

Entanglement sudden death in qubit–qutrit systems

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Abstract

We demonstrate the existence of entanglement sudden death (ESD), the complete loss of entanglement in finite time, in qubit–qutrit systems. In particular, ESD is shown to occur in such systems initially prepared in a one-parameter class of entangled mixed states and then subjected to local dephasing noise. Together with previous results, this proves the existence of ESD for some states in *all* quantum systems for which rigorously defined mixed-state entanglement measures have been identified. We conjecture that ESD exists in all quantum systems prepared in appropriate bipartite states.

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1. Introduction

An important pursuit common to both the investigation of quantum information processing and quantum measurement is fully understanding the behavior of entangled systems in their environments. This is so because entanglement both enables important quantum protocols and arises in the quantum measurement, both of which in practice take place in a larger environment that affects their fundamental properties. When studying quantum information processing, it is almost always necessary to take into account the inevitable interaction of the involved entangled systems with their environment. The resulting influence can in many cases be described simply as local dephasing noise, which tends to decohere the quantum subsystems and to affect system entanglement. Even subject only to such simply weak noise, suitable error correction or mitigation methods must typically be implemented for processing to be reliable. An understanding of decoherence and disentanglement is

essential for the assessment of the necessity and scope of such techniques, which tend to be costly in resource terms. Because both entanglement and decoherence are similarly significant in modeling quantum measurement, their study also contributes to our understanding of the measurement process.

The influence of environmental noise on entangled quantum systems differs in general from that on both classical systems and separable quantum systems. For example, recent work by Dodd and Halliwell [1] and Yu and Eberly [2–7] has invalidated the intuition that the effects of weak local noises on composite quantum systems are necessarily additive in general, as it is in the latter two cases. The former authors showed that all bipartite initially Gaussian states of two-particle systems described by continuous valued dynamical variables disentangle in finite times under local decoherence effects only, exponentially reducing composite-system coherence. The latter authors showed that although the influences of weak noises on coherence in the simplest bipartite quantum system, the two-qubit system, are additive, the influence of noise on *entanglement* is not additive. They also found, similarly to the former duo, that a combination of local weak noise influences on two-qubit systems can lead, for some state classes, to the striking phenomenon of the total

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loss of entanglement in finite time, which they termed *entanglement sudden death* (ESD) [4,8], rather exhibiting exponential entanglement decay as the coherence so decays. Moreover, this phenomenon has recently been experimentally confirmed in an all-optical experimental realization of a two-qubit system in a local dephasing noise environment [9]. Here, we show that ESD in finite-dimensional systems is not limited only to two-qubit systems but can occur in composite systems of larger dimension as well.

Here, we demonstrate the existence of ESD in the hybrid qubit–qutrit system, by considering a class of states in which the qubit and qutrit are individually incoherent but may still possess a degree of *composite-system* coherence and entanglement. The two-qubit and the qubit–qutrit system are the only finite-dimensional bipartite systems for which rigorously defined general mixed-state entanglement measures are known; the bipartite Gaussian states are the only infinite-dimensional such systems [10,11]. Given the above results, this result supports our conjecture that entanglement sudden death is a generic phenomenon, in the sense of extending to all bipartite quantum systems when appropriately prepared, because it establishes the occurrence of ESD in all quantum systems in which a rigorous general analysis is possible, given the limited scope of mixed-state entanglement measures currently available.

The remainder of this Letter is organized as follows. In Section 2, we motivate the choice of initial qubit–qutrit state class considered and provide explicit descriptions of local and multi-local dephasing noise in the operator-sum decomposition. In Section 3, we demonstrate entanglement sudden death for this class of states in environments producing either type of dephasing noise. A summary of conclusions and our ESD conjecture are discussed in Section 4.

2. Initial states and noise model

We begin by motivating the choice of class of bipartite mixed states and of the dephasing noise model here used to demonstrate the existence of the phenomenon of entanglement sudden death in qubit–qutrit systems. This class includes entangled states with sufficient susceptibility to local dephasing noise that ESD always occurs in any system initially prepared in them when subject to it.

2.1. Class of qubit–qutrit states of interest

The general density matrix describing the composite qubit–qutrit system is $\rho_{AB} = [\rho_{ij}]$, where $\rho_{ji}^* = \rho_{ij}$, $\sum_i \rho_{ii} = 1$, with $i, j = 1, \dots, 6$. There are two sorts of term in this matrix: that responsible for subsystem coherence and that responsible for joint-system coherence, both in a chosen basis. The latter terms are typically associated with quantum entanglement. It is important to recognize, however, that entanglement, by definition, is basis-independent whereas coherence is not. It is therefore also noteworthy that basis-dependent noise can dramatically influence entanglement. Let us designate the first and second subsystems by ‘A’ and ‘B’, respectively. Each of the individual subsystem states, described by a reduced density matrix, is

obtained by tracing over the variables associated with the other subsystem.

The general reduced density matrix ρ_A for the qubit is thus

$$\rho_A = \begin{pmatrix} (\rho_{11} + \rho_{22} + \rho_{33}) & (\rho_{14} + \rho_{25} + \rho_{36}) \\ (\rho_{41} + \rho_{52} + \rho_{63}) & (\rho_{44} + \rho_{55} + \rho_{66}) \end{pmatrix}. \quad (1)$$

States in which the subsystems are incoherent can be ones in which the composite system nonetheless possesses high joint-state coherence and is entangled. We therefore begin by taking the reduced states obtained from ρ_{AB} to be incoherent in this representation to more directly observe the influence of local dephasing noise on global properties, entanglement in particular. We do so by taking all off-diagonal terms in the reduced density matrices to be zero [12]. Thus, we seek joint-system states consistent with the incoherent reduced-state

$$\rho'_A = \text{diag}((\rho_{11} + \rho_{22} + \rho_{33}), (\rho_{44} + \rho_{55} + \rho_{66})).$$

Similarly, because the qutrit reduced density matrix ρ_B for the qutrit are

$$\rho_B = \begin{pmatrix} (\rho_{11} + \rho_{44}) & (\rho_{12} + \rho_{45}) & (\rho_{13} + \rho_{46}) \\ (\rho_{21} + \rho_{54}) & (\rho_{22} + \rho_{55}) & (\rho_{23} + \rho_{56}) \\ (\rho_{31} + \rho_{64}) & (\rho_{32} + \rho_{65}) & (\rho_{33} + \rho_{66}) \end{pmatrix}, \quad (2)$$

we consider the incoherent qutrit reduced state

$$\rho'_B = \text{diag}((\rho_{11} + \rho_{44}), (\rho_{22} + \rho_{55}), (\rho_{33} + \rho_{66})).$$

The composite-system density matrices that yield reduced states of the forms ρ'_A and ρ'_B are

$$\rho'_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & \rho_{15} & \rho_{16} \\ 0 & \rho_{22} & 0 & \rho_{24} & 0 & \rho_{26} \\ 0 & 0 & \rho_{33} & \rho_{34} & \rho_{35} & 0 \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} & 0 & 0 \\ \rho_{51} & 0 & \rho_{53} & 0 & \rho_{55} & 0 \\ \rho_{61} & \rho_{62} & 0 & 0 & 0 & \rho_{66} \end{pmatrix}, \quad (3)$$

where $\rho_{ij} \in \mathfrak{R}$; the zeroed off-diagonal terms must each be so, given that they have been chosen to be non-negative and real, because for each of them there is an off-diagonal term in either ρ'_A or ρ'_B that is a sum containing it (cf. Eqs. (1)–(2)) that is zero. The nonzero terms in the full joint-system density matrix of Eq. (3) are exclusively those related to joint-system global state coherence and typically also entanglement. Nonetheless, as we see below, they are affected by local noise.

For all mixed states ρ_{AB} of qubit–qutrit systems, entanglement is well quantified by the negativity, $\mathcal{N}(\rho_{AB})$, which is the degree to which a positive map fails to be completely positive [10,13–15]. For our class of states, this is given by the absolute value of the negative eigenvalues $\lambda_k^{\text{TA}(-)}$ of the partial transpose of the full-system density matrix ρ'_{AB} with respect to the smaller dimensional subsystem, when at least one exists, and by zero otherwise. Thus, here,

$$\mathcal{N}(\rho'_{AB}) = \max \left\{ 0, \sum_k |\lambda_k^{\text{TA}(-)}| \right\}. \quad (4)$$

For simplicity and definiteness, let us study the specific one-parameter class, within those with density matrices described

by Eq. (3), of the form

$$\rho'_{AB}(\bar{x}) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \bar{x} \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ \bar{x} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}, \quad (5)$$

where $0 \leq \bar{x} \leq \frac{1}{4}$. This choice is made because taking the diagonal terms to be not all equal prevents the density matrix and its partial transpose from having identical eigenvalues. This is a necessary condition for $\rho'_{AB}(\bar{x})$ to be separable for some physically allowed values of \bar{x} into which initially entangled states of this class evolve, under the time-evolution corresponding noise model chosen below, because all states remain within this class under that evolution, as shown below. This limitation of the range of values of \bar{x} is necessary for the matrix to be a well-defined density matrix, see below. ESD also takes place for a larger class of states of the form of Eq. (3), as it does for this class, for example, when the conditions (i) ρ_{24} , ρ_{34} and ρ_{35} are all zero, (ii) the diagonal elements are as chosen here, and (iii) one takes $\rho_{15} = \rho_{16} = \rho_{26} = \bar{x}$, are all satisfied. However, in that case, the expressions for eigenvalues are more complex without obvious added benefit for exhibiting or describing the phenomenon. The matrices described by Eq. (5) are well-defined density matrices because they satisfy the following necessary conditions for any matrix ρ to be a density matrix: (1) Hermiticity of ρ , (2) unitarity of $\text{tr } \rho$, and (3) positive semi-definiteness of ρ . Conditions (1) and (2) are obviously satisfied by this matrix. Condition (3) is satisfied because all eigenvalues of $\rho'_{AB}(\bar{x})$ remain non-negative only within the above range of values of \bar{x} , as will be shown below where these are exhibited.

2.2. Noise model

In order to exhibit ESD in this class, it suffices to consider local dephasing noise alone acting on subsystems that are dynamically isolated from each another. The most general time-evolved open-system density matrix expressible in the operator-sum decomposition is

$$\rho(t) = \mathcal{E}(\rho(0)) = \sum_{\mu} K_{\mu}(t) \rho(0) K_{\mu}^{\dagger}(t), \quad (6)$$

where the $K_{\mu}(t)$, which satisfy a completeness condition guaranteeing that the evolution be trace-preserving, represent the influence of statistical noise which can be global or local in scope, and where the index runs over the number of elements required for the decomposition. For local and multi-local dephasing environments, the $K_{\mu}(t)$ are of the form $K_{\mu}(t) = F_j(t) E_i(t)$, so that

$$\begin{aligned} \rho_{AB}(t) &= \mathcal{E}(\rho(0)) \\ &= \sum_{i=1}^2 \sum_{j=1}^3 F_j(t) E_i(t) \rho_{AB}(0) E_i^{\dagger}(t) F_j^{\dagger}(t), \end{aligned} \quad (7)$$

where

$$\begin{aligned} E_1(t) &= \text{diag}(1, \gamma_A) \otimes \text{diag}(1, 1, 1) \\ &= \text{diag}(1, 1, 1, \gamma_A, \gamma_A, \gamma_A), \end{aligned} \quad (8)$$

$$\begin{aligned} E_2(t) &= \text{diag}(0, \omega_A) \otimes \text{diag}(1, 1, 1) \\ &= \text{diag}(0, 0, 0, \omega_A, \omega_A, \omega_A), \end{aligned} \quad (9)$$

$$\begin{aligned} F_1(t) &= \text{diag}(1, 1) \otimes \text{diag}(1, \gamma_B, \gamma_B) \\ &= \text{diag}(1, \gamma_B, \gamma_B, 1, \gamma_B, \gamma_B), \end{aligned} \quad (10)$$

$$\begin{aligned} F_2(t) &= \text{diag}(1, 1) \otimes \text{diag}(0, \omega_B, 0) \\ &= \text{diag}(0, \omega_B, 0, 0, \omega_B, 0), \end{aligned} \quad (11)$$

$$\begin{aligned} F_3(t) &= \text{diag}(1, 1) \otimes \text{diag}(0, 0, \omega_B) \\ &= \text{diag}(0, 0, \omega_B, 0, 0, \omega_B), \end{aligned} \quad (12)$$

$\gamma_A(t) = e^{-t(\Gamma_A/2)}$, $\gamma_B(t) = e^{-t(\Gamma_B/2)}$, $\omega_A(t) = \sqrt{1 - \gamma_A^2(t)}$, $\omega_B(t) = \sqrt{1 - \gamma_B^2(t)}$ ($X = A, B$). The $E_i(t)$ and $F_j(t)$ induce local dephasing in the qubit and qutrit states, respectively, and individually satisfy the usual completeness condition for the operator-sum decomposition of CPTP maps [11], the Γ_X quantifying the rate of local exponential dephasing. The noise parameters in Eqs. (10)–(12) have been chosen such that the rate of dephasing from levels 1 and 2 relative to the state 0 are equal, to simplify resulting expressions. The time-dependence of $\gamma(t)$'s is also often left implicit in the sequel to lend compactness to expressions, particularly in the explicit forms of density matrices.

3. Entanglement sudden death

Here, we show that qubit–qutrit systems initially prepared in the simple one-parameter family of states $\rho'_{AB}(\bar{x})$ motivated and constructed above exhibit entanglement sudden death. In order to find the entanglement of the qubit–qutrit system, we first find the eigenvalues $\{\lambda_k^{\text{TA}}(x, t)\}$ ($k = 1, \dots, 6$) of the partial transpose with respect to the qubit of $\rho'_{AB}(x, t)$, and then sum over the absolute values of the negative ones to obtain the negativity. The three specific environmental noise situations now considered in turn are: (1) qubit dephasing only, (2) qutrit dephasing only, and (3) combined local dephasing.

3.1. Qubit (A) dephasing noise only

The time-dependent solution of Eq. (7), with the initial density matrix given in Eq. (5), in the case when the $E_i(t)$ are of the general form of Eqs. (8)–(9) and when no noise from the environment of the qutrit is present, so that $F_i(t) = I$, is

$$\rho'_{AB}(x, t) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & x\gamma_A \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ x\gamma_A & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}. \quad (13)$$

Its eigenvalues are $\{\lambda_k(x, t)\} = \{\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{4}(1 + 4x\gamma_A), \frac{1}{4}(1 - 4x\gamma_A)\}$. This is a well-defined density matrix as long as its eigenvalues are non-negative. The only eigenvalue of this matrix that can possibly become negative for positive values of x is the last one, which occurs when $\bar{x} = x\gamma_A > \frac{1}{4}$, which values lie outside the range of values of \bar{x} of our class, since $\gamma_A(t) \leq 1$ for all times t and x is strictly less than or equal that $\frac{1}{4}$, so it remains a well-defined density matrix, as required and discussed above. Taking into account the form of $\gamma_A(t)$, one sees that decoherence takes the form of the exponentially decay of off-diagonal terms of the density matrix, which terms tend to zero only in the large-time limit $t \rightarrow \infty$: both nonzero off-diagonal elements of the matrix of Eq. (14) decay asymptotically to zero as a function of time for all values of x , in particular within the restricted range of physically allowed values. Because previous analyses have shown that disentanglement proceeds at least as fast as decoherence in a broad range of states of two-qubit and two-qutrit systems [2,16], we expect this result to hold for the hybrid qubit–qutrit system as well. This is indeed the case. To see that this is so, consider the eigenvalues of the partial transpose of $\rho'_{AB}(x, t)$ with respect to qubit A, namely, $\{\lambda_k^{TA}(x, t)\} = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}(1 + 8x\gamma_A), \frac{1}{8}(1 - 8x\gamma_A)\}$. Clearly, the only one of these eigenvalues that can become negative is the last one. Thus, in accord with Eq. (4), one finds the degree of entanglement as a function of time to be

$$\mathcal{N}(\rho'_{AB}(x, t)) = \max\{0, x\gamma_A(t) - 1/8\}, \tag{14}$$

for the range of values of x specifying states within the class of states under consideration. Thus, one sees that all such states which are initially entangled, that is for which $x > \frac{1}{8}$, become separable for all values of $x\gamma_A(t) \leq \frac{1}{8}$, that is, irreversibly disentangled as soon as $\gamma_A(t)$ reaches $\frac{1}{8x}$. By contrast, decoherence occurs only asymptotically in the large-time limit $t \rightarrow \infty$, in which $\gamma_A(t) \rightarrow 0$.

3.2. Qutrit (B) dephasing noise only

For noise acting on qutrit B alone, the time evolved density matrix is similarly

$$\rho'_{AB}(x, t) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & x\gamma_B \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ x\gamma_B & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}. \tag{15}$$

The eigenvalues $\{\lambda_k^{TA}(x, t)\}$ of the partial transpose of $\rho'_{AB}(x, t)$ with respect to qubit A are $\{\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}(1 + 8x\gamma_B), \frac{1}{8}(1 - 8x\gamma_B)\}$. As in the previous case, the only eigenvalue that can potentially be negative is the last one. Thus, we similarly find the degree of entanglement of the composite system to be

$$\mathcal{N}(\rho'_{AB}(x, t)) = \max\{0, x\gamma_B(t) - 1/8\}. \tag{16}$$

One sees that the state to irreversibly become separable once $x\gamma_B \leq \frac{1}{8}$, that is, as soon as $\gamma_B(t)$ reaches $\frac{1}{8x}$. Entanglement

sudden death therefore also takes place in the case of local dephasing noise acting on the qutrit alone. By contrast, and as in the case of qubit dephasing, full decoherence of the composite system occurs only in the large-time limit, $t \rightarrow \infty$.

3.3. Multi-local dephasing noise

Finally, as one might now expect given the above results for local dephasing noise, multi-local dephasing noise affecting both components of the system gives rise to the time evolved state

$$\rho'_{AB}(x, t) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & x\gamma_A\gamma_B \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ x\gamma_A\gamma_B & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}. \tag{17}$$

The timescales of decay of off-diagonal terms, that is, of decoherence are simple products of terms appearing due to individual local dephasing noise contributions shown in the previous two subsections separately, so that decoherence is additive, a result anticipated on the basis of previous study of two-qubit systems under local noise [16]. One finds that the off-diagonal terms decay at an even faster rate than in the previous cases of dephasing noise on either qubit or qutrit subsystem alone but, again, the coherence decays exponentially to zero only in the limit $t \rightarrow \infty$. The eigenvalues $\{\lambda_k^{TA}(x, t)\}$ of the partial transpose with respect to qubit A are $\{\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}(1 + 8x\gamma_A\gamma_B), \frac{1}{8}(1 - 8x\gamma_A\gamma_B)\}$. As before, the only eigenvalue that can be negative is the last one. Thus, we find

$$\mathcal{N}(\rho'_{AB}(x, t)) = \max\{0, x\gamma_A(t)\gamma_B(t) - 1/8\}, \tag{18}$$

so that, by the same argument as made in the previous cases, multi-local dephasing noise is seen to induce *entanglement sudden death* for the chosen state class, and does so even more quickly than in the single local dephasing noise cases considered above, namely, the state of the qubit–qutrit system irreversibly becomes separable as soon as $\gamma_A(t)\gamma_B(t) \leq \frac{1}{8x}$.

4. Conclusion

We have shown that entanglement sudden death (ESD) can take place in hybrid qubit–qutrit systems, in particular, for a straightforwardly motivated and simple one-parameter class of mixed states. This result extends those of Dodd and Halliwell and of Yu/Eberly on ESD, and supports the conjecture we have made here that ESD is a generic phenomenon in the sense of existing in all bipartite quantum states, by showing that it occurs also in the remaining case not considered by them that is capable of general examination within the current limitations of entanglement measure theory. In addition to quantum information processing applications, this result has implications for the decoherence program for addressing the measurement problem: the abrupt loss of entanglement between subsystems may shed light on how the environment picks out a preferred basis and

how a quantum system transitions towards classical behavior under the decoherence model of quantum measurement.

The definitive demonstration of the phenomenon of entanglement sudden death in all finite-dimensional bipartite quantum systems remains an open problem, one which can only be quantitatively addressed if one possesses an entanglement measure for mixed states of qu- d -it–qu- d' -it systems for general values of d , d' and a class of states of such systems to which it would apply. We anticipate demonstrating entanglement sudden death in such systems in a future publication. Finally, we note that, in a very recent preprint, Chełkińska and Wódkiewicz have studied entangled qutrit-pairs in noisy atomic channels and found evidence that ESD may occur in that situation, which would represent additional progress toward this goal [17].

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