Solutions to HW 1
PY 105

1)
The animation above shows the interesting pattern generated via the addition of two rotating vectors. In the animation, the longer vector has a constant length of 6 units, while the shorter vector has a constant length of 3 units. The pattern is then confined to the space bounded by two circles - the inner circle has a radius of 3 units, and the outer circle has a radius of 9 units.
Let's say we were to re-draw the pattern, but this time the longer vector has a length of 11 units, and the shorter vector has a length of 5 units. In this case,

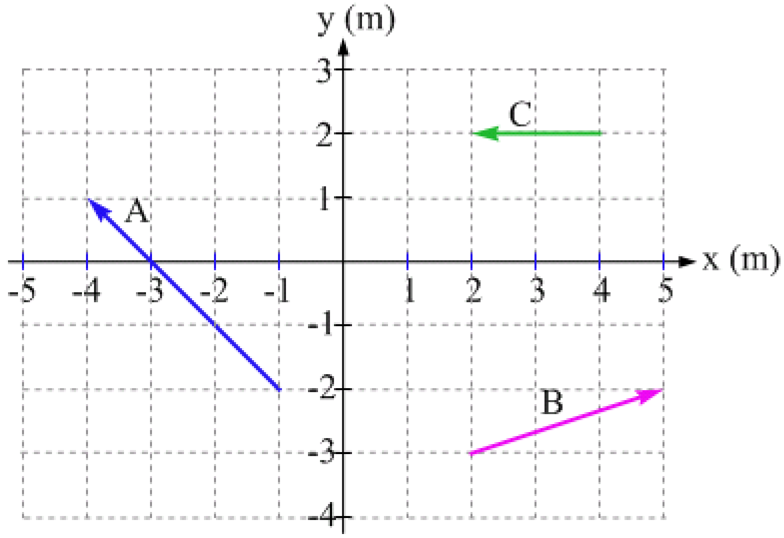
(a) what is the radius of the inner circle?

Define the length of the longer of the two vectors to be R_L and the length of the shorter to be R_S. Then, from the illustration, the minimum distance away from the origin occurs when R_L and R_S are opposite in direction, therefore the total length of the resultant vector is the difference or $R_{inner} = R_L - R_S$.
In the specific case illustrated above, this is $6 = R_{inner} = 11 - 5$.

(b) what is the radius of the outer circle?

Applying the same reasoning as in part (a), the maximum radius, Router, occurs when the two radii are aligned. That is $R_{outer} = R_L + R_S$ or here $11 + 5 = 16$.

2)
Three vectors are shown in the figure below, along with an x-y coordinate system.



Define the vector **R** to be the vector sum of the three vectors **A**, **C**, and the negative of vector **B**.
In equation form, **R** = **A** - **B** + **C**.

(a) Find the magnitude of the vector **R**:

To find the magnitude of the resultant, the x and y components must be found. The total x is simply the sum of the individual x components or:

$X_{total} = A_x - B_x + C_x = (-3) - (3) + (-2) = -8$

In similar fashion:

$Y_{total} = A_y - B_y + C_y = (3) - (1) + (0) = 2$

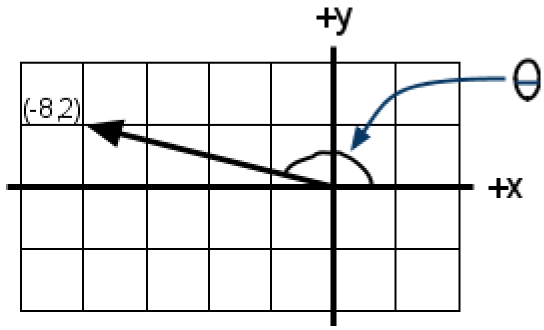
The Magnitude of the resultant from the Pythagorean Theorem is:

$R = (X_{total}^2 + Y_{total}^2)^{1/2}$ or in this case $(64 + 4)^{1/2} = (68)^{1/2}$ which has a numerical value of

R = 8.25 m

(b) Find the direction of the vector **R**. Specify the direction as an angle measured counterclockwise from the positive x-axis.

In order to determine the angle from the positive x-axis a picture is extremely helpful:

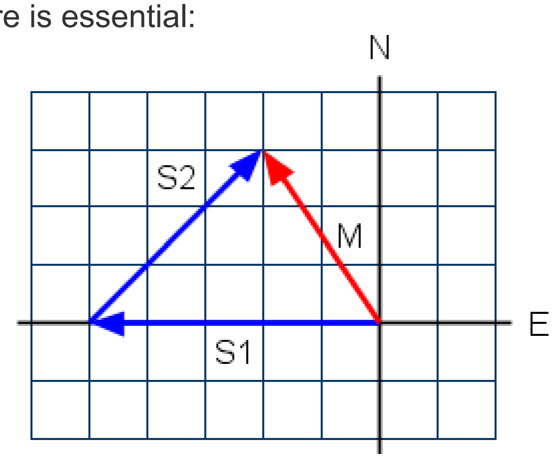


Therefore θ can be found as 180^o minus the angle defined between the -x axis and R. This angle is also defined by $\tan^{-1}(2/8) = \tan^{-1}(1/4) = 14.036^o$.
 Hence:
$\theta = 180 - 14.036$ or 165.96^o.

3)
In the sport of orienteering, participants must plan carefully to get from one checkpoint to another in the shortest possible time. In one case, starting at a particular checkpoint, Sam decides to take a straight path that goes west for 530 meters, and then go northeast for 540 meters on another straight path to reach the next checkpoint. Northeast, by the way, means 45 degrees to both north and east.
Between the same two checkpoints, Mary decides to take the shortest distance between the two checkpoints, traveling off the paths through the woods instead.

(a) What is the distance that Mary travels between the checkpoints? (Note that Sam travels a total distance of 1070 m, although his speed is probably significantly higher than Mary's.)

Here, a good picture is essential:



Where S1 is the distance that Sam travels west, S2 is the distance that Sam travels Northeast and M is the distance that Mary travels.

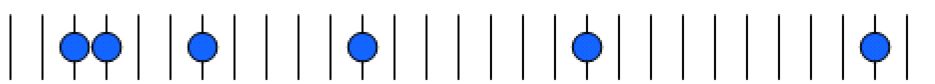
Hence, the total X component of Mary's travels is:
M_x = - S1 + $S2_x$ = - S1 + S2*cos(45) = (in this case) = - 530 + 540*$2^{1/2}$/2 = -148.16m
While the total Y component for Mary is:
M_y = $S2_y$ = S2*cos(45) = 381.84m

Therefore, by the pythagorean theorem, M = $(M_x^2 + M_y^2)^{1/2}$ = 409.58m

(b) In what direction does Mary go? Specify her direction as a positive angle, measured from east.

As in problem 2, we can define the smaller angle in terms of $\tan^{-1}(M_y/M_x)$ and therefore the angle from East can be defined as 180 - $\tan^{-1}$(381.84/ 148.16) = 111.2^o

4)
Consider the motion diagram below, showing the position of an object at regular time intervals as it moves in one dimension. Assume that the object's acceleration is constant throughout the time interval covered by the motion diagram and that it does not reverse direction during the motion.



Select all the correct statements from the list below. Note that you must select every correct statement, and no incorrect statements, to get credit for the question.

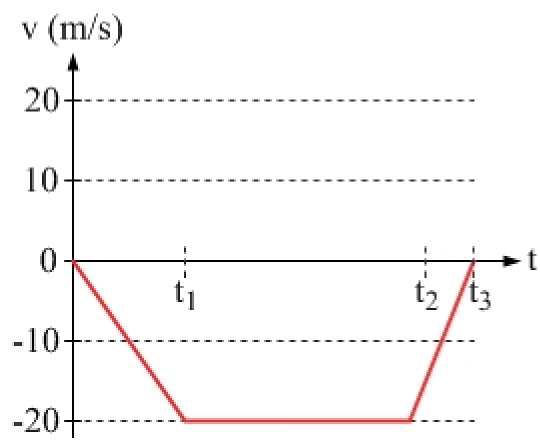
The only thing that can be defined from the motion diagram is:
IF the object originates at the left-most position at rest than is accelerates to the right and continues off to infinity.
IF however, the object originates at the right with some initial velocity directed left, than it is accelerating to the right and comes to a stop at the left most position marked.

Therefore the answers are:

The object is definitely moving to the left.
INCORRECT
The object is definitely moving to the right.

INCORRECT
The object's acceleration is definitely directed to the left.
INCORRECT
The object's acceleration is definitely directed to the right.
CORRECT
A real-life situation this motion diagram could correspond to is a car moving at constant velocity to the right.
INCORRECT
A real-life situation this motion diagram could correspond to is a sprinter traveling to the left but slowing down and coming to a stop at the end of a race.
CORRECT
A real-life situation this motion diagram could correspond to is an overhead view of a toy car being released from rest and rolling to the right down an incline.
CORRECT

5)
At t = 0, a stuntwoman jumps off the top of a tall building. She is wearing a special suit, which increases the air resistance she experiences so that, after falling with constant acceleration for a time of t_1 = 5.00 s, she then falls with a constant velocity of 20 m/s directed down. At a time of t_2 =17.0 s after leaving the top of the building, the stuntwoman reaches a safety net, which provides a constant upward acceleration that brings the stuntwoman to rest at a time of t_3 = 18.0 s after leaving the top of the building. The graph below shows the vertical component of the stuntwoman's velocity throughout the entire process. See if you can use the graph to help you solve the problem.

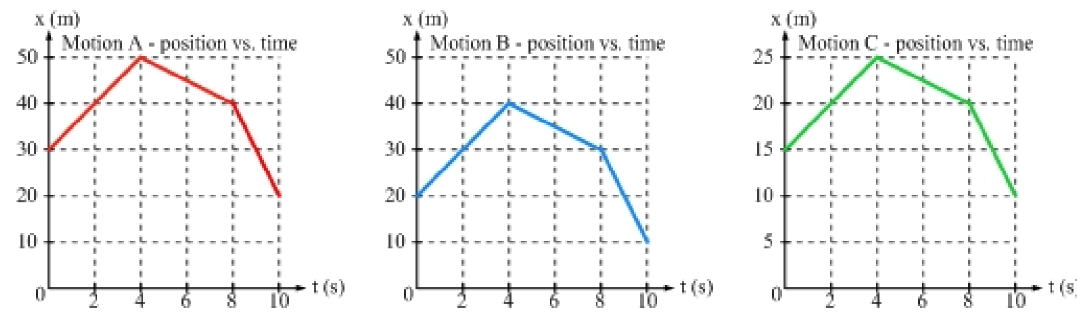


Assuming the stuntwoman comes to rest at ground level (this was a very carefully planned stunt!), how tall is the building? For reference, the John Hancock Tower in Boston, the tallest building in New England, is 241 m tall.

The height of the building is equal to the total distance traveled by the stunt woman during the fall. Since this is a Velocity vs. Time graph, the distance is equal to the area under the curve. This is calculated by taking the area of two triangles and one rectangle:

$A = (1/2\ b_1 * h) + (b_2 * h) + (1/2\ b_2 * h)$
$h = 20$ m/s, $b_1 = t_1$ and $b_2 = t_2 - t_1$ and $b_3 = t_3 - t_2$
which gives:
Building Height = (1/2 * 5 * 20) + (12 * 20) + (1/2 * 1 * 20) = 300 m

6)



The position vs. time graphs for three objects experiencing one-dimensional motion are shown above. Note that the scale on the vertical axis for motion C is different from the scale on the vertical axes for motions A and B.

(a) Rank these situations based on the speed of the objects at *t* = 6 s, from largest to smallest. Use only > and/or = signs in your answer (e.g., B>A=C).

The speed is the magnitude of the slope of the graph at the time indicated. Therefore, for graph A the speed at t = 6 is 10/4 or 2.5. For graph B the speed is also 2.5 m/s. For graph C the speed is 5/4 or 1.25m/s. Hence:
A = B > C

(b) Calculate the speed of the object, at *t* = 1 s, for motion B.

Again, the speed at any time is the magnitude of the slope at that time.

For t = 1 on graph B, the speed is 5m/s

(c) Over the entire 10-second period, determine the total distance traveled by the object for motion A.

During the time interval from 0 to 4 seconds the object6 moves 20m. Between 4 and 8 seconds the object moves an additional 10m. Finally between 8 and 10 seconds the object moves a final distance of 20m. The total distance traveled by the object is therefore the sum of these three parts and is 50m for motion A.

(d) For the entire 10-second period, determine the average speed of the object for motion C.

Average Speed is determined by taking the total distance traveled and dividing by the total time. By the same method for part (c) the total distance is 10m (between 0 and 4 seconds) + 5m (between 4 and 8 seconds) + 10m (between 8 and 10 seconds) which equals a total of 25m. The total time is 10 seconds therefore the average speed is 25m/10s = 2.5m/s

(e) For the entire 10-second period, determine the average velocity of the object for motion B. Note: use a plus or minus sign, as appropriate.

Average velocity is determined by taking the displacement and dividing by the total time. Displacement is defined as the final position minus the initial position or here:
10m - 20m = -10m. Again the total time is 10 seconds and therefore the average velocity is -10m/10s = -1m/s

7)
On August 16, 1960, Joe Kittinger of the United States Air Force jumped from a helium balloon from a height of 102,800 feet. After being in free fall for 4 minutes and 36 seconds, and falling for 85,300 feet, he opened his parachute and eventually landed safely on the ground. To analyze this situation, we will assume that Kittinger's initial velocity was zero, and that his acceleration was constant throughout the free fall. (This was most certainly not the case, but it gives us some idea about the motion.) Note the units we're looking for in the questions below.

(a) What was the magnitude of the acceleration during the free fall?

To determine the acceleration the total distance and time along with the initial velocity should be known. Here t = 4 min 36 s = 276s. Distance = 85,300ft and initial velocity = 0.
In General:
$\Delta x = v_o t + 1/2\ a\ t^2$
Specifically here:
$a = 2\Delta x / t^2$ or $2(85300) / (276)^2$ [ft/s^2]

$a = 2.24$ ft/s^2 converting to the correct units gives 0.683 m/s^2

(b) At the end of the free fall part of the motion, what was Kittinger's speed?

The final velocity can be determined using the equation
$v_f = v_o + a\,t$
Or here
$v_f = 0.683\,(276) = 188.44$ m/s

8)
You are competing in a duathlon, an event that involves running and cycling. This duathlon involves running once around a particular loop, cycling twice around the same loop, and then finishing the race by again running once around the loop. If your average speed when running is 3.00 m/s and your average speed when cycling is 7.50 m/s, what was your average speed for the race?

The average speed is the total distance traveled divided by the total time that the trip took. Take the length of the track to be X. Then the total length of the race is 4X. The time spent running is the distance divided by the speed or:
$T_r = 2X / V_r$
In similar fashion, the total time spent on the bike is:
$Tb = 2X / Vb$
Using the definition of average speed:
$S_{avg} = 4X / (T_r + T_b) = 4X / (2X/V_r + 2X/V_b) = 4X / [2X(1/V_r + 1/V_b)]$
This shows that the length of the track, X, cancels out leaving:

$S_{avg} = 2 / (1/V_r + 1/V_b)$ or $2V_rV_b / (V_r + V_b)$

In this case: Savg = 2(3)(7.5) / (3 + 7.5) = 4.29 m/s

9)
Constant velocity versus constant acceleration
Two balls have a race over a distance of 20.0 meters. The red ball travels at a constant velocity while the blue ball starts at rest and has a constant acceleration. For this problem set the blue ball's acceleration to be 4.00 m/s^2. Note that the simulation may stop slightly after one of the balls reaches the finish line rather than at the instant it reaches the line. Thus, you should calculate all times yourself, rather than relying on the time from the simulation, which is only accurate to plus or minus 0.05 seconds.

IMPORTANT INFORMATION FROM THE SIMULATION:
The red ball has a constant velocity of 5 m/s.
The blue ball has a constant acceleration given above.

(a) How far is the second ball from the finish line at the instant the first ball reaches the line?

Without relying on the simulation for anything more than the velocity of the red ball, the time required for each ball to complete the 20 meters can be determined using the equation:
$\Delta x = v_o t + 1/2\ a\ t^2$

Therefore, for the constant acceleration ball:
$20 = 1/2\ a\ t_{blue}^2$ and solving for t yields
$t_{blue} = (40 / a)^{1/2}$
or in this specific case $t_{blue} = 10^{1/2}$s which is approximately 3.16 seconds

Similarly, the time required for the red ball to complete the course is found with
$20 = v_o\ t_{red}$ or $t_{red} = 20 / v_o$
In this specific case, $t_{red} = 4$ seconds

So, clearly, the blue ball completes the course before the red. To find the position of the red ball when the blue ball completes the course, again the following equation is applied with a = 0 and now with $t = t_{blue}$.
$\Delta x = v_o t + 1/2\ a\ t^2$
$\Delta x = v_o\ t_{blue} = 5 *(10)^{1/2}$ or approximately 15.81 m

The question however, asks how far the red ball is from the end so,
20 - 15.81 = 4.19 m

(b) Assume the motion continues for both balls until the second ball reaches the finish line. What is the distance between the two balls at that time?

We know that when the red ball crosses the line it occurs at tred = 4s. Therefore we can simply use this time in our equation for the location of the blue ball:
$\Delta x = v_o t + 1/2\ a\ t^2$
with $v_o = 0$, a given by the problem and $t = t_{red}$:
$\Delta x = 1/2\ a\ t^2 = 1/2\ (4)\ (4)^2 = 32$m
Again, the wording asks for the distance between the two balls which is $x_{blue} - x_{red}$ or for this problem 32 - 20 = 12m.

10)

You give a toy car an initial velocity of 1.10 m/s directed up a ramp. The car takes a total of 2.00 s to roll up the ramp and then roll back down again into your hand. Assume that you catch the car at the same point from which you released it, and that the acceleration is constant through the entire motion.

(a) Determine the magnitude of the car's acceleration.

The car's acceleration may be determined by applying the equation:

$\Delta x = v_o t + 1/2\ a\ t^2$

For this case, the car's initial velocity is given, as is the time required for the round trip. The displacement (Δx) is equal to zero since the car returns to its starting location. Therefore, by solving for the acceleration:

$a = -2v_o / t$ or in this specific case $-2\ (1.10) / 2 = -1.10\ m/s^2$ where the negative indicates that the acceleration is directed opposite to the initial velocity or downward.

As an alternate method, it may be used that the car comes to rest at the top of its motion, prior to falling back down the track. This must occur halfway through the trip.

Therefore, for the half trip the initial and final velocities are known so the following equation may be applied:

$v_f = v_o + at$

this yields $0 = v_o + at$ or, solving for the acceleration:

$a = -v_o / t$

It should be noted that this value of t is exactly one half of the t used in the previous solution and therefore the form for acceleration is identical.

Note: the question asks for the magnitude of the acceleration and therefore the correct answer is given without direction specific negative sign.

(b) Determine the total distance traveled by the car in this 2.00 second period.

The total distance traveled can be determined by noting that the car travels as far up the ramp as it does down. Also, this must take one half of the total time of the trip which, by applying the same equation as in part (a)

$\Delta x = v_o t + 1/2\ a\ t^2$

$\Delta x_{up} = v_o t_{up} + 1/2\ a\ t_{up}^2 = 1.10\ (1) + 1/2\ (-1.10)\ (1)^2 = 0.55m$ up the ramp

It should be pointed out here that the value of acceleration must be given as negative. This is because the initial velocity is up the ramp and the acceleration pulls the car down. This difference in direction along the axis must be accounted for in the equation.

The symmetry argument to get this equation now states that the total distance must be the distance up plus the equivalent distance down or 1.10 m.