

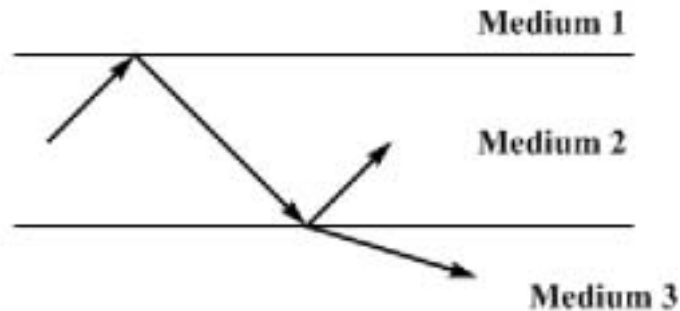
Solutions

Problem 1.

What you will need: Snell's Law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$. The law of reflection.

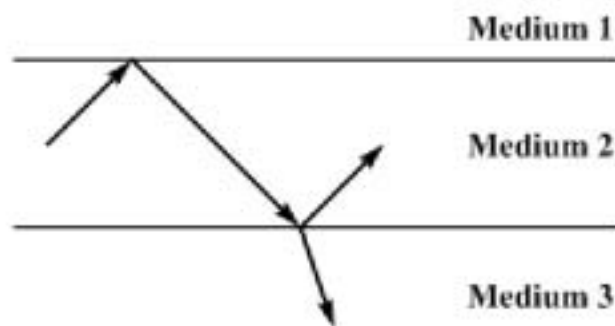
Critical angle for total internal reflection: $\sin(\theta_c) = \frac{n_2}{n_1}$

a)



There is a total internal reflection on the medium 1/2 surface. Since light is coming from medium 2 $n_2 > n_1$. When the reflected light comes to the medium 2/3 surface it's incident angle is the same as that at medium 1/2 due to the reflection law. Since there's no total internal reflection in this case $n_3 > n_1$. However, $n_3 < n_2$ since the refracted angle is greater than the incident one. Thus, $n_2 > n_3 > n_1$.

b)



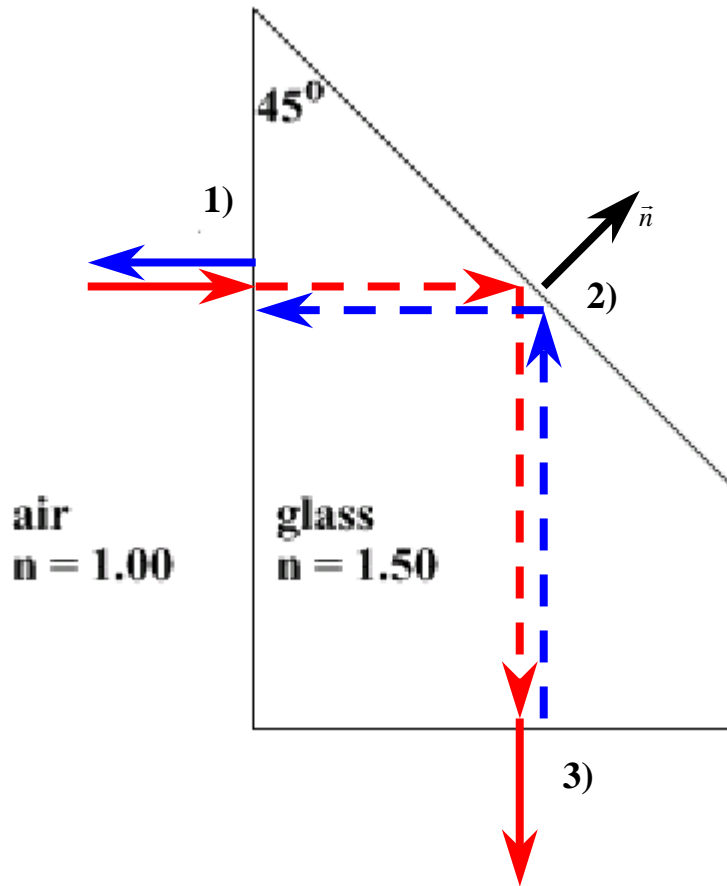
Similarly to case a) $n_2 > n_1$. However, $n_3 > n_2$ since the refracted angle at medium 2/3 surface is less than the angle of incidence. Thus, $n_3 > n_2 > n_1$.

Problem 2.

What you will need: Snell's Law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$. The law of reflection.

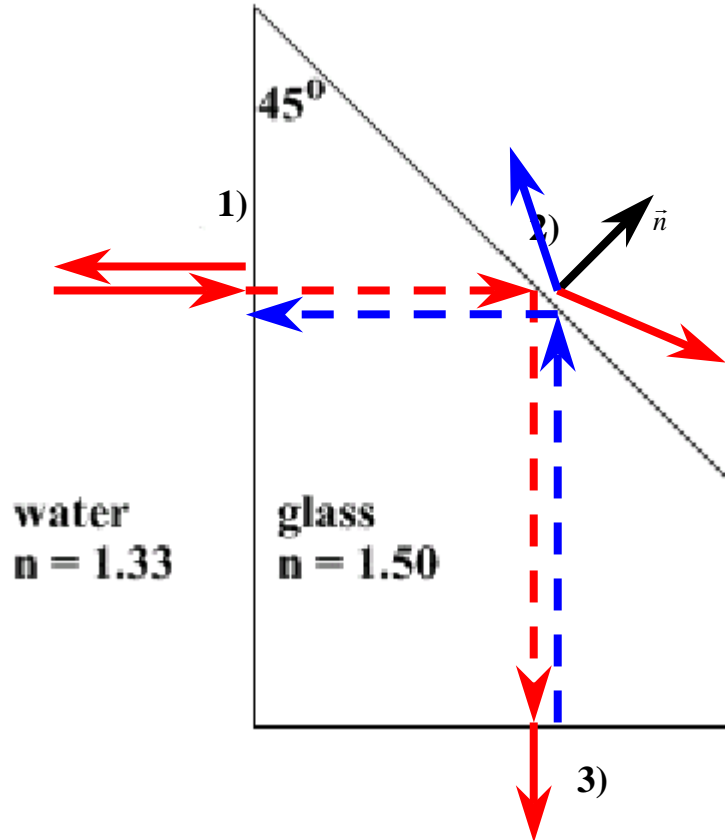
Critical angle for total internal reflection: $\sin(\theta_c) = \frac{n_2}{n_1}$

a)



When laser beam is incident on the first surface certain part of it is reflected straight back (by the reflection law) while the rest is refracted at zero angle since incident angle is zero. At the second surface there is no refraction due to the total internal reflection. The incident angle is 45° due to the geometry of the problem and $\frac{n_{air}}{n_{glass}} = 2/3 < \sin(45^\circ)$. Reflected beam comes to surface 3 and now one part of it is reflected straight back while the rest is straight forward. Reflected part of the beam comes back to surface 2 and experiences total internal reflection again since the incident angle is again 45° . Then reflected light comes to surface 1 and the circle closes.

b) The only difference here is that there will be refraction on surface 2 since $\frac{n_{\text{water}}}{n_{\text{glass}}} = 0.89 < \sin(45^\circ)$. Note that there will be 2 refractions at surface 2!



Refracted angles at surface 2 are $\theta_r = \text{ArcSin}\left(\frac{n_{\text{glass}}}{n_{\text{water}}}\sin(45^\circ)\right) \approx 52.9^\circ$

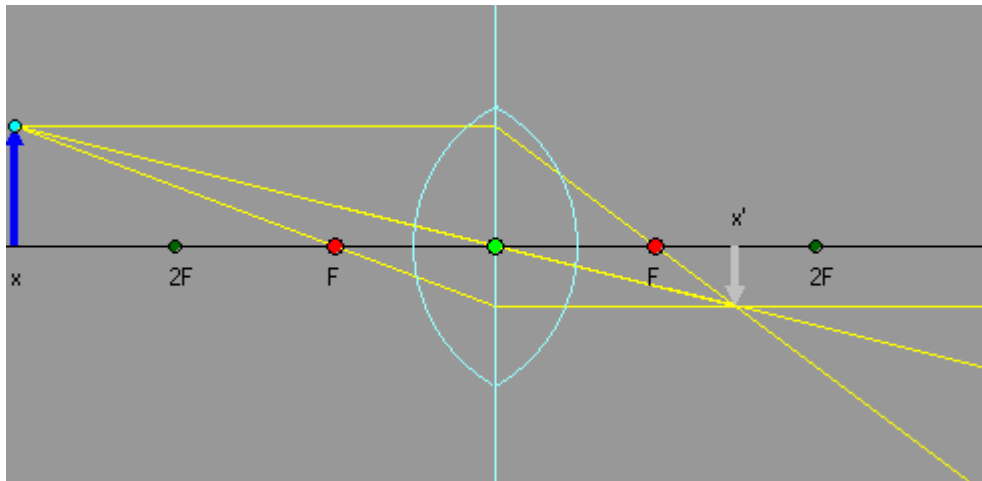
Problem 3.

What you will need: Thin-Lens Equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$. Definition of

Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$.

a) In one of the solutions we use convex mirror and place the object beyond double focus. The real inverted image is formed on the right hand side.

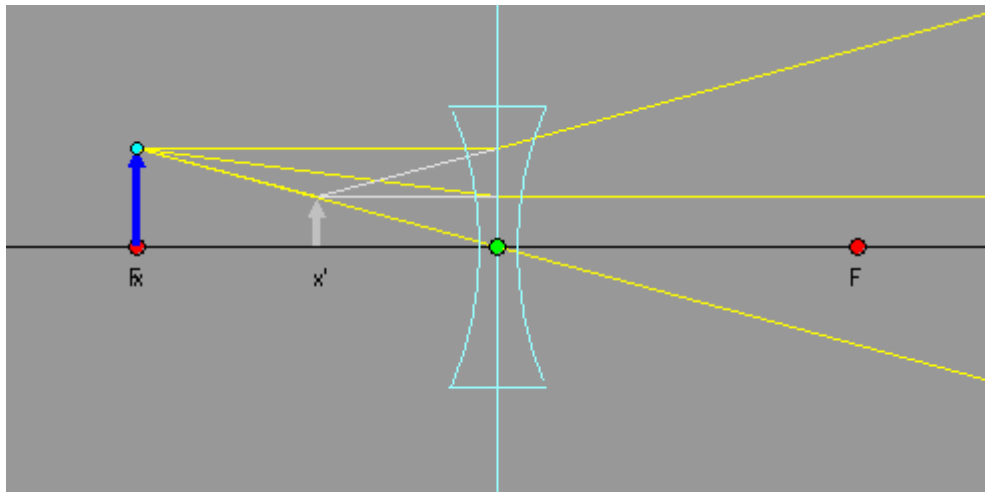
b)



c) $m = -\frac{1}{2} = -\frac{d_i}{d_o}$. $d_i = \frac{1}{2}d_o$. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{2}{d_o} = \frac{3}{d_o} = \frac{1}{f}$. Thus, $d_o = 3f$,
 $d_i = \frac{3}{2}f$.

d) Alternatively, we can use concave lens. The formed on the left hand side image will be virtual and upright

e)



f) $m = \frac{1}{2} = -\frac{d_i}{d_o}$. $d_i = -\frac{1}{2}d_o$. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{2}{d_o} = -\frac{1}{d_o} = \frac{1}{f}$. Thus, $d_o = -f$,
 $d_o = \frac{f}{2}$.

Problem 4.

What you will need: *Thin-Lens Equation*: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$.

It's also good to know that a normal near point is 25 cm and a normal far point is infinity.

Case 1) The young student is a bit farsighted since his near point is further than 0.25m. He needs corrective lens that will create image of a 0.25m object at 0.5m. Thus, $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.25 - 0.02} - \frac{1}{0.5 - 0.02} = \frac{1}{0.44} = \frac{1}{f}$. Here we took into account that the lens is placed at 0.02cm from the eye.

Case 2) The university student is nearsighted since his far point is closer than infinity. He needs corrective lens that will create image of a remote object placed at 0.5m. Thus, $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} - \frac{1}{0.5 - 0.02} = \frac{1}{0.48} = \frac{1}{f}$. Here we took into account that the lens is placed at 0.02cm from the eye.

Case 3) The athlete is fine.

Case 4) Professor is lucky his far and near points don't overlap! He needs a double lens (i.e., bifocals) to fix his vision. Similarly to cases 1) and 2) he needs dual lens glasses. For farsightedness we have

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.25 - 0.02} - \frac{1}{1.0 - 0.02} = \frac{1}{0.30} = \frac{1}{f}. \text{ For nearsightedness}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} - \frac{1}{0.9 - 0.02} = -\frac{1}{0.98} = \frac{1}{f}.$$