Earthquake networks based on similar activity patterns

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Earthquakes are a complex spatiotemporal phenomenon, the underlying mechanism for which is still not fully understood despite decades of research and analysis. We propose and develop a network approach to earthquake events. In this network, a node represents a spatial location while a link between two nodes represents similar activity patterns in the two different locations. The strength of a link is proportional to the strength of the cross correlation in activities of two nodes joined by the link. We apply our network approach to a Japanese earthquake catalog spanning the 14-year period 1985–1998. We find strong links representing large correlations between patterns in locations separated by more than 1000 kilometers, corroborating prior observations that earthquake interactions have no characteristic length scale. We find network characteristics not attributable to chance alone, including a large number of network links, high node assortativity, and strong stability over time.

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I. INTRODUCTION

Despite the underlying complexities of earthquake dynamics and their complex spatiotemporal behavior [1,2], celebrated statistical scaling laws have emerged describing the number of events of a given magnitude (Gutenberg-Richter law) [3], the decaying rate of aftershocks after a main event (Omori law) [4–6], the magnitude difference between the main shock and its largest aftershock (Bath law) [7], as well as the fractal spatial occurrence of events [8–11]. Recent work has shown that scaling recurrence times according to the above laws results in the distribution collapsing onto a single curve [12,13]. However, while the fractal occurrence of earthquakes incorporates spatial dependence, it appears to embed isotropy in the form of radial symmetry, while the occurrence of real-world earthquakes is usually anisotropic [14].

To better characterize this anisotropic spatial dependence as it applies to such heterogeneous geography, network approaches have been recently applied to study earthquake catalogs [15–22]. These recent network approaches define links as being between successive events, events close in distance [19], or being between events which have a relatively small probability of both occurring based on three of the above statistical scaling laws [23]. These methods define links between singular events. In contrast, we define links between locations based on the long-term similarity of earthquake activity. While earlier approaches captured the dynamic nature of an earthquake network, they did not incorporate the characteristic properties of each particular location along the fault. Various studies have shown [4,24–26,28] that the interval times between earthquake events for localized areas within a catalog have distributions not well described by a Poisson distribution [29], even within aftershock sequences [28]. This demonstrates that each area not only has its own statistical characteristics [30], but also retains a memory of its events [24–26]. As a result, successive events may not be just the result of uncorrelated independent chance, but instead might be dependent on the history particular to that location. If prediction is to be a goal of earthquake research, it makes sense to incorporate interactions due to long-term behavior inherent to a given location rather than by treating each event independently. We include long-term behavior as such in this paper by considering a network of locations (nodes) and interactions between them (links), where each location is characterized by its long-term activity over several years.

II. DATA

For our analysis, we utilize data from the Japan University Network Earthquake Catalog (JUNEC) [27]. We choose the JUNEC catalog because Japan is among the most active and best observed seismic regions in the world. Because our technique is novel, this catalog provided the best avenue for employing our analysis. In the future, it may be possible to fine-tune our approach to more sparse catalogs.

The data in the JUNEC catalog span 14 years from 1 July 1985 through 31 December 1998 and are depicted in Fig. 1. Each entry in the catalog includes the date, time, magnitude, latitude, and longitude of the event. We found the catalog to obey the Gutenberg-Richter law [31] for events of magnitude 2.2 or larger. By convention, this is taken to mean that the catalog can be assumed to be complete in that magnitude range. However, because catalog completeness cannot be guaranteed for shorter time periods over a 14-year span, we also examine Gutenberg-Richter statistics for each nonoverlapping two-year period (Fig. 2) [31]. We find that, though absolute activity varies by year, the relative occurrences of quakes of varying magnitudes does not change significantly for events between magnitude 2.2 and 5, where there is the greatest danger of events missing from the catalog.

Additionally, the data are spatially heterogeneous, as shown in Fig. 1. Most events take place either over land or off Japan’s east coast. We remark that this is not an artifact of more detection equipment being located on land. The primary means for locating and detecting earthquake events involves using the S waves and P waves that emanate from the events. Seismic stations are capable of detecting these waves a great distance from their source. Both S waves and P waves [32] travel...
through the Earth’s mantle, and the characteristic absorption distance, defined as the distance for a wave amplitude to drop to 1/e of its original value, for body waves is on the order of 10,000 km [33]. Any event of magnitude 5.5 or larger, for example, is detectable anywhere on Earth. Hence, the location of the detection equipment does not affect how accurately events are catalogued. Additionally, because the location of the Japanese archipelago is a consequence of seismic activity of the Japanese archipelago is a consequence of seismic activity, it is not surprising that most seismic events take place on or near the islands themselves.

We partition the region associated with the JUNEC catalog as follows: We take the northernmost, southernmost, easternmost, and westernmost extrema of all events in the catalog as the spatial bounds for our analysis. We partition this region into a 23 × 23 grid which is evenly spaced in geographic coordinates. Each grid square of approximate size 100 × 100 km is regarded as a possible node in our network. The results do not qualitatively differ when the fineness of the spatial grid is modified, in agreement with analogous work carried out by the authors of Ref. [20], using a different technique from ours [18]. However, 100 km boxes are a more physical choice, as 100 km is on the order of rupture length associated with larger earthquakes [34], which in turn is roughly equivalent to the aftershock zone distance for larger earthquakes \( M \geq 7.0 \) [35].

For a given measurement at time \( t \), an event of magnitude \( M \) occurs inside a given grid square. Similar to the method of Corral [28], we define the signal of a given grid square to form a time series \( \{ s_i \} \), where each series term \( s_i \) is related to the earthquake activity that takes place inside that grid square within the time window \( \Delta t \), as described below.

Because events do not generally occur on a daily basis in a given grid square, it is necessary to bin the data to some level of coarseness. How coarse the data are treated involves a trade-off between precision and data richness. We refer to the time series \( \{ s_i \} \) belonging to each grid cell \( ij \) as that grid cell’s signal. We define the signal that is related to the energy released in the the \( ij \) grid cell by

\[
s_{ij}(t) = \sum_{\ell=1}^{N_{ij}} 10^{-2M_{ij}(t)},
\]

where \( N_{ij}(t) \) denotes the number of events that occur in the \( t \)th time window in grid square \( ij \). We choose this definition because the term \( 10^{-2M_{ij}(t)} \) is proportional to the energy released from an earthquake of magnitude \( M \) [36]. The signal therefore is proportional to the total energy released at a given location in a 90-day time period [38].

To define a link between two grid squares, we calculate the Pearson product-moment correlation coefficient \( r_{x,y} \) between the two time series \( \{ x_t \}, \{ y_t \} \) associated with those two grid squares [41]

\[
r_{x,y} = \frac{\langle XY \rangle - \langle X \rangle \langle Y \rangle}{\sigma_X \sigma_Y},
\]

where \( \langle \ldots \rangle \) indicates the mean and \( \sigma_X, \sigma_Y \) the standard deviations of the time series \( \{ x_t \}, \{ y_t \} \).

We consider the two grid squares linked if \( r_{x,y} \) is larger than a specified threshold value \( r_c \), where \( r_c \) is a tunable parameter. As is standard in network-related analysis, we define the degree \( k \) of a node to be the number of links the node has. Note that our

FIG. 1. (Color online) Number of events by location in the JUNEC catalog, shown in a 23 × 23 mesh. Larger circles with brighter colors denote more events. The JUNEC catalog clusters spatially, with most activity occurring on the eastern side of Honshu, Japan’s main island.

FIG. 2. (Color online) Demonstrating that the magnitude above which the Gutenberg-Richter law is obeyed is approximately constant from year to year. To this end, we provide Gutenberg-Richter statistics for the JUNEC catalog over separated two-year periods. The Gutenberg-Richter law states that the number \( N \) of events greater than a given magnitude \( M \) obeys \( \log_{10} N = a - bM \), with \( b \approx 1 \).
FIG. 3. (Color online) Examples of highly correlated signals, as defined in Eq. (1), with values of \((i, j)\) marked above. (a) Two signals with Pearson correlation coefficient \(r = 0.9617\), associated with locations 878-km apart. (b) The corresponding \(\tilde{r}\) as a function of time offset as defined by Eq. (3). (c) Corresponding scatterplot of (a) with signal \((i, j) = (10, 12)\) plotted against signal \((i, j) = (2, 14)\). Each point corresponds to a single point in time for the simultaneous signals of \((10, 12)\) and \((2, 14)\). Note that because the signal is defined in terms of exponentiation that large events dominate the correlation, just as large events dominate the total energy released in an earthquake catalog.

signal definition Eq. (1) involves an exponentiation of numbers of order 1. This means that the energy released, and therefore the cross correlation between two signals, is dominated by large events. Examples of signals with high correlation are shown in Fig. 3.

To confirm the statistical significance of \(r_{x,y}\), we compare \(r_{x,y}\) of any two given signals with \(r_{x,y}\) calculated by shuffling one of the signals. We also compare \(\tilde{r}_{x,y}\) with the cross correlation \(\tilde{r}_{x,y}(\tau)\) we obtain by time shifting one of the signals by varying time increments \(\tau\)

\[
\tilde{r}_{x,y}(\tau) = r(s_{x,t}, s_{y,t+\tau}),
\]

where \(\tau\) is in units of 90 days. Further, we impose periodic boundaries

\[
t + \tau \equiv (t + \tau) \mod t_{\text{max}},
\]

where \(t_{\text{max}}\) is the length of the series. Our justification for these boundaries is that events in the distant past (>10 years) should have nominal effects on the present.

FIG. 4. (Color online) Testing the statistical significance of cross correlations to demonstrate that the correlations observed are stronger than ambient noise. For each pair of signals, \(i_1,j_1\) and \(i_2,j_2\), with a cross correlation \(r \geq r_c\), we shift one of the signals in time by \(\tau\) time periods and calculate the new correlation coefficient. Each colored line is a comparison of a pair of signals, as described by Eq. (3). Note the strong peak at \(\tau = 0\) corresponding to signals being compared at the same time. Offsetting the signals in time results in lower cross correlation, dropping to the level of noise in the actual data. As a control, we shuffle all the signals for each time shift and calculate the spread of cross-correlation values at that value of \(\tau\) (shown below each figure). Cross correlation between various pairs of signals vs. time offset. Shown are links for which (a) \(\tilde{r}(0) \geq r_c = 0.7\) and (b) \(\tilde{r}(0) \geq r_c = 0.9\).

where \(t_{\text{max}}\) is the length of the series. Our justification for these boundaries is that events in the distant past (>10 years) should have nominal effects on the present,
FIG. 5. (Color online) Demonstration that empirical data show far more links than time-shuffled data. (a) In black is the distribution of the number of links obtained in the network after time shuffling the data many times. A link corresponds to a correlation coefficient between two signals of \( r \geq r_c \). Shown is the case \( r_c = 0.8 \). Actual results, shown in gray (red), are greater than 5σ from the mean of the shuffled distribution, about 17% more links than the mean of the shuffled distribution. (b) Results are similar for other values of \( r_c \). We note that the fraction of links we can regard “real” or meaningful in general increases with \( r_c \), while they also provide typical background noise for comparison.

We note that over the 14-year time period 1985 to 1998, the overall observed activity increases in the areas covered by the catalog. To ensure that the \( r_{x,y} \) values we calculate are not simply the result of trends in the data, we compare our results to those obtained with linearly detrended data [39]. We find that the trends do not have a significant effect. For example, using \( r_c = 0.7 \), we obtain 815 links, while detrending the data results in only three links dropping below the threshold correlation value. For \( r_c = 0.6 \), we obtain 1003 links, while detrending results in only three links dropped. Additionally, after detrending, 94% of correlation values stay within 2% of their values.

IV. RESULTS

As described above, we compare \( r_{x,y} \equiv \tilde{r}_{x,y}(0) \) of Eq. (3) between signals at different locations at the same point in time with \( \tilde{r}_{x,y}(\tau) \) and with the correlation coefficient obtained by shuffling one of the series. Shuffling or time shifting by a single time step (representing 90 days) reduces \( \tilde{r}_{x,y} \) to within the margin of significance, as shown in Fig. 4. We find a large number of links with cross correlations far larger than their shuffled counterparts. The number of links exceeds that of the time-shuffled data by roughly 3σ – 8σ, depending on the choice of \( r_c \) as shown in Fig. 5(a). However, as shown, there are still many links that can be regarded as the result of noise. We therefore further examine the difference between the number of links found in the time-shuffled data and the number found in the original data [Fig. 5(b)]. We find that the fraction of “real” links in general increases with \( r_c \).

A significant fraction of these links connect nodes farther apart than 1000 km, as can be seen in Fig. 6. This is consistent with the finding that there is no characteristic cutoff length for interactions between events [20,23], corroborated by Fig. 7, showing the number of links a network has at a given distance as a fraction of the number of links that are possible from choosing any two nodes in the potential network. Distances shorter than 100 km have sparse statistics due to the coarseness of the grid while distances greater than 2300 km have sparse statistics due to the finite spatial extent of the catalog. Within this range, the fraction of links observed drops off
approximately no faster than a power law. We find qualitatively similar results when we adjust the grid coarseness.

Our results, shown in Fig. 6, are anisotropic, with the majority of links occurring at approximately 37.5° east of north. This is roughly along the principal axis of Honshu, Japan’s main island, and parallel to the highly active fault zone formed by the subduction of the Philippine and Pacific tectonic plates under the Amurian and Okhotsk plates, respectively. High degree nodes (i.e., nodes with a large number of links) tend to be found in the northeast and northcentral regions of the JUNEC catalog and are notably not strongly associated with the locations in the catalog that are most active, which we discuss in further detail below.

In network physics, we often characterize networks by the preference for high degree nodes to connect to other high degree nodes. The strength of this preference is quantified by the network’s assortativity, defined as

\[ A \equiv r_{k_1,k_2} \]  

(5)

where \( r \) is the Pearson correlation coefficient given by Eq. (2). The series \( k_1 \) and \( k_2 \) are found as follows: Iterating through all entries \( i,j \) in the adjacency matrix [40], the degree of each node \( i \) is appended to the series \( \{k_1\} \) and the degree of the node \( j \) that \( i \) is linked to is appended to the series \( \{k_2\} \). The assortativity coefficient thus gives a correlation of the node degree within the network. If each node of degree \( k \) connects only to nodes of the same degree, the two series \( \{k_1\} \) and \( \{k_2\} \) will be identical and \( A = 1 \). Networks like the network of paper coauthorship have positive assortativity, while those of the World Wide Web and of many ecological and biological systems have negative assortativity [37].

Figure 8 shows that the networks resulting from our procedure are highly assortative with assortativity generally increasing with \( r_c \). The finding of a positive correlation between the degree of a node and the degree of its neighbors is consistent with an analogous finding [20] with Iranian data, using a different technique from ours [18]. For comparison we show the assortativity obtained by using time-shuffled networks. Since the assortativity of the original networks is far higher than those of shuffled systems, the high assortativity cannot be due to a finite size effect or to the spatial clustering displayed in the data since time shuffling preserves the location. We investigate the nature of the high degree nodes and find that a high degree is not a matter of more events being nearby, as there is a slight tendency for higher degree nodes to actually have longer distance links on average than low degree nodes. Additionally, we find that the node degree is essentially independent of both maximum earthquake size and number of events.

Because Fig. 5 shows, as mentioned above, that many links can be regarded as the result of noise, we investigate the stability of links over time (Fig. 9). The similarity of the network between the first seven years (1985–1992) and the second seven years (1992–1998) in the catalog is found as follows. We find the set of links that satisfy \( r \geq r_c \) in both the...
1985–1992 network and the 1992–1998 network, and create a series out of the respective link strengths (correlations) in the 1985–1992 network. We create another series using the same links, now using the corresponding strengths from the 1992–1998 network. We then correlate the two series using the Pearson correlation coefficient given by Eq. (2). We find that the network is far more stable over time than the counterpart results given by shuffling the time series (Fig. 9). Because one would expect large correlations that arise purely from noise to have no “memory” from one time period to another, the finding of network stability over several years is consistent with our result that these links are not simply the result of chance.

V. DISCUSSION AND CONCLUSION

To summarize our results, we have introduced a method for analyzing earthquake activity through the use of networks [42]. The resulting networks (i) display links with no characteristic length scale, (ii) display far more links than expected from chance alone, (iii) are far more assortative, and (iv) display significantly more link stability over time. The lack of a characteristic length scale is consistent with previous work and underscores the difficulty in making accurate predictions. The statistically significant nature of all of these results is consistent with the possibility of the presence of hidden information in a catalog, not captured by existing models or previous earthquake network approaches.

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[27] Available online at http://wwwiec.eri.u-tokyo.ac.jp/CATALOG/junecl/.

[42] For comparison, we also carried out our analysis with other three signal definitions that we omit here. (1)’Average magnitude”: \( s^{(1)}_{ij}(t) \equiv N_{ij}^{-1} \sum_{\ell=1}^{N_{ij}} M_{\ell}^t(ij) \), where \( M_{\ell}^t(ij) \) is the magnitude of the event and \( N_{ij}(t) \) is the total number of events occurring in the 90-day time window \( t \) in the grid square \( ij \). (2)’Number of events”: \( s^{(2)}_{ij}(t) \equiv N_{ij}(t) \), with the symbols as defined in (1). (3)’Magnitude sum”: \( s^{(3)}_{ij}(t) \equiv \sum_{\ell=1}^{N_{ij}(t)} M_{\ell}^t(ij) \). All three of these alternative definitions fail to give results significantly better than the shuffled data that are robust with respect to the various adjustable parameters.

