SC528 Sample Test 1

Simple Harmonic Motion

$x(t) = A\cos(\omega t)$ $\quad v(t) = -A\omega\sin(\omega t)$ $\quad a(t) = -A\omega^2\cos(\omega t)$

Angular frequency: $\omega = 2\pi f = \dfrac{2\pi}{T}$

For a mass on a spring: $\omega = \sqrt{\dfrac{k}{m}}$ For a simple pendulum: $\omega = \sqrt{\dfrac{g}{L}}$

Springs: $F = -kx$ $\quad U = \dfrac{1}{2}kx^2$

Energy Conservation: $U_1 + K_1 + W_{nc} = U_2 + K_2$

Waves

$y(x,t) = A\sin(kx \pm \omega t)$ $\quad k = \dfrac{2\pi}{\lambda}$

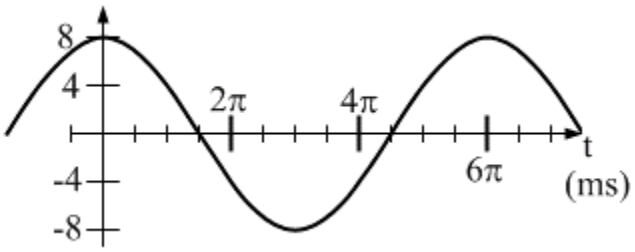
Wave speed: $v = f\lambda$ For a wave on a string: $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{m/L}}$

<u>Standing waves</u>

For a string fixed at both ends, or a tube open at both ends: $f_n = \dfrac{nv}{2L}$, (n = 1, 2, 3, …)

For a tube open at one end only: $f_n = \dfrac{nv}{4L}$, (n = 1, 3, 5, …)

PROBLEM 1 – 15 points



[6 points] (a) Let's say the graph above shows the acceleration as a function of time for a Thanksgiving turkey experiencing simple harmonic motion (oscillating along a line parallel to the x-axis), and the units on the vertical scale are in cm/s^2. Note that the units on the time axis are milliseconds. Come up with an equation describing the x-position of the turkey as a function of time. Hint: if you find the amplitude and the angular frequency that should make it easier to write an appropriate equation.

[3 points] (b) If we define A as the amplitude of the oscillation, and define x = 0 as the turkey's equilibrium position, at what location on the x-axis is the turkey at t = 0?

[ ] x = -A [ ] x = -A/2 [ ] x = 0 [ ] x = +A/2 [ ] x = +A

[6 points] (c) Sketch a graph of the **x-velocity** of the turkey as a function of time from t = 0 to t = 6π ms. What is the turkey's maximum speed?

PROBLEM 2 – 15 points

A 2.0-kg bowl of stuffing oscillates left and right with simple harmonic motion on a frictionless horizontal table. The spring attached to the bowl has a spring constant k = 8.0 N/m. At time t = 0, the mass is released from rest at a position x_o = +3.0 meters to the right of its equilibrium position. **You can leave answers in terms of π if necessary.**

[3 points] (a) What is the magnitude and sign of the force F_o exerted by the spring at x_o? What is the magnitude and sign of resulting acceleration a_o?

[3 points] (b) What is the magnitude and sign of the velocity v_{eq} of the bowl as it first passes through the equilibrium position?

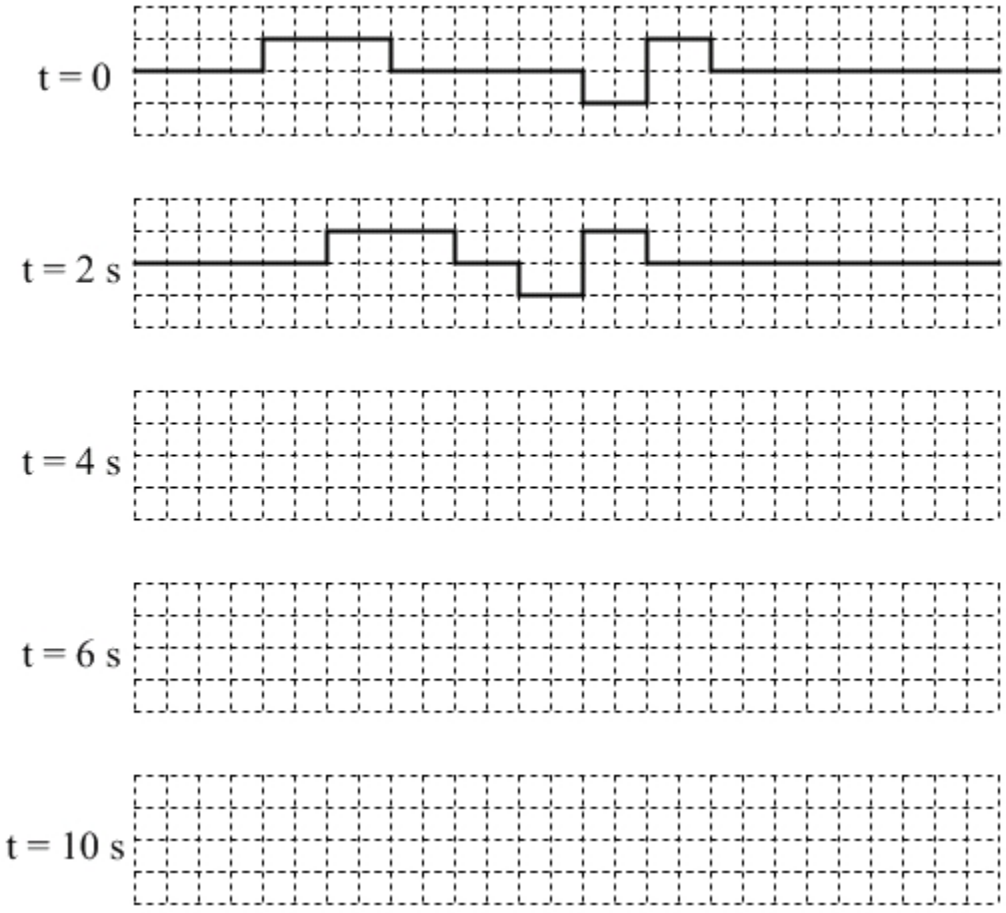
[3 points] (c) What is the magnitude and sign of the acceleration a_{eq} of the bowl as it first passes through the equilibrium position?

[3 points] (d) At what times t_+ does the bowl return to the release point? State the first four times after t = 0.

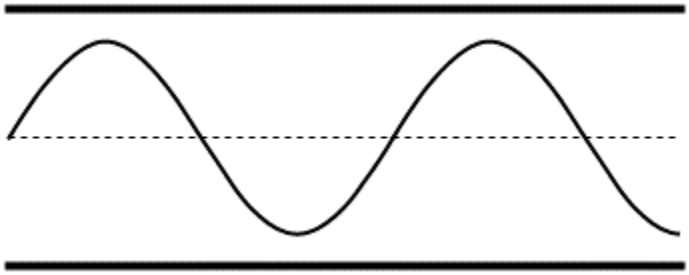
[3 points] (e) At what times t_{eq} does the bowl pass through the equilibrium position? State the first four times after t = 0.

PROBLEM 3 – 12 points

The top graph below shows a picture of a string at t = 0, showing the position of two pulses moving along the string. The second graph shows a picture of a string at t = 2 s. Show what the string looks like at t = 4 s, t = 6 s, and t = 10 s.



PROBLEM 4 – 15 points



[3 points] (a) The diagram above represents a standing wave in a tube at a particular instant in time. The frequency of the wave is one of the resonant frequencies for the tube. What kind of tube is it? (The diagram shows the outline of the sides of the tube but does not indicate whether the ends are open or closed.)

[ ] a tube open at both ends [ ] a tube open at one end and closed at the other

[4 points] (b) The next time the standing wave in the tube can be represented by the diagram above is a time T after the time shown above. Assuming the diagram shows the standing wave at its maximum amplitude, sketch (on the diagram above) the standing wave at a time t = T/2 and a time t = 5T/4 after the time represented above. Clearly indicate which sketch is which.

[4 points] (c) Assume the speed of sound is 340 m/s, and the fundamental frequency of this tube is 170 Hz. What frequency does the wave shown above correspond to?

[4 points] (d) How long is the tube?