Subject: Work and Energy

Used for: As an alternative to using forces. The Work-Energy Theorem often makes a problem a lot easier to solve (no need for vectors).

Prior knowledge: -

Kinetic energy:

Often you can use kinetic energy to find the speed of an object: $K = \frac{1}{2}mv^2$

Work:

If an object moves a distance d while a force F is applied to it, the work done by the force is:

$$W = Fd\cos\theta$$

Here θ is the angle between the direction of force F and the direction of displacement d.

- Friction always does negative work, because f_k is in the opposite direction to the displacement of the object, so: $W_f = f_k d\cos(180°) = -f_k d$.
- The normal force never does any work, because it is perpendicular to the displacement: $W_N = F_N d\cos(90°) = 0$.

Potential energy:

Use potential energy whenever you work with gravity or springs:

Equation	Name	Notes
$U_g = mgh$	Gravitational potential energy	$g = 9.8$ m/s^2 on the Earth. Set the lowest point in the problem (the ground) to $h = 0$ such that there $U_g = 0$.
$U_s = \frac{1}{2}kx^2$	Spring potential energy	x is indicates how much the spring is compressed or stretched, and $x = 0$ if it's not stretched or compressed at all. k is called the spring constant.

Mechanical energy:

Simply sum of kinetic energy and potential energy: $E_{mech} = K + U$

Work-Energy Theorem:

Use this equation to compare to situations, the initial state and the final state. The total work done on an object is equal to the total change of its mechanical energy:

$$W = \Delta U + \Delta K = U_f - U_i + K_f - K_i$$

Power:

The amount of work per time is power: $P = W/t$ which has units of J/s or W (Watt).

Extended example:
A 1-kg ball is dropped from a height of 6 meters. As it falls, it is constantly acted upon by air resistance, whose average force on the ball is 3.3 N. Taking this into account, calculate the speed with which the ball hits the ground.

A. 9.0 m/s   B. 10.0 m/s   C. 10.6 m/s   D. 11.1 m/s

This problem can be solved using forces and kinematics, but because they ask for the *speed* (and not acceleration for example) we could use the Work-Energy Theorem. Finding the kinetic energy means we've found the speed.

$$W = U_f - U_i + K_f - K_i$$

It is here easy to identify the two different states:

Initial state
The ball is at a height of 6 meters and practically still at rest.

$$U_i = mgh_i = (1)(10)(6) = 60 \text{ J} \qquad K_i = \tfrac{1}{2} m v_i^2 = \tfrac{1}{2}(1)(0)^2 = 0 \text{ J}$$

Final state
The ball is about to reach the ground with a speed v (which were are trying to find).

$$U_f = mgh_f = (1)(10)(0) = 0 \text{ J} \qquad K_f = \tfrac{1}{2} m v_f^2 = \tfrac{1}{2}(1)v^2 = \tfrac{1}{2}v^2$$

We should not forget the work done by the friction. The displacement while going from the initial state to the final state is 6 m (going down). The (average) force is equal to 3.3 N (directed upwards). The angle between the force and displacement is 180 degrees, and thus

$$W_f = f_k d \cos\theta = (3.3)(6)\cos(180°) = -19.8 \approx -20 \text{ J.}$$

Putting everything together, we find

$$W = U_f - U_i + K_f - K_i \qquad = -20 = 0 - 60 + \tfrac{1}{2}v^2 - 0$$

from which follows $v = \sqrt{80} \approx 9.0$ m/s.