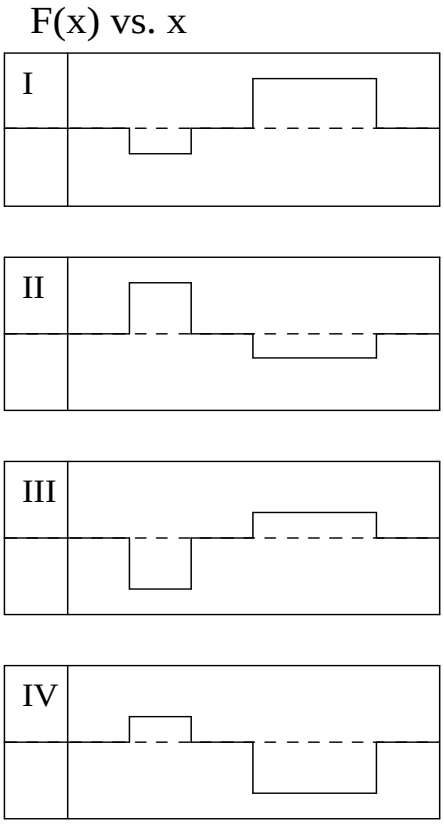




1. The plot on the left above shows a **potential energy** function, $V(x)$ versus x, for a limited range of x values. Which of the plots on the right is most likely the plot of the corresponding **force** function, $F(x)$ versus x, over the same range.

(a) I

(b) II

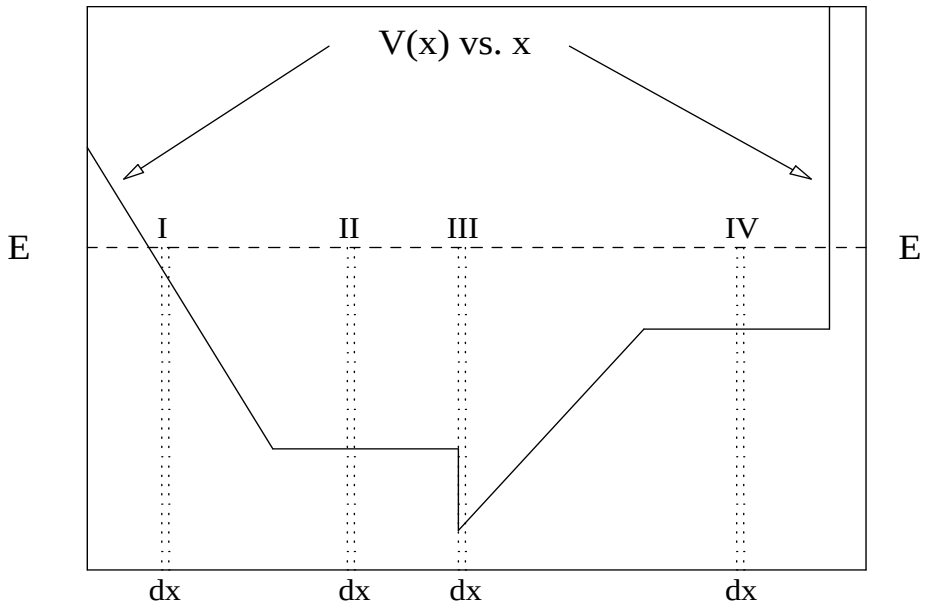
(c) III

(d) IV

(e) None of them is consistent with being the force function.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



2. The plot above shows a function of **potential energy** versus position ($V(x)$ versus x) for a one-dimensional system. A dashed horizontal line indicates the value of energy E for a particle moving in this potential well, corresponding to a bound state system. Small regions, each of equal width dx, are indicated at several locations in the well. Order the **time spent** in each small position bin (dx), $t(I)$, $t(II)$, $t(III)$, and $t(IV)$, from **longest** to **shortest** as the particle moves back and forth in the well.

(a) $t(III) > t(II) > t(IV) > t(I)$

(b) $t(I) > t(IV) = t(II) > t(III)$

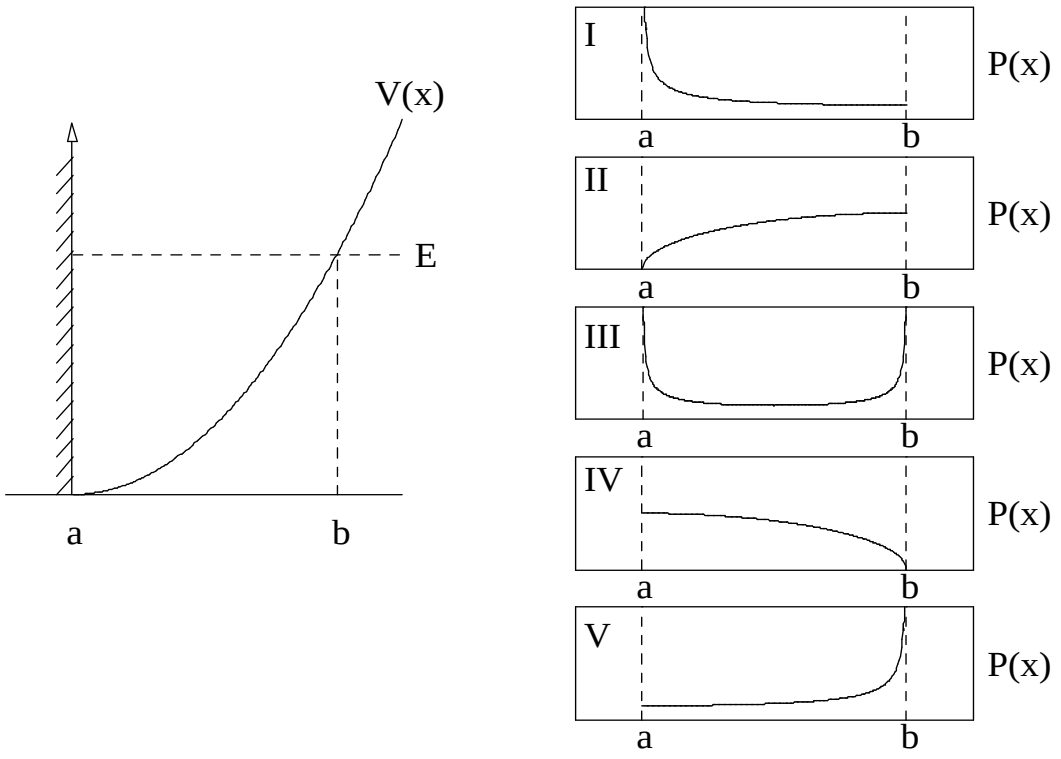
(c) $t(III) > t(II) = t(IV) > t(I)$

(d) $t(I) > t(IV) > t(II) > t(III)$

(e) $t(I) = t(II) = t(III) = t(IV)$ since E is constant

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



3. A particle of energy E moves between the classical turning points, a and b, in a one-dimensional potential as shown in the figure on the left above. The turning point at a is defined by an impenetrable wall, while the one at b is given by the intersection with the $V(x)$ curve shown. Which of the **classical probability distributions**, $P(x)$, shown on the right corresponds to this system. (Recall that the classical probability distribution is defined such that the probability of finding the particle in the small interval $(x, x+dx)$ is given by $d\text{Prob}(x, x+dx) = P(x)\,dx$.)

(a) I

(b) II

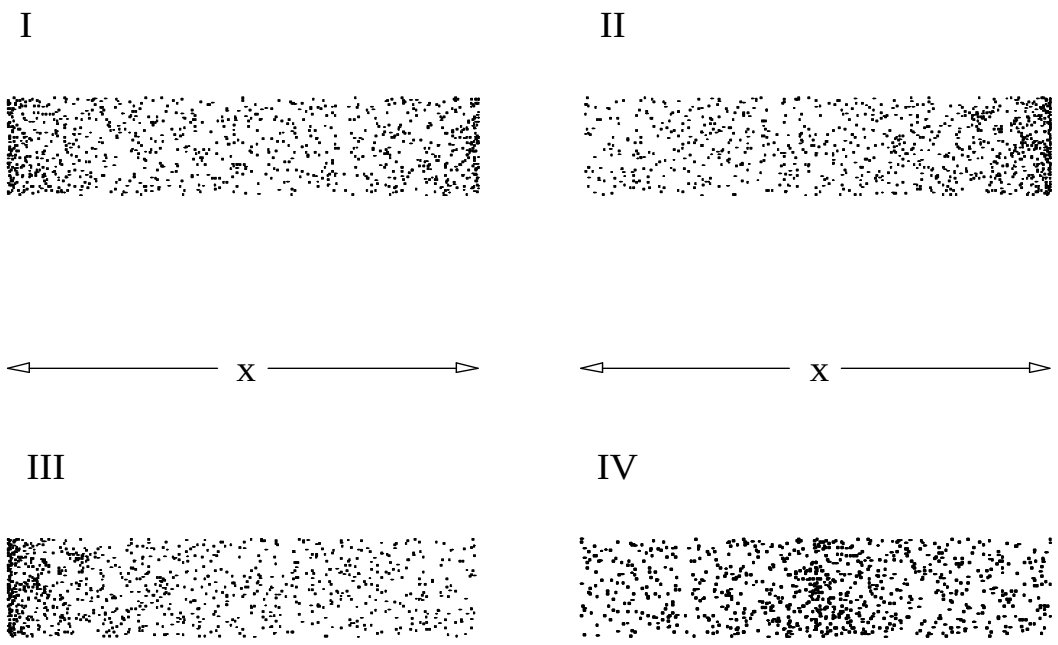
(c) III

(d) IV

(e) V

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



4. The position of four objects with differing types of motion in one-dimension are captured at a large number of random times in the computer-generated 'snapshots' shown above. (The vertical spread in the dots is simply added to make the density of dots more clearly visible.) Two possible one-dimensional motions are:

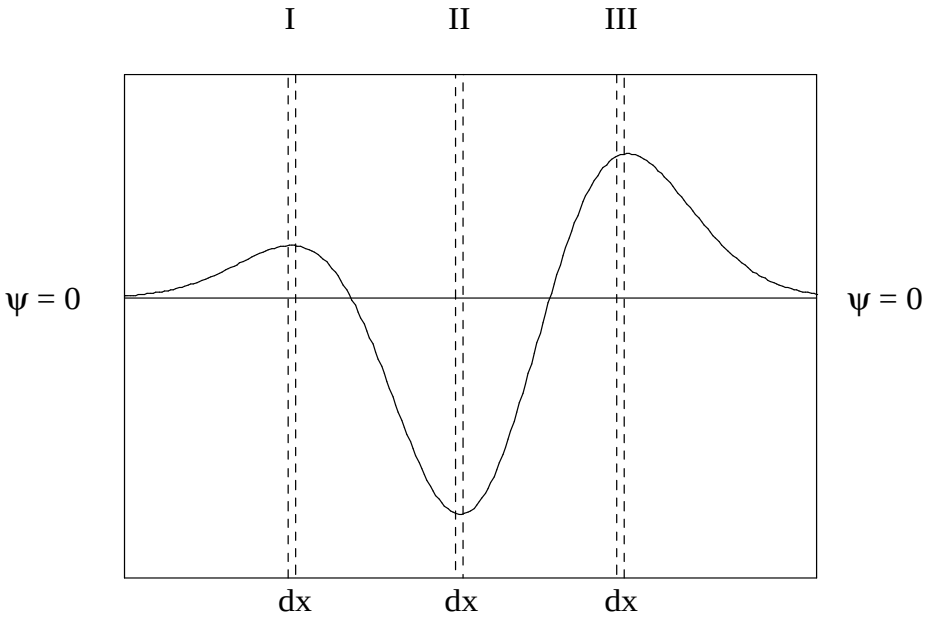
A. A mass oscillating at the end of a spring
B. A mass undergoing uniform acceleration to the right, starting from rest

Which motion (A, B) goes with which 'snapshot' (I, II, III, IV)?

(a) A: IV B: II
(b) A: IV B: III
(c) A: I B: II
(d) A: I B: III
(e) None of the above

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain | somewhat certain | somewhat uncertain | very uncertain



5. The plot above shows a one-dimensional wave function, $\psi(x)$ versus x. The labels, I, II, and III, indicate regions about which measurements of the position of the particle can be made. Order the probabilities of finding the particle described by $\psi(x)$ in a small region of width dx near each labeled point, from **biggest** to **smallest**.

(a) $P(III) > P(I) > P(II)$

(b) $P(II) > P(I) > P(III)$

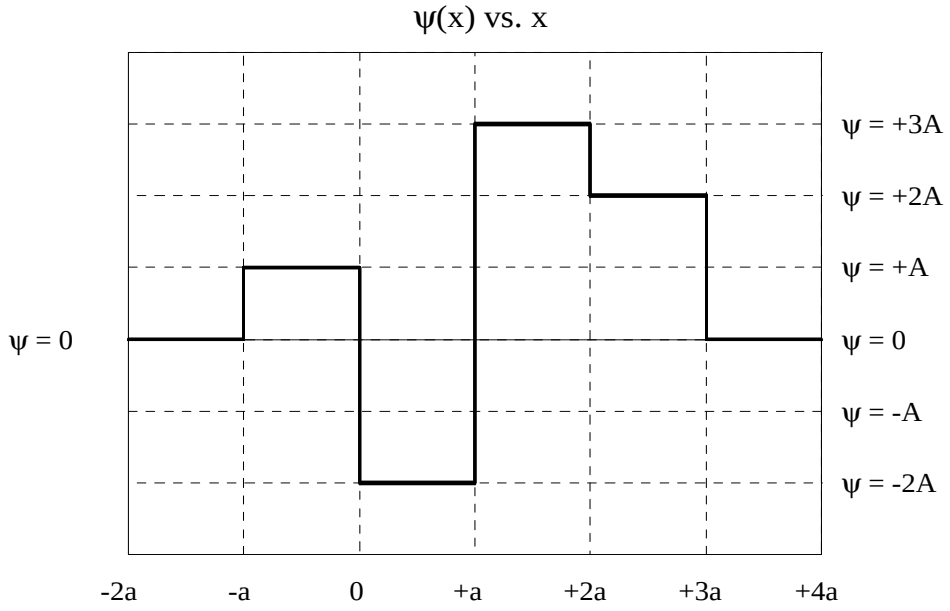
(c) $P(III) > P(II) > P(I)$

(d) $P(I) > P(II) > P(III)$

(e) $P(II) > P(III) > P(I)$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



6. The picture above shows a plot of a (rather artificial) one-dimensional wave function, $\psi(x)$ versus x, over the range $(-2a, +4a)$. The wave function vanishes for all other values of x. What is the probability that a measurement of the position of the particle would find it in the range $(+2a, +3a)$, i.e, from $x = +2a$ to $x = +3a$?

(a) 2/9

(b) 1/6

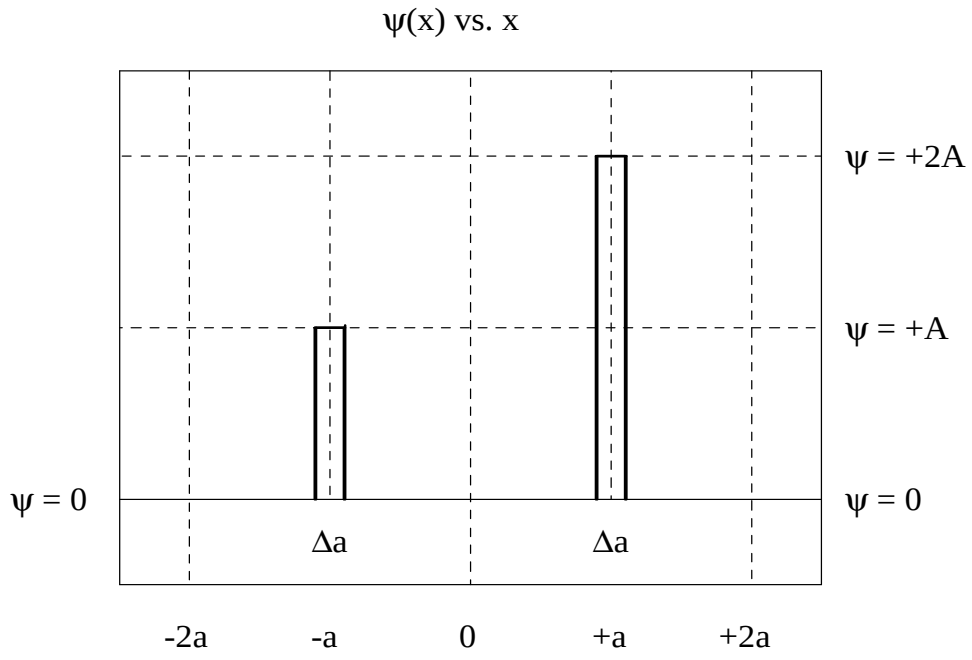
(c) 1/4

(d) 1/2

(e) 1/3

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



7. The figure above shows a plot of a (rather artificial) wave function, $\psi(x)$ versus x, given by the two 'spikes' shown, each of width Δa. The wave function vanishes for all other values of x not shown. Which of the expressions below is closest to the value of the **spread** or **uncertainty** in the position variable, namely Δx. Recall that

$$\Delta x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

where $\langle \cdots \rangle$ denotes **average** or **expectation value.**

(a) $\Delta x = 3a/5$

(b) $\Delta x = 2a$

(c) $\Delta x = 2\Delta a$

(d) $\Delta x = 4a/5$

(e) $\Delta x = \Delta a$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain

8. The position-space wave function, $\psi(x)$, of a particle at some instant is given by the functional form

$$\psi(x) = \frac{1}{[L^2 + (x - x_0)^2/\alpha^2]}$$

where L, x_0, α are all constants. Which expression below would be closest to the value of the **spread** or **uncertainty** in **momentum**, i.e., Δp, at that time. (You are not expected to evaluate any integrals for this problem.)

(a) $\Delta p \sim \hbar/(\alpha L)$

(b) $\Delta p \sim \hbar/\alpha$

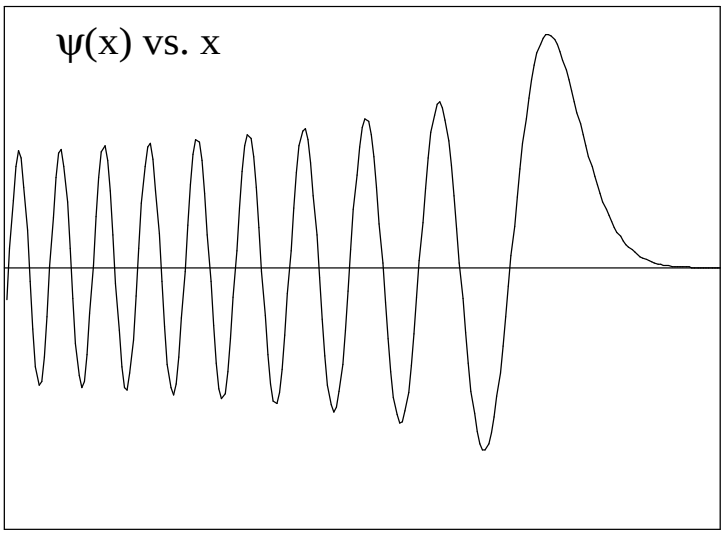
(c) $\Delta p \sim \hbar/(\alpha^2 x_0)$

(d) $\Delta p \sim \hbar/(\alpha^2 L^2)$

(e) $\Delta p \sim \hbar/(\alpha x_0)$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



9. A plot of a position-space energy eigenstate, $\psi(x)$ versus x, is shown above. To which classical system would this solution of the time-independent Schrödinger equation most likely correspond?

(a) A free particle, moving at constant velocity to the right.

(b) A free particle, moving at constant velocity to the left.

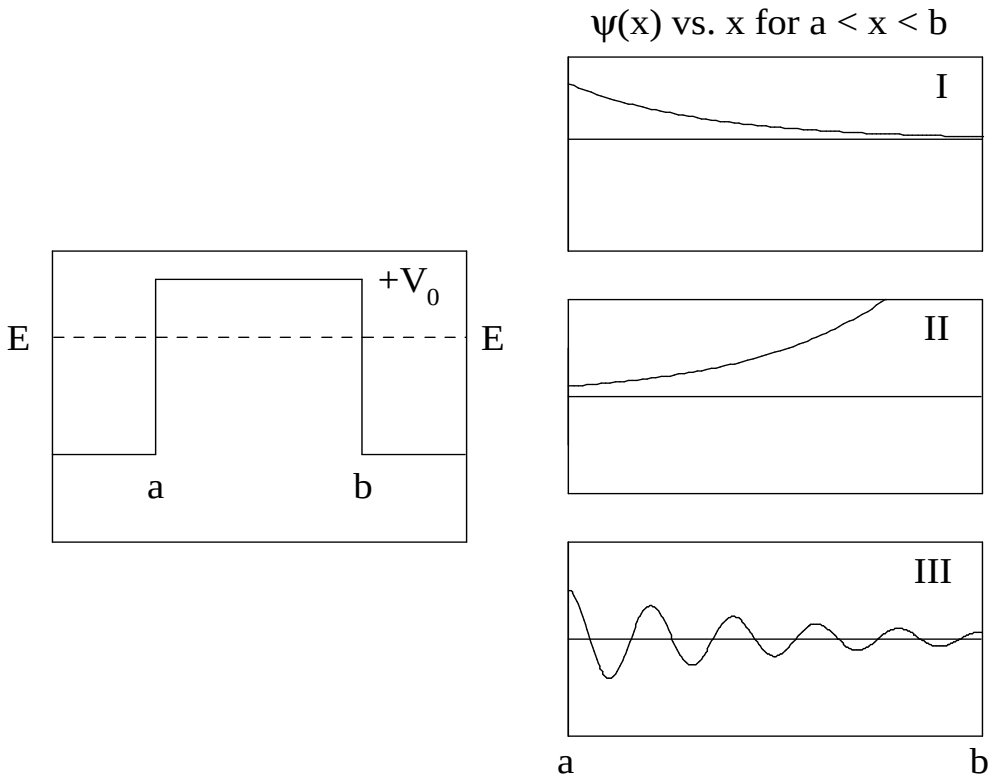
(c) A particle undergoing uniform acceleration to the right, starting from rest.

(d) A particle undergoing uniform acceleration to the left, starting from rest.

(e) An otherwise free particle, moving to the right, but colliding with an infinite wall.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



10. The figure on the left above indicates a particle of energy E near the boundary of a 'step up, then down' or rectangular barrier potential located between $x = a$ and $x = b$ as shown, with $0 < E < +V_0$. If you solve the time-independent Schrödinger equation in the region (a, b) using this potential and this energy, which of the possible wave functions on the right could you find?

(a) I and II only.

(b) III only.

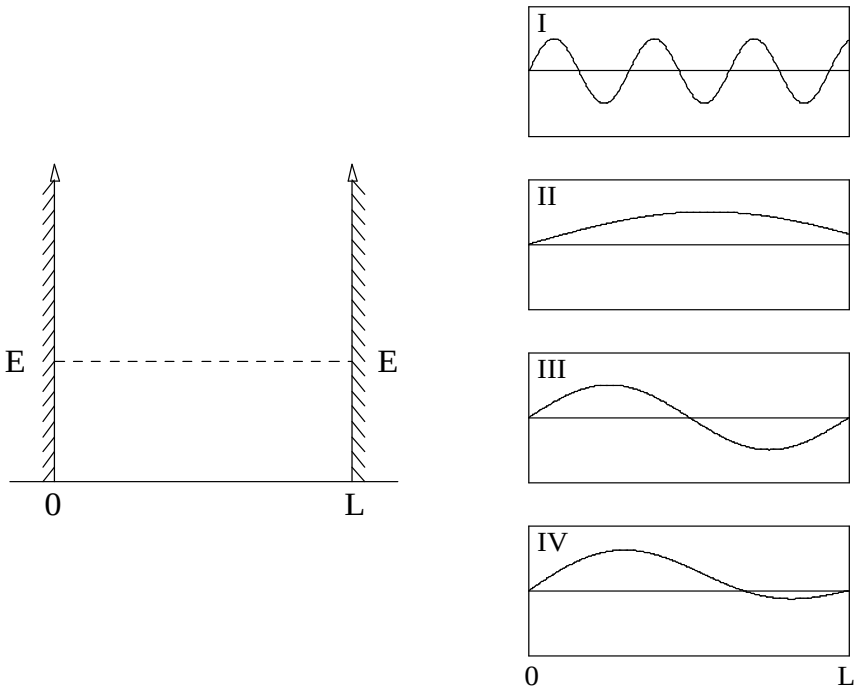
(c) I only.

(d) I and III only.

(e) I, II and III are all possible.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



11. You are given a computer program which is designed to solve the time-independent Schrödinger equation (TISE)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

for a particle inside the infinite well shown above (defined by impenetrable walls at $x = 0, L$ and with $V(x) = 0$ in between) for any value of $E > 0$. The program finds solutions to this particular TISE which also satisfy the boundary condition $\psi(0) = 0$ and outputs wavefunctions in a graphical format. A convenient reference value of energy is defined to be $E_{REF} = \hbar^2\pi^2/2mL^2$. Consider the following statements about the waveforms I – IV shown above:

I: You could get Fig. I as output for some value of $E >> E_{REF}$.
II: You could get Fig. II as output for some value of $E < E_{REF}$.
III: You could get Fig. III as output for $E = 2E_{REF}$.
IV: You could get Fig. IV as output for some value of E.

Which of these statements is (are) true?

(a) III and IV only

(b) III only

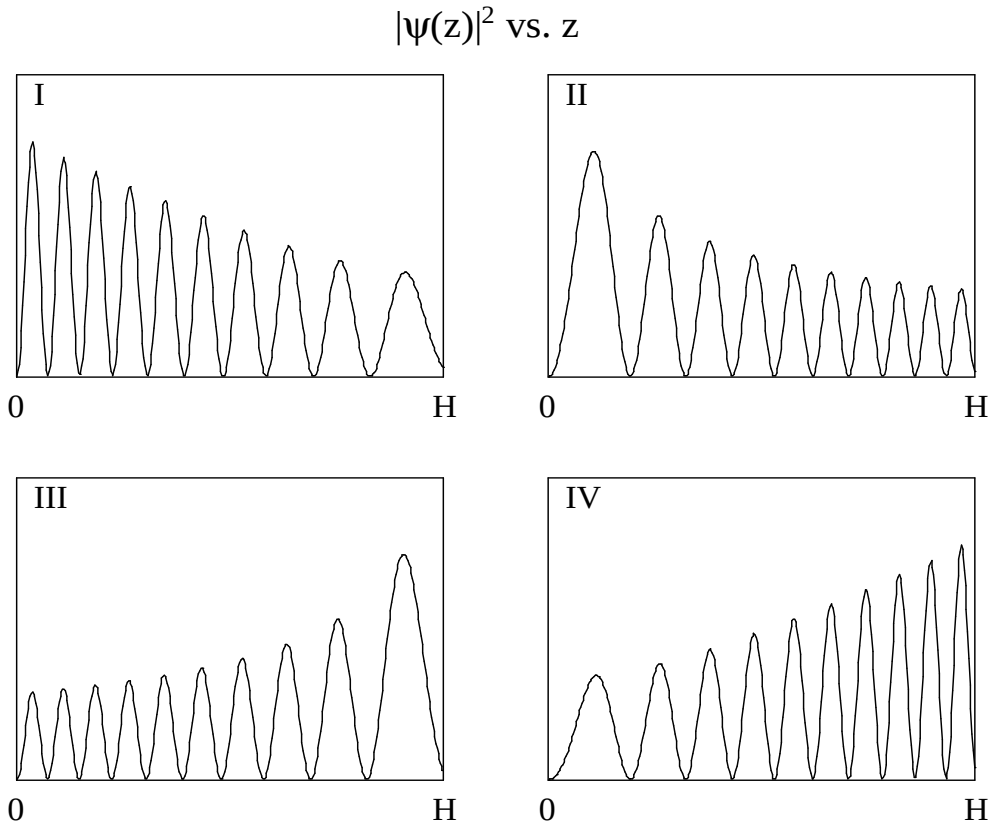
(c) I and II only

(d) I and III only

(e) None of them are true

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



12. A particle is dropped from a height H under the influence of gravity and bounces, without loss of energy, from a flat surface (at $z = 0$) back up to the same height. Which of the plots above would be the best representation of the quantum mechanical position-space probability density, $|\psi(z)|^2$ versus z, of an energy eigenstate for this system.

(a) I

(b) II

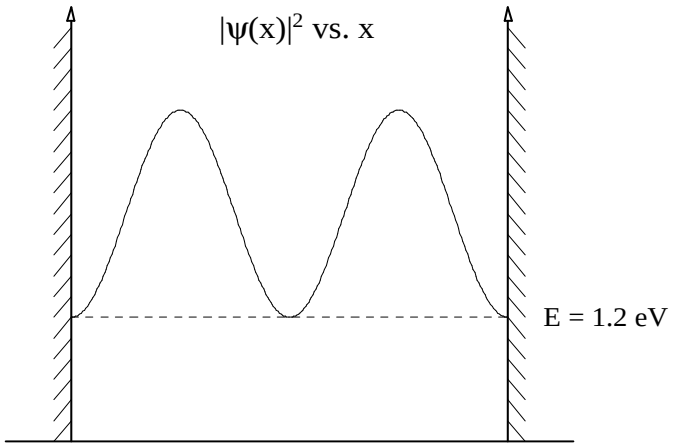
(c) III

(d) IV

(e) None of them are possible solutions of this problem.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



13. A particle in an infinite well (where $V(x) = 0$ between the otherwise impenetrable walls) is in the energy eigenstate corresponding to the probability density $|\psi(x)|^2$ versus x shown above. The energy of this state is $E = 1.2\,eV$. What is the energy of the lowest possible allowed energy state in this well?

(a) $0.0\,eV$

(b) $0.3\,eV$

(c) $0.4\,eV$

(d) $0.6\,eV$

(e) $1.2\,eV$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain

14. A beam of particles of energy E is incident, from the left, on a step potential of height $V_0 < E$ as shown in the figure on the next page. This problem is often analyzed using plane-wave solutions of the time-independent Schrödinger equation with incident (I), reflected (R) and transmitted (T) components represented by the solution

$$\psi(x) = \begin{cases} Ie^{ikx} + Re^{-ikx} & \text{for } x \leq 0 \\ Te^{iqx} & \text{for } 0 \leq x \end{cases}$$

where $k = \sqrt{2mE/\hbar^2}$ and $q = \sqrt{2m(E - V_0)/\hbar^2}$.

Which of the waveforms shown on the left and below the figure on the next page (namely I, II, III, IV, or V) best represents the true solution in this approach?

(a) I

(b) II

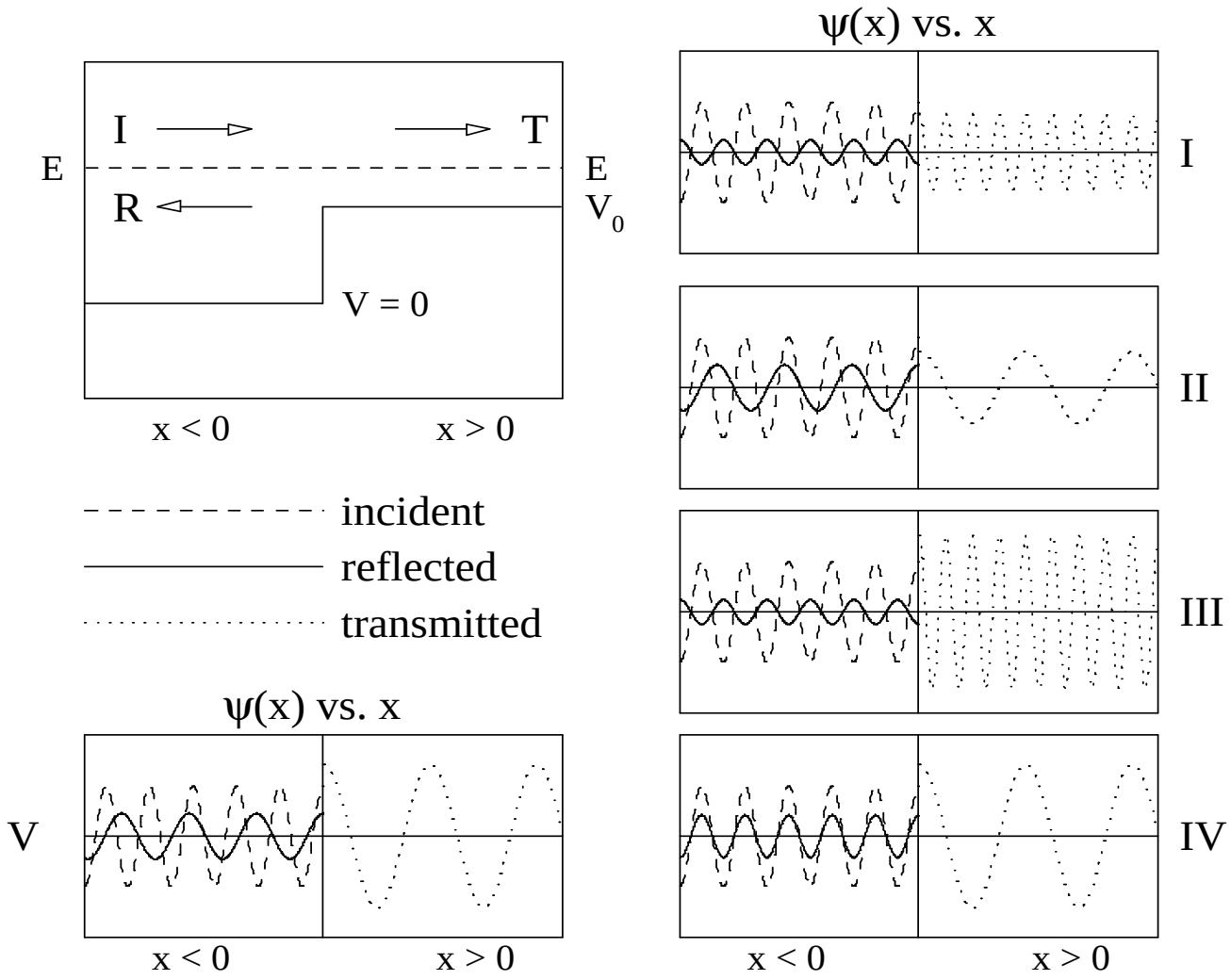
(c) III

(d) IV

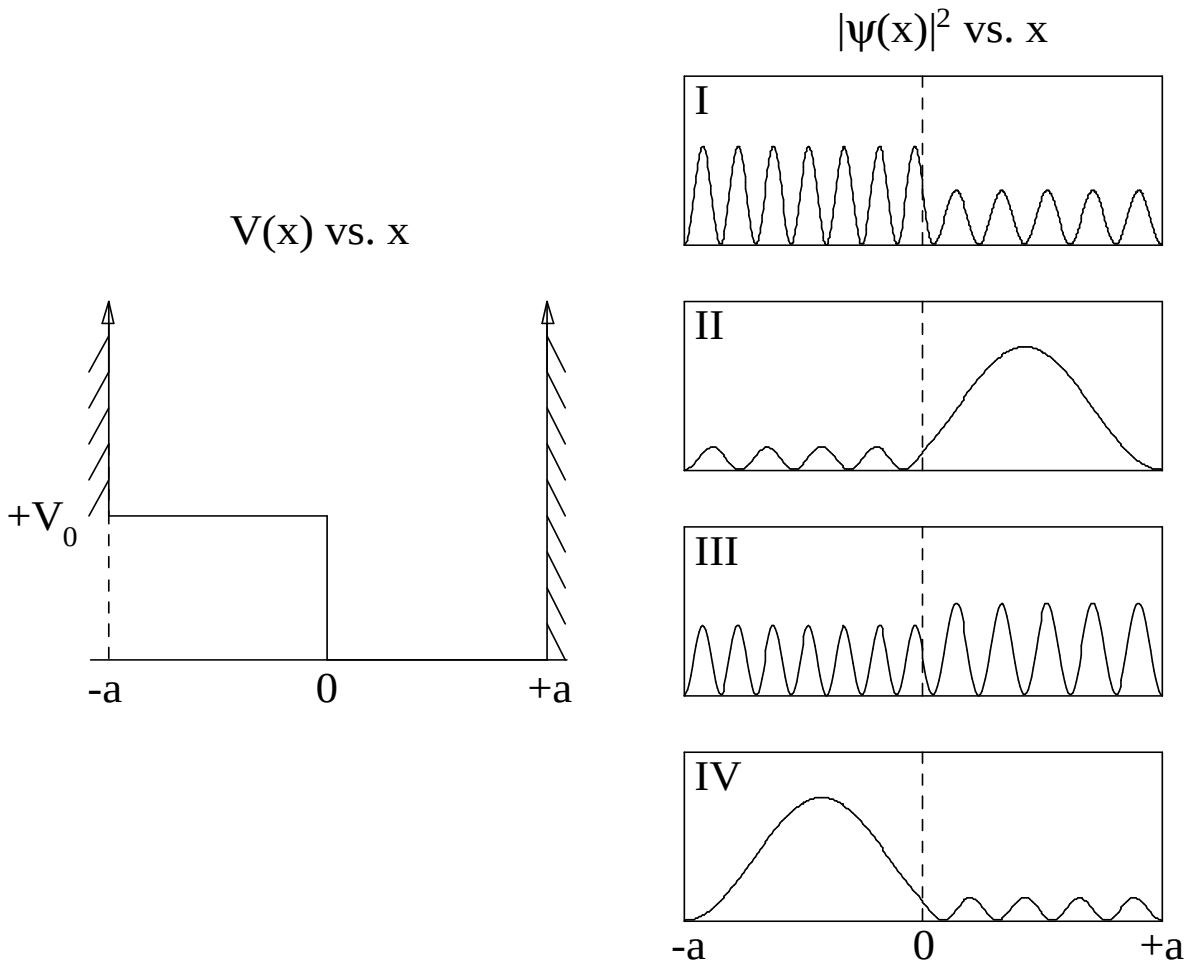
(e) V

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



Problem **14.** Figures I, II, III, IV, and V show possible solutions to the scattering problem illustrated in the upper left corner in terms of incident (dashed), reflected (solid) and transmitted (dotted) plane wave solutions.



15. The plot on the left shows a potential energy function, $V(x)$ versus x, corresponding to an 'asymmetric' infinite well. The infinite well is of width $2a$, with impenetrable walls at $x = \pm a$, but where $V(x) = +V_0$ for $-a < x < 0$ and $V(x) = 0$ for $0 < x < +a$. Of the figures on the right, which is/are most likely to be physically acceptable energy eigenstate solutions for the time-independent Schrödinger equation for this well?

(a) I and IV only

(b) I only

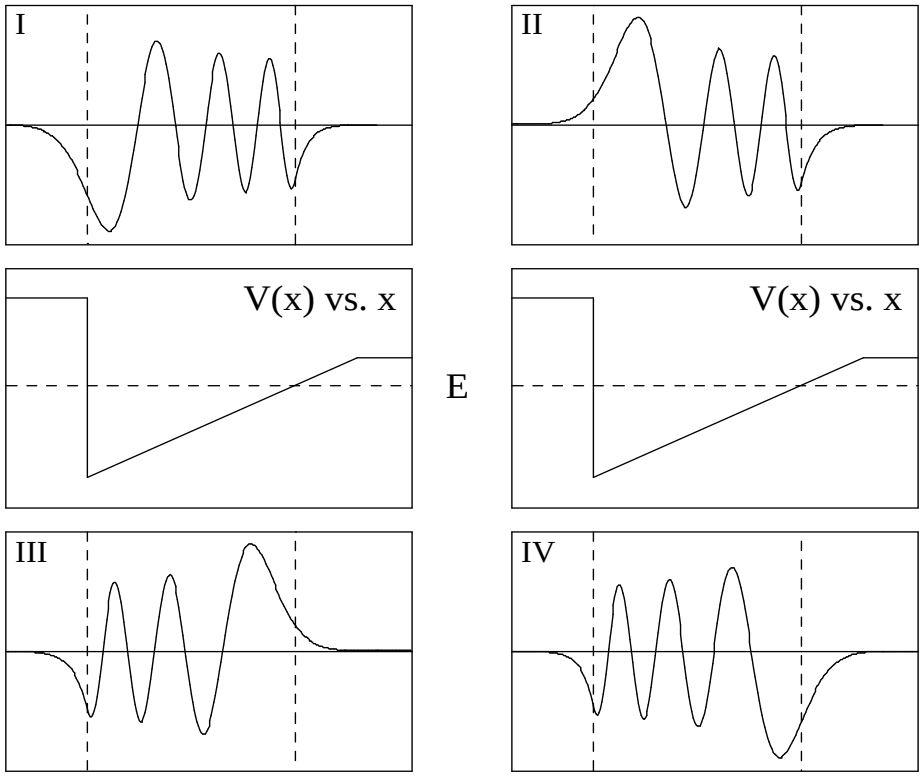
(c) II and III only

(d) II only

(e) IV only

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



16. The two identical pictures in the center of the figure above show a one-dimensional potential, $V(x)$ versus x, as well as a horizontal dashed line indicating a quantized energy level corresponding to the **sixth excited state**. Which of the wave function plots (shown on the top or bottom rows) is most likely to correspond to this state? Note that the vertical dashed lines in the $\psi(x)$ versus x pictures correspond to the locations of the classical turning points in the $V(x)$ plot. (Recall that the lowest energy state is conventionally called the **ground state**, the next higher energy level corresponds to the **first excited state** and so forth.)

(a) *I*

(b) *II*

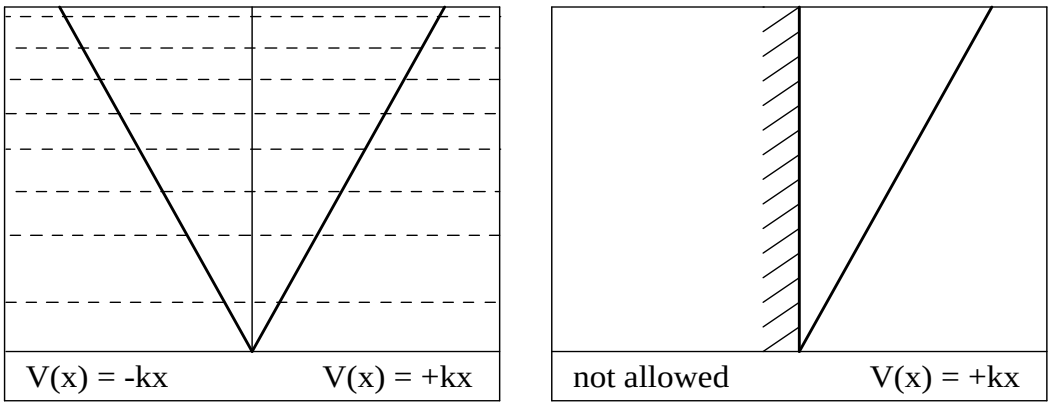
(c) *III*

(d) *IV*

(e) There is something inconsistent about each one so that none of them is an acceptable wave function for this state.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



17. The picture on the left above shows a one-dimensional potential energy function given by $V(x) = k|x|$ (solid lines) as well as the eight lowest allowed energy eigenstates (dashed horizontal lines.) These energies are given, in terms of a quantity $\mathcal{E}$, by the values

$$E = (1.0\,,\ 2.3\,,\ 3.2\,,\ 4.1\,,\ 4.8\,,\ 5.5\,,\ 6.2\,,\ 6.8\,, ...)\,\mathcal{E}$$

The plot on the right shows the 'half-well' version of this potential, namely that given by

$$V(x) = \begin{cases} \infty & \text{for } x < 0 \\ +kx & \text{for } x > 0 \end{cases}$$

so that the potential is the same for $x > 0$, but the particle is not allowed in the region $x < 0$. The most likely pattern of energy eigenvalues in the 'half-well' on the right is given by

(a) $E = (2.3\,,\ 4.1\,,\ 5.5\,,\ 6.8\,, ...)\mathcal{E}$

(b) $E = (1.0\,,\ 3.2\,,\ 4.8\,,\ 6.2\,, ...)\mathcal{E}$

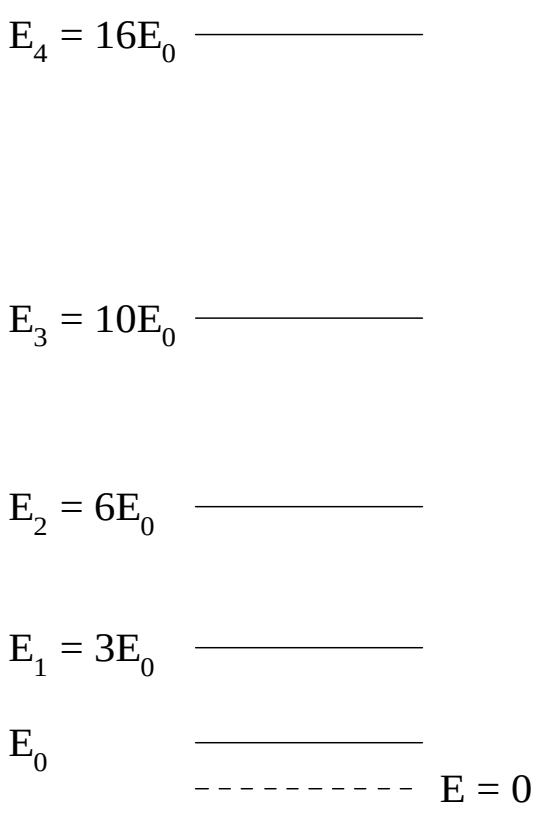
(c) $E = (1.0\,,\ 2.3\,,\ 3.2\,,\ 4.1, ...)\mathcal{E}$ (i.e., the same energy spectrum)

(d) $E = (1.0\,,\ 2.3\,,\ 3.2\,,\ 4.1, ...)(2\mathcal{E})$ (i.e., the energies are all doubled)

(e) $E = (1.0\,,\ 2.3\,,\ 3.2\,,\ 4.1, ...)(\mathcal{E}/2)$ (i.e., the energies are all halved)

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



18. The spectrum of a one-dimensional bound state system is shown in the figure above, starting with the ground state (E_0) up through the 4th excited state (E_4). Five indistinguishable electrons are known to be in this well; their mutual interactions can be neglected. What is the lowest possible total energy that such a state can have?

(a) 0

(b) $5E_0$

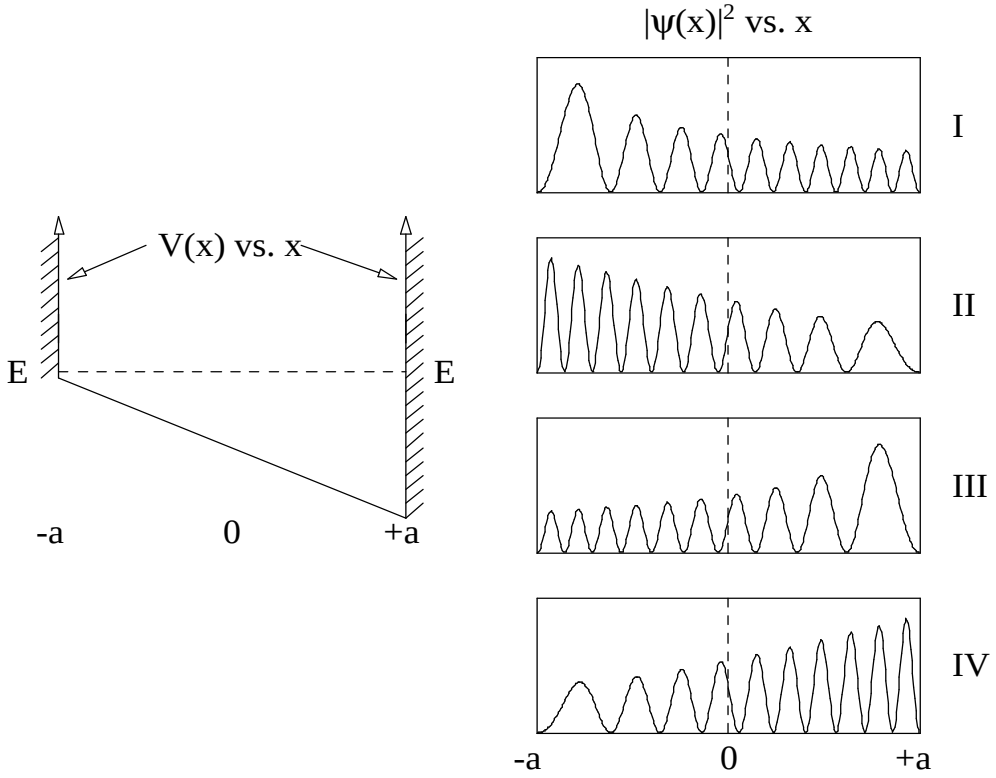
(c) $14E_0$

(d) $36E_0$

(e) $80E_0$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



19. On the left above is a picture of an infinite well potential, with impenetrable walls at $\pm a$, but with a slanted bottom so that one end is deeper than the other. A particular quantized energy level is indicated by the horizontal dashed line. Which of the wave functions on the right (plots of $|\psi(x)|^2$ versus x are shown) is most likely to correspond to this energy level? (For the wave function pictures on the right, the vertical dashed line indicates the center of the well at $x = 0$.)

(a) I

(b) II

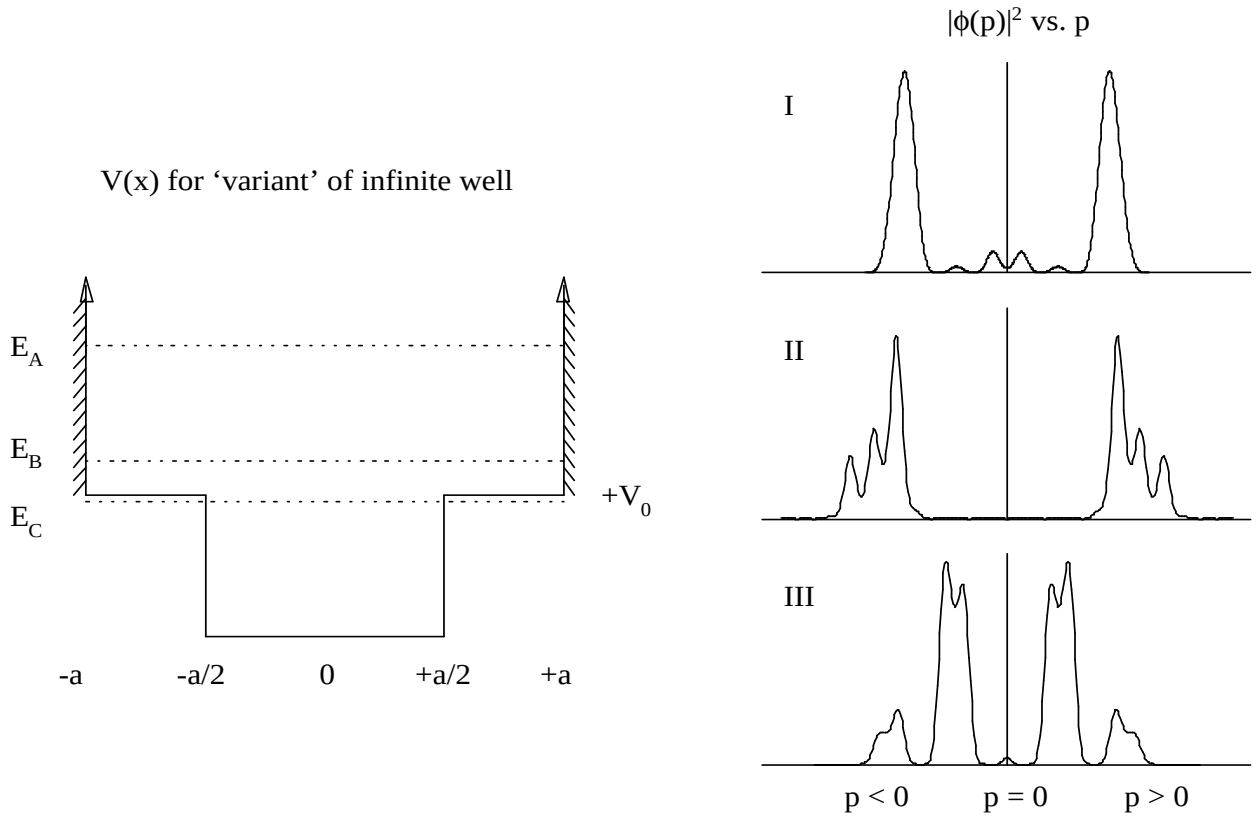
(c) III

(d) IV

(e) They are all possible solutions, depending on the exact numerical parameters of the well shown.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain

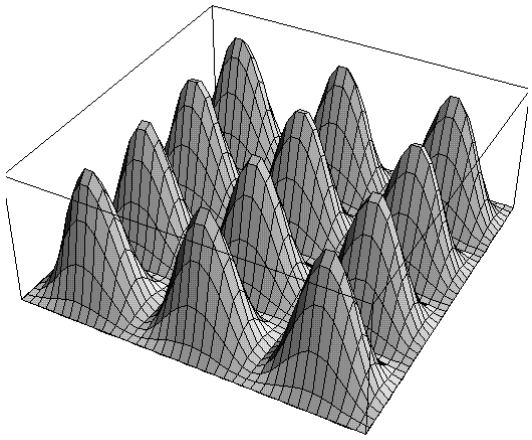


20. On the left above, there is a picture of three different quantized energy levels (E_A, E_B, E_C) in a variant of an infinite square well: it's an infinite well of width $2a$ with impenetrable walls at $x = \pm a$, but where $V(x) = 0$ for $-a/2 < x < +a/2$ and $V(x) = +V_0$ in the rest of the well. On the right are three different **momentum-space probability distributions**, plots of $|\phi(p)|^2$ versus p, corresponding to the three levels given by E_A, E_B, and E_C. Which momentum distribution (I, II, III) goes with which energy level? (Recall that the probability of finding the particle with momentum in the range $(p, p + dp)$ is given by $d\text{Prob}(p, p + dp) = |\phi(p)|^2\, dp$.)

(a) $E_A : I$ $E_B : III$ $E_C : II$
(b) $E_A : II$ $E_B : I$ $E_C : III$
(c) $E_A : III$ $E_B : II$ $E_C : I$
(d) $E_A : I$ $E_B : II$ $E_C : III$
(e) $E_A : II$ $E_B : III$ $E_C : I$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain | somewhat certain | somewhat uncertain | very uncertain



21. The plot above shows the position-space probability density, $|\psi(x,y)|^2$ versus (x,y), for an energy eigenstate of a particle of mass m in a **two-dimensional infinite square well** of length L on each side. The potential energy for such a well is given by

$$V(x,y) = \begin{cases} 0 & \text{for } 0 < x, y < L \\ \infty & \text{for } x > L,\ x < 0,\ y > L, \text{ or } y < 0 \end{cases}$$

What is the energy eigenvalue corresponding to this state?

(a) $E = 7\hbar^2\pi^2/2mL^2$

(b) $E = 6\hbar^2\pi^2/2mL^2$

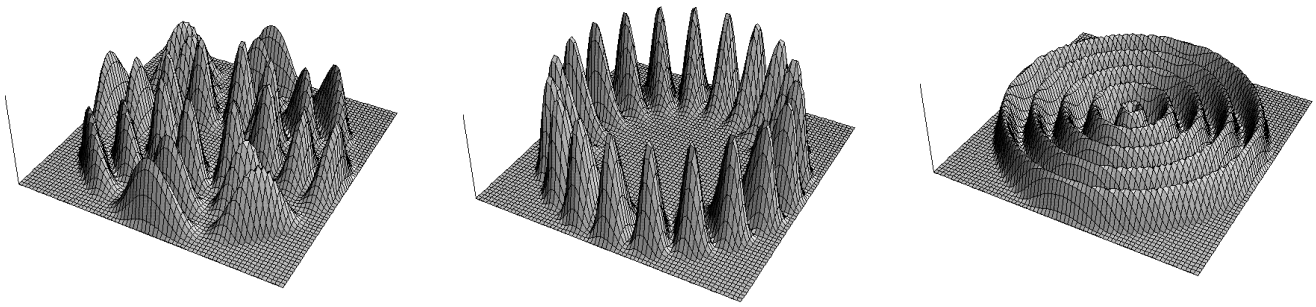
(c) $E = 13\hbar^2\pi^2/2mL^2$

(d) $E = 12\hbar^2\pi^2/2mL^2$

(e) $E = 25\hbar^2\pi^2/2mL^2$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



I *II* *III*

22. The plots above illustrate the position-space probability density for three different states for a particle in a **two-dimensional infinite circular well**. Such a potential is defined by

$$V(x,y) = V(r) = \begin{cases} 0 & \text{for } r < a \\ \infty & \text{for } r > a \end{cases}$$

The three different energy eigenstates above have the same **total energy**. Order the three states, I, II, and III, in terms of their **rotational** or **angular kinetic energy**, RKE, from **highest** to **lowest**.

(a) $RKE(II) > RKE(III) > RKE(I)$

(b) $RKE(III) > RKE(I) > RKE(II)$

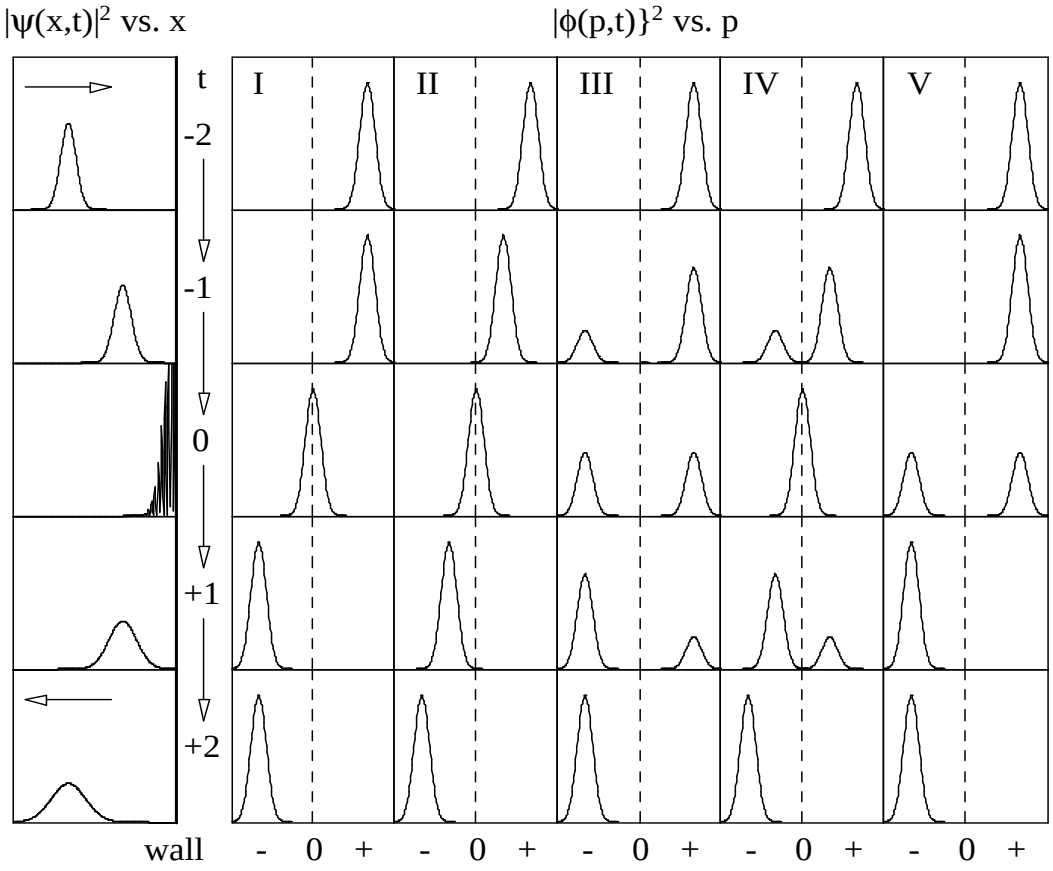
(c) $RKE(I) = RKE(II) = RKE(III)$

(d) $RKE(II) > RKE(I) > RKE(III)$

(e) $RKE(III) > RKE(II) > RKE(I)$

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



23. A position-space wave packet is constructed which represents an otherwise free particle which moves to the right, hits an impenetrable wall, 'bounces', and eventually moves freely to the left (i.e., $V(x) = 0$ to the left of the infinite wall.) Five 'snapshots' of the behavior of $|\psi(x,t)|^2$ versus x are shown in the left most column for times before ($t < 0$), during ($t \approx 0$), and after ($t > 0$) the collision. Which of the corresponding 'time-lapse' plots of the **momentum-space probability distributions**, $|\phi(p,t)|^2$ versus p, shown in the other vertical columns (labeled I, II, III, IV, and V) best represents the corresponding collision as seen in momentum-space? (Positive (+) and negative (-) values of momentum are indicated, with $p = 0$ denoted by the vertical dashed lines.)

(a) I

(b) II

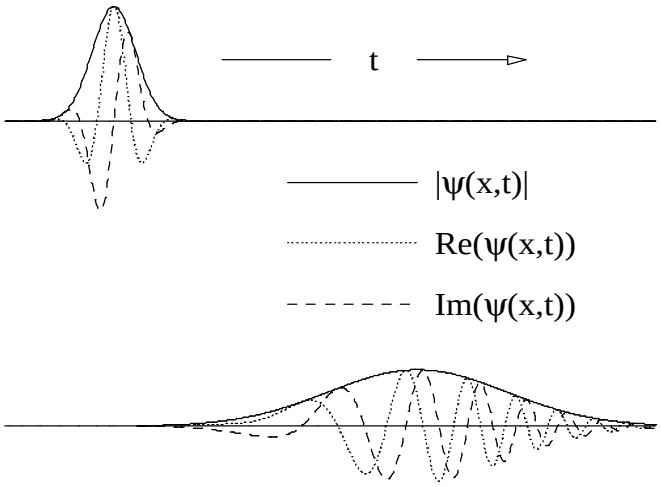
(c) III

(d) IV

(e) V

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



24. The real (dotted) and imaginary (dashed) parts of a position-space wave function representing a free particle wave packet, along with the modulus or absolute value (solid) of $\psi(x,t)$ is shown in the figure above. As time progresses, and the wave packet spreads, the real and imaginary parts of the wave packet appear to be more 'wiggly' in the leading edge of the wave than in the trailing edge. Which statement below best expresses what is shown in the figure above?

(a) All position-space wave functions spread with time because their real and imaginary parts have different phase velocities.

(b) All position-space wave functions spread with time because their real and imaginary parts get increasingly 'out of phase' with each other.

(c) The different momentum components which are used to construct a free particle wave packet travel at different speeds.

(d) As the spread in the position-space wave function, Δx_t, increases with time, the spread in the corresponding momentum-space wave function, Δp_t, must decrease with time to ensure that the uncertainty principle relation is maintained.

(e) The real and imaginary parts of any wave packet must spread in such a way that the modulus or absolute value $|\psi(x,t)|$ has the same **shape** as $|\psi(x,0)|$, but is just wider.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain

25. The pictures on the next page illustrate the time evolution of a wave packet in a one-dimensional infinite square well of width L, formed from energy eigenstates $u_n(x)$ with quantized energies E_n, by the linear combination

$$\psi(x,t) = \sum_{n=1}^{\infty} a_n u_n(x) e^{-iE_n t/\hbar}$$

with appropriate coefficients, a_n. (Plots of $|\psi(x,t)|^2$ versus x for various times are shown.) The wave packet is designed to travel initially to the right, returning to something like its initial state after a time t_{CL} (its classical period) and two bounces with the walls, as shown by the figure on the left. The behavior of the wave packet after 4 and 8 classical periods is also shown, on the right. For even later times, the wave packet will:

(a) Permanently reform into two, distinct peaks, reflecting the presence of left-moving and right-moving momentum components.

(b) Reform exactly into the initial state after a time which depends on the initial spread (Δx), average energy ($\overline{E}$) of the wave packet, and Planck's constant ($\hbar$), repeating the cycle thereafter.

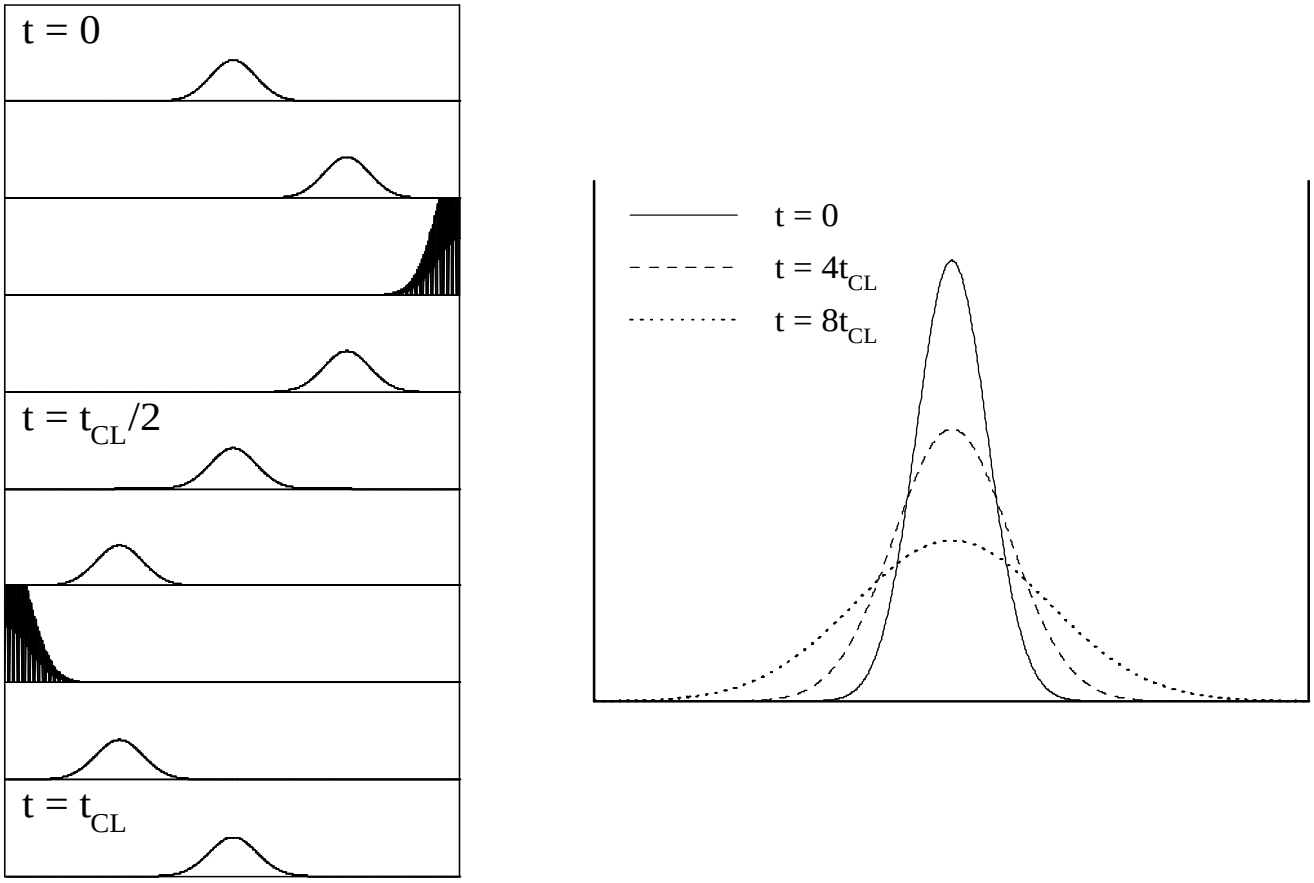
(c) Reform exactly into the initial state after a time which is determined by the width of the well (L), the mass of the particle (m), and Planck's constant ($\hbar$), repeating the cycle thereafter.

(d) Continue to bounce, spread, and flatten, eventually approaching a **non-zero**, constant value of $|\psi(x)|^2$ over the entire well for all later times.

(e) Continue to bounce, spread, and flatten, eventually approaching a **vanishing** value of $|\psi(x)|^2$ over the entire well for all later times.

↑ Explain your answer in 1-2 sentences in the space above. ↑
↓ Circle the statement below which describes how you feel about your answer. ↓

very certain	somewhat certain	somewhat uncertain	very uncertain



Problem **25.** Plots of $|\psi(x,t)|^2$ versus x for a wave packet in an infinite well. Plots over various times during the first classical period (left) and after 4 and 8 periods (right) are shown.

November 2000
Version 0.4

Quantum Mechanics Visualization Instrument (QMVI)

The survey you are about to complete is designed to probe students' understanding of concepts in quantum theory, especially their ability to visualize quantum mechanical ideas and phenomena, and how this ability changes over the undergraduate career. This test is being given to students ranging from undergraduates in their sophomore year (modern physics type classes) to first year graduate students. Because of this, the range of difficulty you encounter in the instrument may seem to vary dramatically from question to question. Look at each item carefully and use whatever knowledge you can bring to bear to answer it as best you can.

The test consists of 25 multiple choice questions. For each item, please read the question, using the accompanying graphic, if appropriate, and then

- Circle the best answer (a) , (b), (c), (d), or (e).
- Briefly explain (in one or two sentences) your reasoning in picking that answer in the space provided.
- Circle the statement at the bottom of the page which best describes how you feel about your answer, judging how confident you are in your response.

Good luck and have fun!

Name: _____

Soc-Sec-No: _____

Course: _____