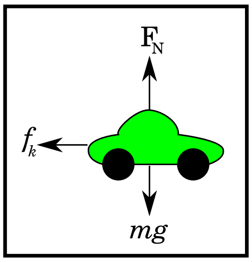
In the figure, the streaks below the road represent the braking distance of each car. So the following distance of the car has to be equal to the distance the driver travels before reacting. As the car has a constant velocity before applying brakes, this distance can be found using $v_0 = \frac{d}{t}$.

$$
\begin{aligned}
d &= v_0 t = 90 \text{ km/hr} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} 0.46 \text{ s} \\
&= 0.0115 \text{ km} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \\
&= \boxed{11.5 \text{ m}}
\end{aligned}
$$

(b) If the length of a car is 3.0 meters, d can easily be expressed in terms of car lengths:

$$d = 11.5 \text{ m} \cdot \frac{1 \text{ car length}}{3.0 \text{ m}} = \boxed{3.83 \text{ car lengths}}$$

(c) Now we must find the distance between the two cars at the instant you (the follower) apply the brakes. To do this, we have to understand the motion of the two vehicles. Since you have yet to apply your brakes, you are still going at a constant velocity v_0. However, the leader has already begun to brake, which means that a force is being applied to a car to slow it down. We can get a clear picture of what's going on using a force diagram.



Of course, in the vertical direction, we have the force due to gravity mg pulling the car down and the normal force F_N which the road pushes onto the car. In the horizontal direction we have the force due to kinetic friction, f_k, which is what is slowing the car down. We can then write Newton's Second Law in the horizontal direction to figure out the leading car's acceleration (taking right to be positive):

$$a = \frac{F_{net}}{m} = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$$

It becomes clear that the acceleration of the car is constant while it's braking. That means we can use the constant acceleration equations to solve our problem! First, it's useful to set $x = 0$ at the location where you apply your brakes. Let's list all the information we know in order to chose the proper constant acceleration equation to find Δx with.

- The initial velocity is v_0
- The acceleration is $a = \mu_k g$
- The time elapsed is t_R, as we're finding the distance the leading car travels from when it begins to brake until when you do, so that time interval is your reaction time.

With this information, the following equation works perfectly:

$$\begin{aligned}\Delta x &= v_0 t + \frac{1}{2}at^2 = v_0 t_R - \frac{1}{2}\mu_k g t_R^2 \\ &= 25 \text{ m/s } \cdot 0.46 \text{ s} - \frac{1}{2} 0.87 \cdot 10 \text{ m/s}^2 \cdot (0.46 \text{ s})^2 \\ &= \boxed{10.58 \text{ m}}\end{aligned}$$

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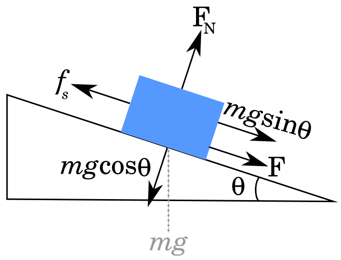
3. (a) **The force directed parallel to the slope**: Let's first draw a diagram for this situation:



Why do we choose to direct the force to the right? We're trying to minimize the force that we need in order to make the block slide. Pushing it up the ramp would basically be pushing it uphill while pushing it down would be pushing the block downhill, which is easier. Physically, the force is joining with a component of gravity to act against static friction, while if the force were directed up the ramp, it would be fighting both gravity and static friction (because static friction always opposes motion and would be directed downwards if we tried to push the block upwards).

Alright, now we should balance forces in order to discover what F is. Since we are trying to minimize F to get the block moving, we know that it should balance with the maximum possible static friction force, $f_{s,\max}$.

So, Newton's second law in the "horizontal" direction gives us:

$$F + mg\sin\theta - f_{s,\max} = 0$$

And in the "vertical" direction it gives us:

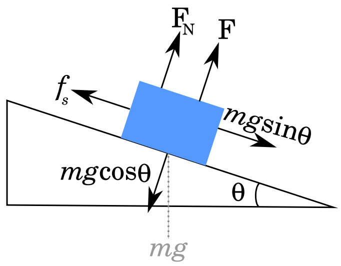
$$F_N - mg\cos\theta = 0$$

Putting this together (and remembering $f_m = \mu_s F_N$) gives us:

$$\begin{aligned}F &= f_{s,\max} - mg\sin\theta = \mu_s F_N - mg\sin\theta \\ &= \mu_s mg\cos\theta - mg\sin\theta = mg(\mu_s\cos\theta - \sin\theta) \\ &= 3 \text{ kg } \cdot 9.8 \text{ m/s}^2 [0.7 \cdot \cos(20^\circ) - \sin(20^\circ)] \\ &= \boxed{9.28 \text{ N}}\end{aligned}$$

(b) **The force directed perpendicular to the slope**:

We will essentially follow the exact same steps as in part (a), which were to draw the force diagram, apply Newton's second law in the two directions, and then to solve for F.



Here we choose the force to come out of the slope rather than down into it. Intuitively, if you press an object against a surface, it'll tend to stick more (you can even keep an object from falling down by pressing it against a vertical wall), so that seems to be the wrong direction to push it.
Let's apply Newton's 2nd law "vertically":

$$F_N + F = mg\cos\theta$$

And now Newton's 2nd law "horizontally":

$$mg\sin\theta = f_s$$

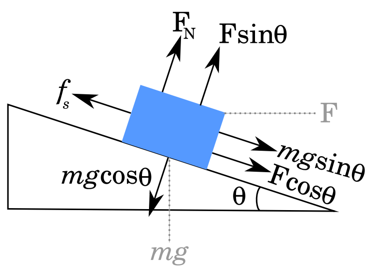
Again, we have the situation where $f_s = fs,_{\text{max}} = \mu_s F_N$ as we are trying to get the block to move.

$$mg\sin\theta = fs,_{\text{max}} = \mu_s f_N = \mu_s(mg\cos\theta - F)$$

Solving for F gives us:

$$\begin{aligned} F &= mg\cos\theta - \frac{mg\sin\theta}{\mu_s} = mg\left[\cos\theta - \frac{1}{\mu_s}\sin\theta\right] \\ &= 3\text{ kg}\cdot 9.8\text{ m/s}^2\left[\cos(20^\circ) - \frac{1}{0.7}\sin(20^\circ)\right] \\ &= \boxed{13.26\text{ N}} \end{aligned}$$

(c) **The force directed horizontally**:
Here we have to break the force up in order to make it jive with our angled coordinate system.



Newton's 2nd law, "vertically":

$$F_N + F\sin\theta = mg\cos\theta$$

Newton's 2nd law, "horizontally":

$$F\cos\theta + mg\sin\theta = f_s$$

Again, in this situation, we use $f_{s,\max} = \mu_s F_N$:

$$F\cos\theta + mg\sin\theta = \mu_s F_N = \mu_s(mg\cos\theta - F\sin\theta)$$

Now this just becomes an algebra problem—we want to solve for F:

$$F\cos\theta + \mu_s F\sin\theta = \mu_s mg\cos\theta - mg\sin\theta$$

$$F(\cos\theta + \mu_s\sin\theta) = mg(\mu_s\cos\theta - \sin\theta)$$

$$\begin{aligned} F &= mg\frac{\mu_s\cos\theta - \sin\theta}{\cos\theta + \mu_s\sin\theta} = 3\text{ kg} \cdot 9.8\text{ m/s}^2\frac{0.7\cos(20^\circ) - \sin(20^\circ)}{\cos(20^\circ) + 0.7\sin(20^\circ)} \\ &= \boxed{7.87\text{ N}} \end{aligned}$$

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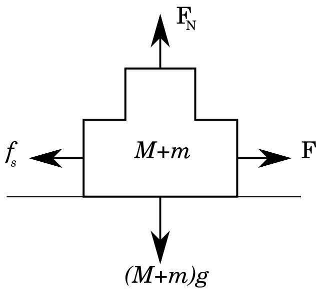
4. (a) To find the force needed to get the system moving, we can look at the system as a whole.

Force body diagram:



Our force must balance with $f_{s,\max}$ in order to start the system moving. Newton's 2nd law in the horizontal yields: $F - f_{s,\max} = 0$

In the vertical: $F_N - (M+m)g = 0$

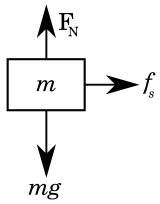
Altogether:

$$\begin{aligned} F &= f_{s,\max} = \mu_s F_N = \mu_s(M+m)g \\ &= 0.3(14\text{ kg} + 5\text{ kg}) \cdot 10\text{ m/s}^2 \\ &= \boxed{57\text{ N}} \end{aligned}$$

(b) In order to find the minimum force to keep the system moving at a constant velocity, all we have to do is change from f_s to f_k, as now we're dealing with a kinetic system.

$$\begin{aligned} F &= f_k = \mu_k(M+m)g = 0.2(14\text{ kg} + 5\text{ kg}) \cdot 10\text{ m/s}^2 \\ &= \boxed{38\text{ N}} \end{aligned}$$

(c) When you are beginning to push the larger block, the smaller block doesn't want to move (due to inertia). If you're standing on the big block, it will look like the small block is slipping to the left, so the force of friction will be to the right, as shown below.



That means that the acceleration of the small block in the horizontal direction is due entirely to its static friction. When the static friction maxes out ($f_s = \mu_s F_N$), so will the block's acceleration, so that's when it will start to slide off the big block.

$$f_{s,\max} = \mu_s F_N = \mu_s mg$$

From Newton's 2nd law, we know $f_{s,\max} = ma$, which gives us $a = \mu_s g$

Now we need to find the largest F such that the acceleration of the entire system doesn't exceed this value. From Newton's 2nd Law (again):

$$F - f_k = (M+m)a$$

And solving for F gives us:

$$\begin{aligned} F &= f_k + (M+m)a = \mu_k F_N + (M+m)a = \mu_k (M+m)g + (M+m)\mu_s g \\ &= (M+m)g(\mu_k + \mu_s) \\ &= (14 \text{ kg } + 5 \text{ kg}) \cdot 10 \text{ m/s}^2 \cdot (0.3 + 0.2) \\ &= \boxed{95 \text{ N}} \end{aligned}$$

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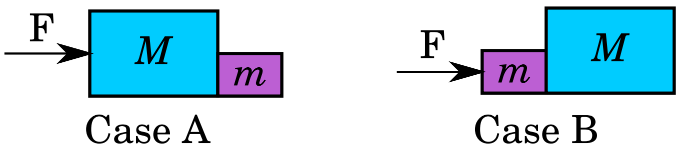
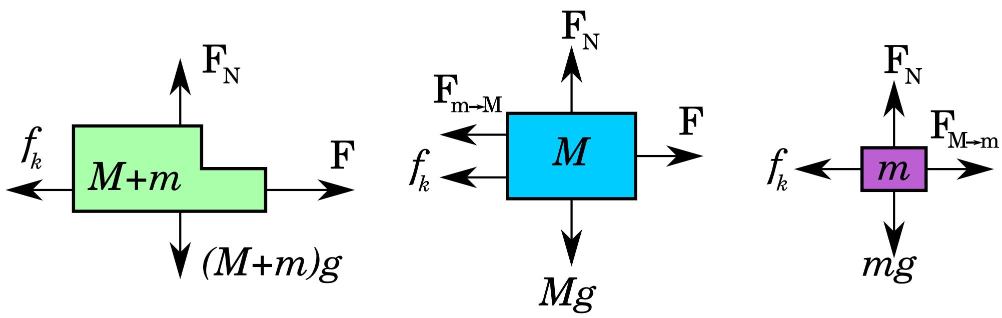
5. Two sliding boxes



(a) **What is the magnitude of the force exerted by the larger box on the smaller box in case (a)** To figure this out, we do the following:

- Draw the force body diagram for the entire system and the two boxes
- Figure out the acceleration of the entire system (using Newton's 2nd law)
- Use Newton's 2nd law on either the large or small box to find the internal force (noting that Newton's 3rd law tells us that $F_{M\to m} = F_{m\to M}$)

So, the force body diagrams! Note that we are using kinetic friction, as the system is moving.



Now let's calculate the acceleration using the FBD of the entire system and Newton's 2nd law:

$$F - f_k = (M+m)a$$

$$\begin{aligned} a &= \frac{F - f_k}{M+m} = \frac{F - \mu_k F_N}{M+m} \\ &= \frac{F - \mu_k g(M+m)}{M+m} \\ &= \frac{120 \text{ N} - 0.4 \cdot 10 \text{ m/s}^2 \cdot (9 \text{ kg})}{9 \text{ kg}} \\ &= 9.33 \text{ m/s}^2 \end{aligned}$$

I'm going to use the FBD of the smaller box to find the internal force, but using the larger one works as well. From Newton's 2nd law applied to the small box, we find:

$$F_{M\to m} - f_k = ma$$

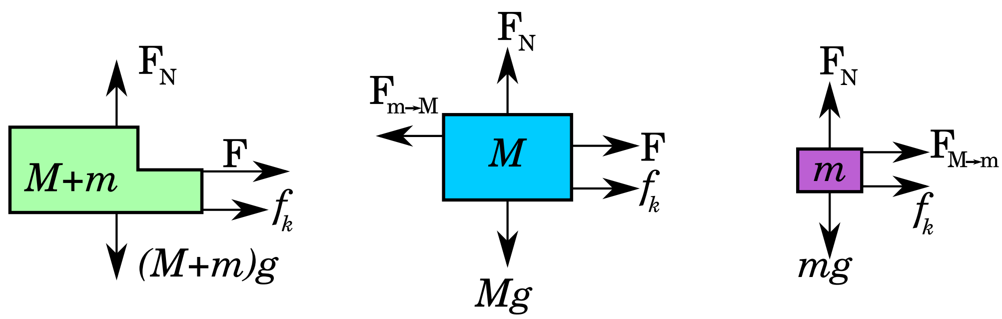
And solving for $F_{M\to m}$ yields:

$$\begin{aligned} F_{M\to m} &= f_k + ma = \mu_k F_N + ma = \mu_k mg + ma \\ &= 0.4 \cdot 3 \text{ kg} \cdot 10 \text{ m/s}^2 + 3 \text{ kg} \cdot 9.33 \text{m/s}^2 \\ &= \boxed{40 \text{ N}} \end{aligned}$$

(b) **Find the magnitude of the acceleration of the system and $F_{M/rightarrowm}$ if the boxes are initially moving to the left**

If the boxes are moving to the left, the frictional force will oppose the motion, so it will be directed to the right, which means it'll actually be aiding the force! As such, we should expect a higher acceleration.

New force diagrams:



Like above, we will apply Newton's 2nd law to the entire system in order to find the acceleration of the blocks:

$$F + f_k = (M + m)a$$

$$\begin{aligned} a &= \frac{F + f_k}{M + m} = \frac{F + \mu_k F_N}{M + m} \\ &= \frac{F + \mu_k (M + m) g}{M + m} \\ &= \frac{120 \text{ N} + 0.4 \cdot (9 \text{ kg}) \cdot (10 \text{ m/s}^2)}{9 \text{ kg}} \\ &= \boxed{17.3 \text{ m/s}^2} \end{aligned}$$

Now, to find the magnitude of the internal force, we can look at the free body diagram of the small block and write Newton's 2nd law, solving for $F_{M \to m}$:

$$F_{M \to m} + f_k = ma$$

$$\begin{aligned} F_{M \to m} &= ma - f_k = ma - \mu_k F_N = ma - \mu_k mg \\ &= 3 \text{ kg} \cdot 17.33 \text{ m/s}^2 - 0.4 \cdot 3 \text{ kg} \cdot 10 \text{ m/s}^2 \\ &= \boxed{40 \text{ N}} \end{aligned}$$

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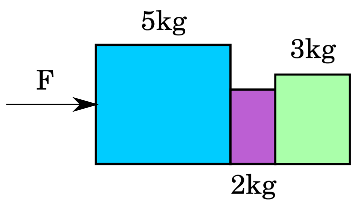
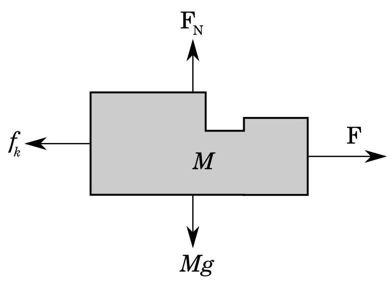
6. Three blocks being pushed by a force, with friction.



(a) Find the magnitude of the acceleration

We do this as above, by looking at the three blocks as one system, drawing the force diagram, and then using Newton's 2nd Law to find the acceleration.

Force body diagram:



We are using kinetic friction as the system is moving, and it is directed to the left as the system is moving to the right. Let's call the sum of all the masses $M = 10$ kg.

$$F - f_k = Ma$$

$$\begin{aligned} a &= \frac{F - f_k}{M} = \frac{F - \mu_k F_N}{M} = \frac{F - \mu_k M g}{M} \\ &= \frac{F}{M} - \mu_k g \\ &= \frac{50 \text{ N}}{10 \text{ kg}} - 0.2 \cdot 10 \text{ m/s}^2 \\ &= \boxed{3 \text{ m/s}^2} \end{aligned}$$

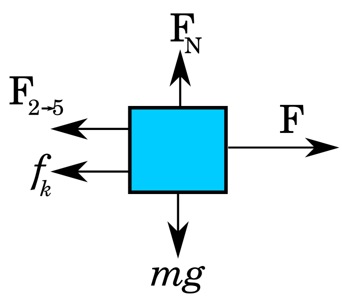
(b) **What's the magnitude of the net force acting on the 2 kg block?**

We can figure this out quickly by realizing that block 2 must accelerate by the same amount as the same system, and then using Newton's 2nd law directly:

$$F_{net} = ma = 2\text{kg} \cdot 3\text{m/s}^2 = \boxed{6 \text{ N}}$$

(c) **What is the magnitude of the force $F_{5\to 2}$ exerted by the 5 kg block onto the 2 kg block?**

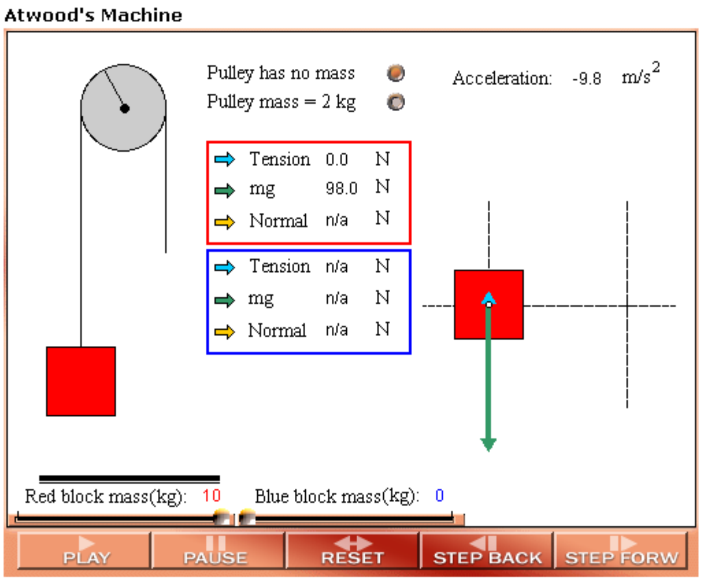
We could look at the force body diagram of the 2nd block, but there are two unknown forces there, which are $F_{5\to 2}$ and $F_{3\to 3}$. Instead, we'll use Newton's 3rd law, that each action has an equal and opposite reaction, and realize that $F_{2\to 5} = -F_{5\to 2}$. This allows us to look at the force body diagram of the first block which only has one unknown force, $F_{2\to 5}$. We can apply Newton's 2nd law there, as we already know the block's acceleration, to find the internal force.

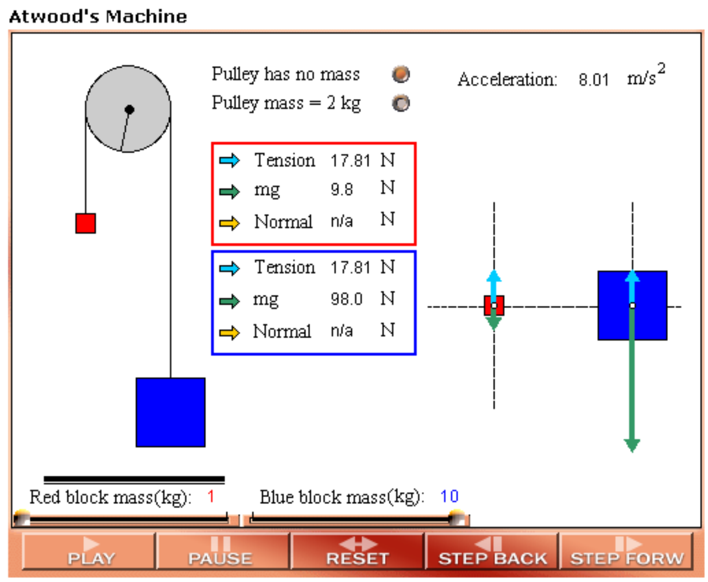


$$F - F_{2\to 5} - f_k = ma$$

$$\begin{aligned} F_{2\to 5} &= F - f_k - ma = F - \mu_k F_N - ma \\ &= F - \mu_k m g - ma \\ &= 50 \text{ N} \; - 0.2 \cdot 5 \text{ kg} \cdot 10 \text{ m/s}^2 - 5 \text{ kg} \cdot 3 \text{ m/s}^2 \\ &= \boxed{25 \text{ N}} \end{aligned}$$

7. (a) There are three correct statements to choose. These are the first, second, and last statements. Actually, the second statement, “Increasing the mass of the larger block results in an increase in the magnitude of the acceleration of the system when it is released from rest.” is true almost all the time – it is true as long as both blocks have a non-zero mass.





One way to find the answers to the acceleration questions is to simply play with the simulation. The acceleration is negative when the red box accelerates down and the blue box accelerates up. To produce the largest acceleration in this direction, we can set the red box to its largest mass (10 kg) and the blue box to its smallest mass (0 kg). In this case, the red box is simply in free-fall, so the acceleration is that due to gravity. This gives an acceleration of –9.8 m/s^2, as shown in the picture on the left, below.

The acceleration is positive when the red box accelerates up, and the blue box accelerates down. To produce the largest positive acceleration, then, we want to set the mass of the red box to its smallest value (1 kg) and the mass of the blue box to its largest value (10 kg). According to the simulation, this gives an acceleration of +8.01 m/s^2, as shown in the picture on the right, above.

We can also find the answers by analyzing the system. Using the simulation’s coordinate system, with the acceleration being positive when the red block accelerates up and the blue one accelerates down, we can apply Newton’s second law to the free-body diagrams that apply when the system is released from rest.

Red block: $F_T - mg = ma$

Blue block: $Mg - F_T = Ma$

Add the equations: $Mg - mg = Ma + ma$

Solve for acceleration: $a = \frac{M - m}{M + m} g$

To get the most negative acceleration, we use the smallest value for the mass of the blue block (M = 0). Because M = 0, the equation reduces to $a = -g$, and with g = 9.8 m/s^2, we get a = –9.8 m/s^2. It doesn't matter what the mass of the red block is set to.

To get the most positive acceleration, we use the largest value for the mass of the blue block (M = 10 kg) and the smallest value for the mass of the red block (m = 1 kg). Solving for the acceleration in this case gives:

$a = \frac{9}{11} g = 8.018 \text{ m/s}^2$.