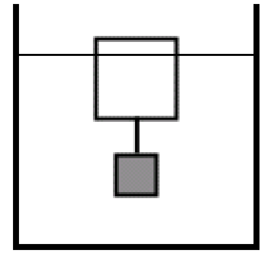
PROBLEM 1 – 20 points



A wooden cube measuring 20.0 cm on each side floats in water with 80.0% of its volume submerged. Suspended by a string below the wooden cube is a metal cube. The metal cube measures 10.0 cm on each side and has a specific gravity of 5.00.
[2 points] (a) Which cube has a larger buoyant force acting on it?

[**X**] the wooden cube [ ] the metal cube [ ] neither, they're equal
The wooden cube displaces more fluid.

[8 points] (b) Taking the density of water to be 1000 kg/m^3, what is the density of the wooden cube?

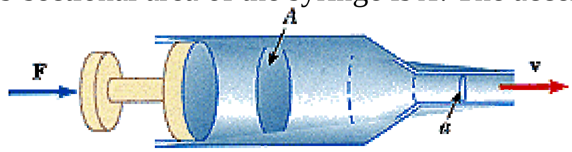
Let's use the subscript 1 for the wooden cube and 2 for the metal cube. If you treat the two cubes as one system, the fact that the forces balance on this system gives:
$$F_b = (m_1 + m_2)g$$
But, from Archimedes' principle we know that: $F_b = \rho_{fluid} V_{disp} g$

Setting these two expressions equal, and using the fact that mass is density*volume:
$$F_b = (\rho_1 V_1 + \rho_2 V_2)g = \rho_{fluid} V_{disp} g$$
Factors of g cancel. Solving for the density of the wooden cube gives:
$$\rho_1 = \frac{\rho_{fluid} V_{disp} - \rho_2 V_2}{V_1} = \frac{1000*(0.8*0.008+0.001)-5000*0.001}{0.008} = \frac{7.4-5}{0.008} = \frac{2400}{8} = 300 kg/m^3$$

[4 points] (c) What is the tension in the string between the cubes? Assume the string itself has negligible mass and volume.
Summing forces on the metal block gives: $F_b + T - m_2 g = 0$

$$T = m_2 g - F_b = \rho_2 V_2 g - \rho_{fluid} V_2 g = 5000*0.001*10 - 1000*0.001*10 = 50.0 - 10.0 = 40.0 N$$
[6 points] (d) The pair of blocks is now placed in a different liquid. In this new liquid the buoyant force acting on the wooden cube is exactly the same as the buoyant force acting on the metal cube. What is the density of this new liquid?
For this we can return to the expression we derived in part (b):
$$F_b = (\rho_1 V_1 + \rho_2 V_2)g = \rho_{fluid} V_{disp} g$$
Factors of g cancel again. If the cubes have the same buoyant force they must displace the same volume of fluid. The metal cube will still be entirely immersed, and the volume displaced by the wooden cube will be equal to the volume of the metal cube.
This gives:
$$\rho_{fluid} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_{disp}} = \frac{300*0.008+5000*0.001}{0.002} = \frac{7.4}{0.002} = \frac{7400}{2} = 3700 kg/m^3$$

PROBLEM 2 – 10 points
A syringe containing liquid with a density ρ sits on a horizontal surface. Initially the pressure throughout the syringe is 1 atmosphere, so no liquid squirts out. The cross-sectional area of the needle is a, while the cross-sectional area of the syringe is A. The acceleration due to gravity is g.



[8 points] (a) With the syringe held at rest a force F is then applied to the plunger as shown above. In terms of the variables stated in the problem (ρ, a, A, g, and/or F), what is the speed v of the liquid emerging from the needle?

Let's apply Bernoulli's equation to this situation. Use the subscript 1 to refer to a point just to the right of the plunger, and 2 to refer to the open end of the needle. Assume both points are at the same vertical position.

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2$$

If we assume the points are at the same height we can reduce this to:

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

The pressure at point 2 is atmospheric pressure, and at point 1 it is larger than atmospheric pressure because of the applied force. It is larger by an amount equal to F/A.

This gives us: $\frac{1}{2}\rho v_1^2 + \frac{F}{A} = \frac{1}{2}\rho v_2^2$

Now is a good time to bring in the continuity equation:

$A v_1 = a v_2$ **so** $v_1 = \frac{a}{A} v_2$

Plug this into the equation above, after multiplying the equation above by 2 on both sides:

$\rho \frac{a^2}{A^2} v_2^2 + \frac{2F}{A} = \rho v_2^2$ **which gives:** $\frac{2F}{A} = \rho v_2^2 \left(1 - \frac{a^2}{A^2}\right)$

Solving this for the speed at point 2 gives: $v_2 = \sqrt{\dfrac{2F}{\rho A\left(1 - \frac{a^2}{A^2}\right)}} = \sqrt{\dfrac{2FA}{\rho\left(A^2 - a^2\right)}}$

[2 points] (b) If the syringe is held vertically instead, with the needle up, and the same force is applied to the plunger, would the speed of the liquid emerging from the needle be larger than, smaller than, or the same as the speed when the syringe is horizontal?

[ ] the speed is larger when the syringe is vertical

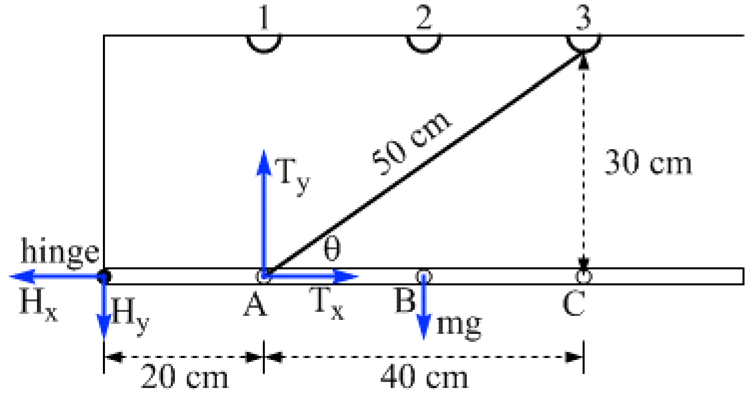
[**X**] the speed is smaller when the syringe is vertical

[ ] the speed is the same

With the needle up some of the extra pressure has to raise the fluid's potential energy, so the kinetic energy is smaller.

PROBLEM 3 - A rod with a length of 80 cm and a mass of 6.0 kg is attached to a wall by means of a hinge at the left end. The rod's mass is uniformly distributed along its length. A string will hold the rod in a horizontal position; the string can be tied to one of three points, lettered A-C, spaced at 20 cm intervals along the rod, starting with point A which is 20 cm from the left end of the rod. The other end of the string can be tied to one of three hooks, numbered 1-3, in the ceiling 30 cm above the rod. Hook 1 is directly above point A, hook 2 is directly above B, etc. **Use g = 10 m/s^2.**

A string is attached from point A to hook 3. Remember that point B is 40 cm from the hinge.



[6 points] (a) What is the tension in the string?

Draw a good free-body diagram of the rod. The downward force of gravity, mg, is applied at point B, in the middle of the rod. In addition to the tension there is an unknown force at the hinge, which is shown broken into components on the diagram. To eliminate these unknowns sum torques about an axis through the hinge. Take counter-clockwise to be positive (you could set clockwise to be positive if you want).

$\sum \tau = 0$ **so** $+(20\ cm)T_y - (40\ cm)mg = 0$

This gives $T_y = 2mg = 2*6.0kg*10m/s^2 = 120\ N$**.**

To find T, recognize that the triangle shown is a 3-4-5 triangle. $T_y = T\sin\theta$**, so:**

$T = \frac{T_y}{\sin\theta} = \frac{T_y}{3/5} = \frac{5T_y}{3} = \frac{5*120}{3} = 200N$**.**

[2 points] (b) What is the horizontal component of the hinge force? (Clearly state whether this is directed left or right.)

The horizontal component of the tension is directed right, so the horizontal component of the hinge force must be directed left to balance it.

Summing forces in the x-direction, taking positive to the right, gives:

$\sum \vec{F}_x = 0$ **so** $+T_x - H_x = 0$

$H_x = T_x = T\cos\theta = 200N*\frac{4}{5} = 160N$ **to the left.**

[2 points] (c) What is the vertical component of the hinge force? (Clearly state whether this is directed up or down.)

Let's assume the vertical component of the hinge force is directed down. Summing forces in the y-direction, taking positive to be up, gives:

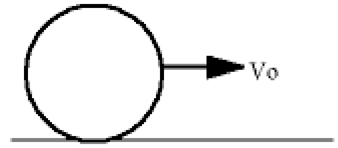
$\sum \vec{F}_y = 0$ **so** $+T_y - H_y - mg = 0$

$H_y = T_y - mg = 120\ N - 60\ N = 60\ N$ **down.**

Summing torques about point A is also a good way to get this answer.

PROBLEM 4 – 25 points

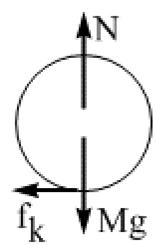
A bowling ball of mass M and radius R = 20.0 cm is released with an initial translational velocity of v_o = 14.0 m/s and an initial angular velocity of $\omega_o = 0$. The ball is a uniform solid sphere with moment of inertia I = (2/5)MR^2.



The coefficient of kinetic friction between the ball and the surface is μ_k = 0.200. The force of kinetic friction causes a linear acceleration, as well as a torque that causes the ball to spin.

The ball slides along the horizontal surface for some time, and then rolls without slipping at constant velocity after that. **Use g = 10 m/s².**

[4 points] (a) Draw the free-body diagram of the ball showing all the forces acting on it while it is sliding.



[4 points] (b) What is the acceleration a of the ball while it is sliding?

Apply Newton's second law horizontally.

$\sum F_x = Ma$ **Define positive as being to the right.**

$-f_k = Ma$

so $a = \frac{-f_k}{M} = \frac{-\mu_k N}{M} = \frac{-\mu_k Mg}{M} = -\mu_k g = -0.200*10.0 = -2.00 m/s^2$

[4 points] (c) What is the angular acceleration α of the ball while it is sliding?

Apply Newton's second law for rotation. Take torques about the center of the ball.

$\sum \tau = I\alpha$ **Define clockwise as positive.**

The only torque acting is from the force of friction, and that is Rf_k

So, $\alpha = \frac{Rf_k}{I} = \frac{R\mu_k N}{\frac{2}{5}MR^2} = \frac{5\mu_k Mg}{2MR} = \frac{5\mu_k g}{2R} = \frac{5*0.200*10.0}{2*0.200} = 25.0 rad/s^2$

You can't use α =a/R because that doesn't apply when there is slipping.

[6 points] (d) How far does the ball travel while it is sliding?

The key here is to figure out when the ball starts to roll without slipping. While it is sliding the linear speed is steadily decreasing and the angular speed is decreasing. The ball rolls without slipping when the linear speed is related by v = rω.

The linear speed is $v = v_o + at$

The angular speed is $\omega = \omega_o + \alpha t = \alpha t$

Rolling without slipping starts when $v_o + at = r\alpha t$

This happens at a time given by $t = \frac{v_o}{r\alpha - a} = \frac{14.0}{0.200 * 25.0 - (-2.00)} = \frac{14.0}{7.00} = 2.00s$

In that time the ball travels a distance of $x = v_o t + \frac{1}{2}at^2 = 14.0 * 2.00 - 1.00 * 4.00 = 24.0m$

[2 points] (e) What is the constant speed of the ball when it rolls without slipping?

One way to get this is to use $v = v_o + at = 14.0 - 2.00 * 2.00 = 10.0m/s$

[5 points] (f) When the ball is rolling without slipping at constant velocity, which way is the force of friction acting on the ball, and what kind of friction is it?

[ ] a force of kinetic friction acts to the left
[ ] a force of kinetic friction acts to the right
[ ] a force of static friction acts to the left
[ ] a force of static friction acts to the right
[**X**] none of the above, there is no force of friction acting

Briefly justify your answer:

If an object rolls without slipping then if there is friction it is static friction. But, if there is a friction force in the direction of motion the velocity would increase and the angular velocity would decrease, inconsistent with rolling without slipping; the opposite would happen if there was a force of friction opposite to the direction of motion and that would also be inconsistent with rolling without slipping. Therefore there must be no force of friction, which is certainly consistent with motion at constant velocity on a horizontal surface.