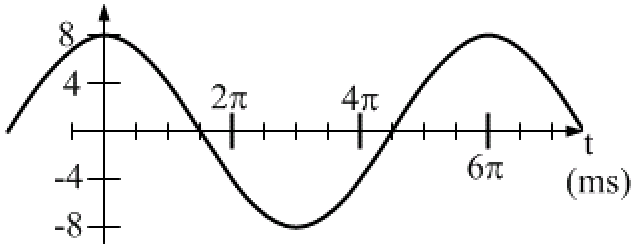
Solution to SC528 Sample Test 1

PROBLEM 1 – 15 points



[6 points] (a) Let's say the graph above shows the acceleration as a function of time for a Thanksgiving turkey experiencing simple harmonic motion (oscillating along a line parallel to the x-axis), and the units on the vertical scale are in cm/s^2. Note that the units on the time axis are milliseconds. Come up with an equation describing the x-position of the turkey as a function of time. Hint: if you find the amplitude and the angular frequency that should make it easier to write an appropriate equation.

From the equations:

$$x(t) = A\cos(\omega t)$$
$$v(t) = -A\omega\sin(\omega t)$$
$$a(t) = -A\omega^2\cos(\omega t)$$

and from the graph, we know that $-A\omega^2$ = 8cm/s^2 = 0.08m/s^2. Therefore,

$$A = -\frac{0.08m/s^2}{\omega^2}$$

From the graph we also know that the period *T* is 6π ms = 6π x10^{-3} s. We can find the angular frequency using:

$$\omega = \frac{2\pi}{\mathrm{T}} = \frac{2\pi}{6\pi x10^{-3}} = \frac{1000}{3s}$$

So $A = -0.08m/s^2 . \frac{1}{\frac{1000^2}{(3s)^2}} = -72x10^{-8}m$

Therefore, $x(t) = -72x10^{-8}\cos(\frac{1000}{3}t)$

[3 points] (b) If we define A as the amplitude of the oscillation, and define x = 0 as the turkey's equilibrium position, at what location on the x-axis is the turkey at t = 0?

[**X**] x = -A [ ] x = -A/2 [ ] x = 0 [ ] x = +A/2 [ ] x = +A

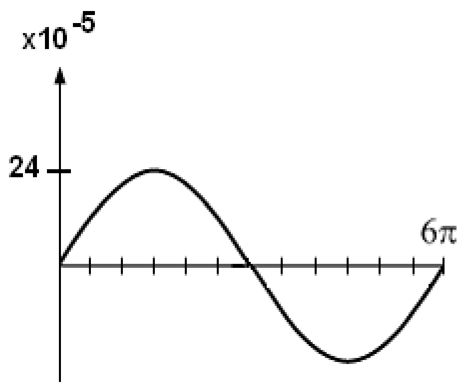
Plug t = 0 in the equation we got for a) to find that the initial position is -72x10^{-8} m. The amplitude A is the absolute value of this number, so x(t=0)=-A

[6 points] (c) Sketch a graph of the **x-velocity** of the turkey as a function of time from t = 0 to t = 6π ms. What is the turkey's maximum speed?

Using again the equations

$$x(t) = A\cos(\omega t)$$
$$v(t) = -A\omega\sin(\omega t)$$
$$a(t) = -A\omega^2\cos(\omega t)$$

we see the difference between the graphs of the position vs time and acceleration vs time. The amplitude of the oscillation is different, the signs are opposite, and instead of a cosine function we have a sine function. So our graph should look like the graph of a positive sine, with amplitude $A\omega = 24x10^{-5}\,m/s$ (which corresponds to the maximum speed) and same period as the accelerations graph:



PROBLEM 2 – 15 points

A 2.0-kg bowl of stuffing oscillates left and right with simple harmonic motion on a frictionless horizontal table. The spring attached to the bowl has a spring constant k = 8.0 N/m. At time t = 0, the mass is released from rest at a position x_o = +3.0 meters to the right of its equilibrium position. **You can leave answers in terms of π if necessary.**

[3 points] (a) What is the magnitude and sign of the force F_o exerted by the spring at x_o? What is the magnitude and sign of resulting acceleration a_o?

$$F = -kx \Rightarrow F_o = -kx_o = -8N/m * 3m = -24N$$

$$a = \frac{F}{m} = -\frac{24N}{2kg} = -12m/s^2$$

[3 points] (b) What is the magnitude and sign of the velocity v_{eq} of the bowl as it first passes through the equilibrium position?

The velocity vs time graph is a sine function. The time it takes for the bowl to reach the equilibrium position is a fourth of the period, and the sine reaches its maximum value at t = T/4. Therefore, the velocity is

$$v\left(t = \frac{T}{4}\right) = -A\omega$$

We know that, for a spring, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{2}} = 2s^{-1}$**, and A = 3m. Therefore,**

$$v\left(t = \frac{T}{4}\right) = -6m/s$$

[3 points] (c) What is the magnitude and sign of the acceleration a_{eq} of the bowl as it first passes through the equilibrium position?

The acceleration vs time graph is a cosine function. The cosine is zero at T/4, and hence the acceleration is zero.
Also, F = -kx = ma. In the equilibrium position, x = 0 and so a = 0.

[3 points] (d) At what times t_+ does the bowl return to the release point? State the first four times after t = 0.

It takes a full period to reach the initial position. We know the angular frequency, so we can calculate the period:

$$T = \frac{2\pi}{\omega} = \pi \text{ s}$$

So the first four times are: π, 2π, 3π and 4π.

[3 points] (e) At what times t_{eq} does the bowl pass through the equilibrium position? State the first four times after t = 0.

We already know that at T/4 the bowl first passes through the equilibrium position. At t=T/2 it reaches its minimum value, and at t=3T/4 it crosses again the equilibrium position. Then the cycle is repeated, so the times are π/4, 3π/4, 5π/4 and 7π/4.

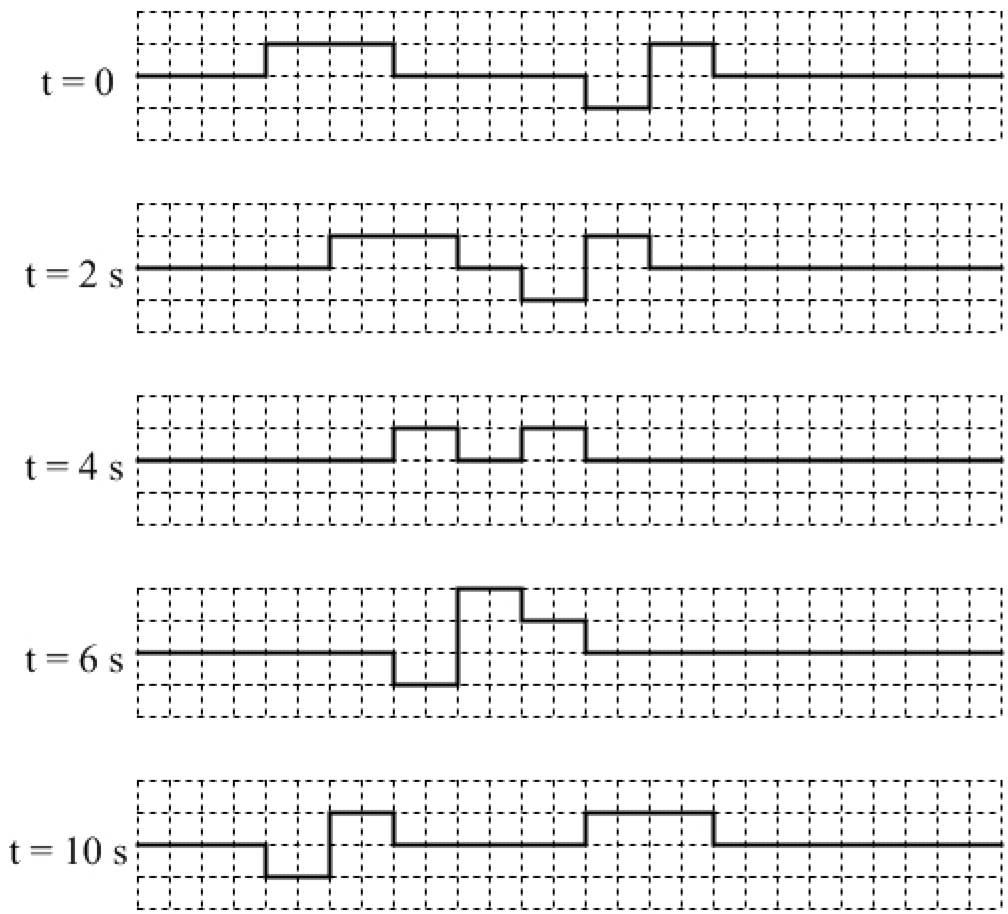
Another way is to use the equation of position,

$$x = A\cos(\omega t) = 0$$
$$\Rightarrow \omega t = \frac{(2n-1)}{2}\pi$$
$$\Rightarrow t = \frac{(2n-1)}{4}\pi$$

with n = 1, 2 , 3 and 4

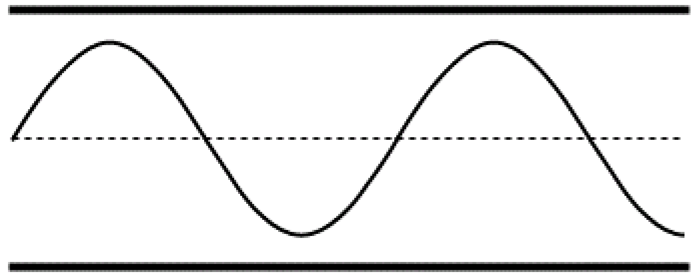
PROBLEM 3 – 12 points

The top graph below shows a picture of a string at t = 0, showing the position of two pulses moving along the string. The second graph shows a picture of a string at t = 2 s. Show what the string looks like at t = 4 s, t = 6 s, and t = 10 s.



The key here is the concept of superposition. At any time to find the shape of the string just add one pulse to the other over the ranges where they overlap. At t = 10 s they have passed through each other and continue on as if they had never met. At 4 s and 6 s, however, they partly overlap. At t = 4 s this leads to some cancellation, while at t = 6 s they reinforce one another.

PROBLEM 4 – 15 points

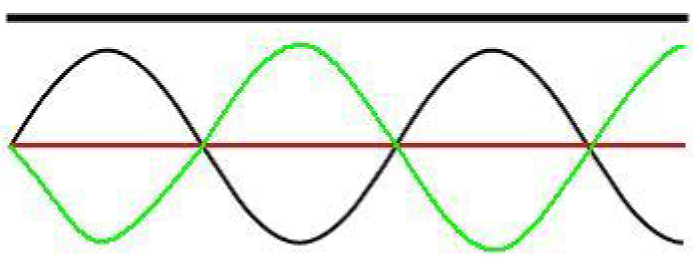


[3 points] (a) The diagram above represents a standing wave in a tube at a particular instant in time. The frequency of the wave is one of the resonant frequencies for the tube. What kind of tube is it? (The diagram shows the outline of the sides of the tube but does not indicate whether the ends are open or closed.)

[ ] a tube open at both ends [**X**] a tube open at one end and closed at the other

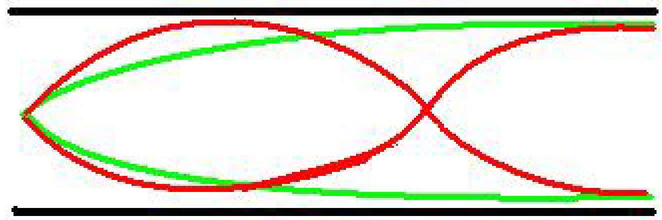
[4 points] (b) The next time the standing wave in the tube can be represented by the diagram above is a time T after the time shown above. Assuming the diagram shows the standing wave at its maximum amplitude, sketch (on the diagram above) the standing wave at a time t = T/2 and a time t = 5T/4 after the time represented above. Clearly indicate which sketch is which.

Since after a time T we have the same wave, T is the period. Then, after a time T/2, the displacements at each point will be opposite of the value seen in the above diagram, which is shown with green below. Aftertime 5T/4 passes, the shape of the wave will be exactly the same as the shape of the wave at T/4 because T is the period and 5T/4 – T = T/4. At time T/4, the shape of the wave is exactly halfway from the shape of it at 0 to the shape of it at T/2, which is shown with red below.



[4 points] (c) Assume the speed of sound is 340 m/s, and the fundamental frequency of this tube is 170 Hz. What frequency does the wave shown above correspond to?

The wave mode corresponding to fundamental frequency is shown with green in the following figure. Next mode is shown with red.



Looking at the previous figure, if we take the length of the pipe L and the wavelength of the wave is λ, their relation can be written as:

$$L = \frac{(2n+1)}{4}\lambda$$

For the fundamental mode:

$$L = \frac{(2.0+1)}{4}\lambda_0 = \frac{\lambda_0}{4}$$

$$\lambda_0 = 4L$$

For the mode given at the beginning:

$$L = \frac{(2.3+1)}{4}\lambda_3 = \frac{7\lambda_3}{4}$$

$$\lambda_3 = \frac{4}{7}L$$

We know that

$$v = \lambda . f$$

Then the fundamental frequency is:

$$f_0 = \frac{v}{\lambda_0} = \frac{v}{4L}$$

and the frequency of the mode given:

$$f_3 = \frac{v}{\lambda_3} = \frac{7v}{4L}$$

$$\frac{f_0}{f_3} = \frac{v}{4L} / \frac{7v}{4L}$$

$$\frac{f_0}{f_3} = \frac{1}{7}$$

$$\underline{f_3 = 7 f_0 = 1190 Hz}$$

[4 points] (d) How long is the tube?

$$\lambda_0 = \frac{v}{f_0} = 4L$$

$$\underline{L = \frac{v}{4 f_0} = \frac{340}{4.170} = 0.5m}$$