

Conservation of Angular Momentum in Impact

PURPOSE

The aim of this experiment is to show that total angular momentum is conserved in a collision on the assumption that a rigid body has angular momentum $\vec{L} = I\vec{\omega}$. You will also show that an object moving in a straight line can have non-zero angular momentum.

THEORY

A particle having linear momentum $\vec{p} = m\vec{v}$ at a displacement \vec{r} from a point has angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ about that point. If the angle between the vectors \vec{r} and \vec{v} is θ , the magnitude of the angular momentum is $mvr\sin\theta = mvd$, where $d = r\sin\theta$ is the perpendicular distance from the point to the line obtained by extending the vector \vec{v} . If the particle is moving directly toward the point, the angular momentum about that point is zero. If the particle is moving along any other path, the angular momentum is given by the expression above. One such path for the apparatus you will use is shown below. The distance d can be found by extending the velocity vector backward in this case and measuring the distance from this line to the pivot at the center of the centerweight.

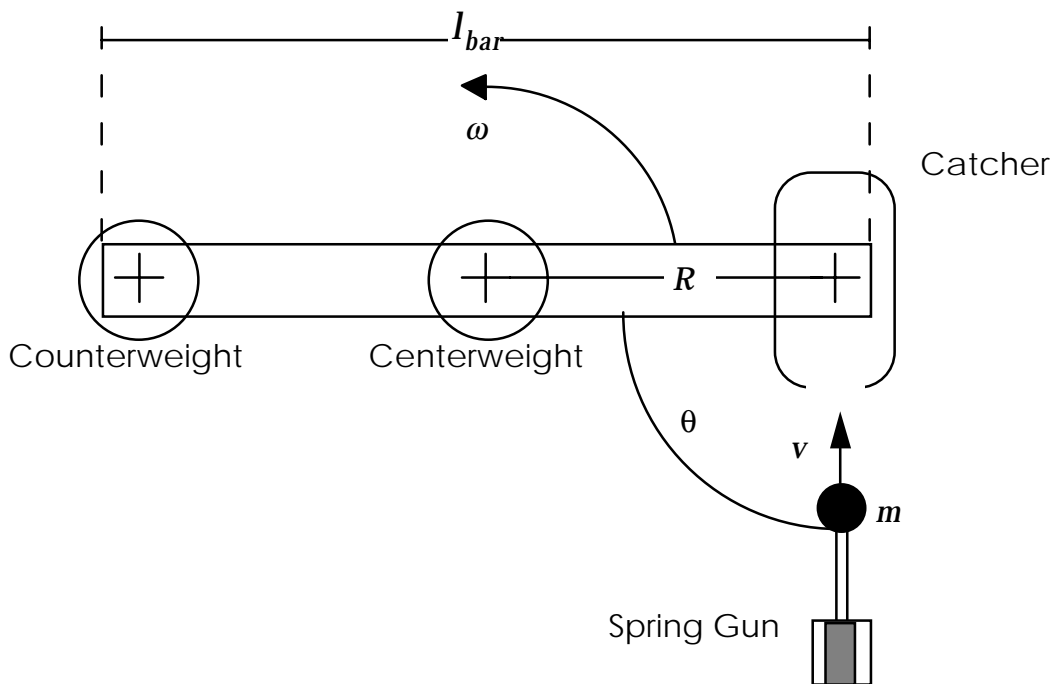


Figure 1: Top View of the Apparatus

A definition of angular momentum for a rigid body can be derived from the above by considering the rigid body as a collection of particles. For a given particle of mass m_i which is a distance r_i from the axis of rotation, the angular momentum is $L_i = m_i r_i v_i$. Suppose the rigid body is rotating with angular velocity ω . For each particle we have $\omega = v_i / r_i$ so that

$L_i = m_i r_i^2 \omega$. Now we add up these L_i to find the total angular momentum. Each particle has the same ω , so we find $L = (\sum m_i r_i^2) \omega$. In the limit of infinitesimal particles the sum in parentheses becomes the integral $\int_{body} m_i r_i^2$ which is the body's moment of inertia I , giving $L = I\omega$. The direction of the vector L is along the axis of the rotation in the direction of a right hand screw (out of the page in the top figure).

In this experiment you will test the above theory by attempting to show that angular momentum is conserved when a 'particle' collides with a rigid body. You will do this by shooting a ball of mass m and velocity v_1 into a cup at the end of a bar that is free to rotate about an axis. The angular momentum $mvr\sin\theta$ of the ball about the axis will be determined, and the angular momentum $I_s\omega$ of the system (bar, catcher, and counterweight) about the same axis will be determined after the ball is caught in the cup. You will compare these values of angular momentum.

The equipment basically consists of a spring gun, and a metal bar free to rotate about an axis. The bar is equipped with a plastic catcher on one end and a counterweight on the other. The spring gun fires a ball of mass m into the catcher and the impact causes the bar to turn. The velocity v_1 of the ball may be determined by measuring the range of the ball when fired horizontally from a measured height.

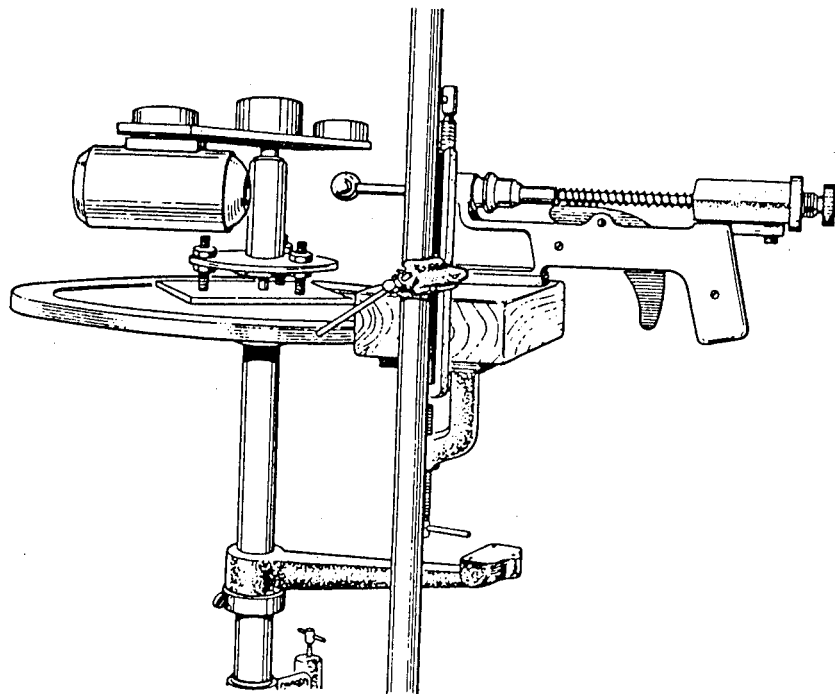


Figure 2: The Angular Momentum in Impact Apparatus

If angular momentum is conserved, the angular momentum of the ball before the collision should equal the angular momentum of the ball and the catcher system right after the collision. Expressed mathematically

$$m v_1 r \sin \theta = I_s \omega_0 \quad (1)$$

The moment of inertia of the system, I_s , can be considered the sum of four different moments of inertia: the moment of inertia of the bar, I_{bar} , of the ball and the catcher, I_{bc} , of the counterweight, I_{co} , and of the centerweight, I_{ce} , all taken around the axis of rotation.

The moment of inertia of the bar, I_{bar} , is given by

$$I_{bar} = \frac{1}{12} M_{bar} l_{bar}^2 \quad (2)$$

M_{bar} , is the mass of the bar and l_{bar} is the overall length of the bar. The moment of inertia of the centerweight, I_{ce} , is given by

$$I_{ce} = \frac{1}{2} M_{ce} r_{ce}^2 \quad (3)$$

where M_{ce} is the mass of the centerweight and r_{ce} is its radius. The moment of inertia of the catcher and ball can be approximated by

$$I_{bc} = M_{bc} R^2 \quad (4)$$

where M_{bc} is the mass of the ball and the catcher and R is the distance from the pivot point to the center of mass of the catcher assembly. The moment of the counterweight can be approximated by

$$I_{co} = M_{co} R^2 \quad (5)$$

M_{co} is the mass of counterweight and R is the counterweight to pivot distance; it has the same value as R in Equation 3.

The velocity of the ball before collision, v_1 , can be determined by the method used in the projectile motion or ballistic pendulum experiment. In short, a ball fired horizontally will have a constant horizontal velocity, so that the horizontal distance covered in time t is:

$$x = v_1 t \quad (6)$$

The initial velocity in the vertical direction is zero. Therefore the vertical distance covered in time t is:

$$y = \frac{1}{2} g t^2 \quad (7)$$

If Eqs. (6) and (7) are combined to eliminate time and then solved for v_1 , the result is

$$v_1 = x \sqrt{\frac{g}{2y}} \quad (8)$$

The remaining quantity to determine is w_o , the angular velocity of the system right after the collision. If the bearing at the axis of the bar has a negligible amount of friction, then the time for one revolution can be measured, and the angular velocity can be calculated. The bearing is good, but the friction is not negligible. We assume for the sake of simplicity that the system constantly decelerates and therefore the following equation is valid:

$$\theta = \frac{w_o + w}{2} t \quad (9)$$

where θ is the total angle the system rotates through in time t ; w is the final angular velocity and w_o is the initial angular velocity. If t_f is the time it takes for the bar to come to rest,

$$\theta = \frac{w_o t_f}{2} \quad (10)$$

Hence measurements of w and t_f yield w_o .

PROCEDURE

1. Weigh the ball, bar, catcher apparatus, counterweight and centerweight. Measure the bar's total length (I_{bar}), the distance from the axis of rotation to the center of the counterweight, and the radius of the centerweight.
2. Rotate the gun so as to shoot out into the room. Measure the height y of the ball from the floor. Determine the approximate range of the ball by firing the gun a couple of times. Then tape a piece of paper to the floor such that the ball strikes the paper approximately in the center. Fire ten shots and find the mean range. Calculate the speed of the ball as it leaves the gun.
3. Rotate the gun so that it is aimed at the catcher apparatus. Make sure the catcher cup is perpendicular to the bar and that the force table is leveled. Before shooting, use tape to mark the angle to which the indicator points. To minimize risk of injury, the gun should be quite close to the catcher cup. After the ball is shot, the gun will have to be rotated out so that the bar will not strike it as it rotates.
4. Fire the gun and start the stopwatch simultaneously. Count the number of revolutions the bar makes and stop the timer when the bar stops rotating. Record the time, number of revolutions, and the number of degrees of the incomplete turn.

5. Repeat Steps 3 and 4 twice more.
6. Reset the catcher cup so that its angle is 45° with respect to the bar.
7. Take three more sets of data with the catcher cup at this new angle.

QUESTIONS

1. In the theory section we approximated the moment of inertia for the counterweight and the catcher and ball. How would you calculate the exact moment of inertia?
2. If the radius of the counterweight is equal to the radius of the centerweight, how much error was introduced in using the approximation for I_{co} ?
3. Explain why Eqs. (9) and (10) follow from the assumption of constant angular deceleration.
4. What is the percent difference between the angular moment before and after?
5. What were the sources of error in this experiment? What could be done to reduce them? Estimate the size of each error and discuss the uncertainty in Iw introduced by them.
6. Was energy conserved in this collision? If not, was more energy lost in the 90° or 45° collision? Why?
7. If the separation between the spring gun and the catcher cup was increased, would w_o change?

DATA SHEET - Angular Momentum in Impact TA []

NAME: _____ INSTRUCTOR: _____

PARTNER: _____ SECTION: _____ DATE: _____

Mass of Bar (M_{bar}) _____

Mass of Centerweight (M_{ce}) _____

Mass of Counterweight (M_{co}) _____

Mass of Ball (m) _____

Mass of Catcher & Ball (M_{bc}) _____

Length of Bar (l_{bar}) _____

Radius of Orbits (R) _____

Moment of Inertia of the System (I_s) _____

Height of Ball (y) _____

Mean Range (\bar{x}) _____

Velocity of Ball (v_I) _____

Catch Angle	L _{initial}	t _f	θ	w _o	L _{final}