Fluids

Pressure: $P = \frac{F}{A}$ Density: $\rho = \frac{m}{V}$

Pressure in a static fluid: $P_2 = P_1 + pgh$

Buoyant Force: $F_B = \rho_{fluid} V_{disp} g$

Continuity Equation: $A_1 v_1 = A_2 v_2$

Bernoulli's Equation: $\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2$

Rotation

$s = r\theta$ $v = r\omega$ $a_T = r\alpha$ centripetal acceleration: $a_C = \frac{v^2}{r}$

Rotational kinematics:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha\, \Delta\theta$$

Static Equilibrium: $\sum \vec{F} = 0$ $\sum \vec{\tau} = 0$

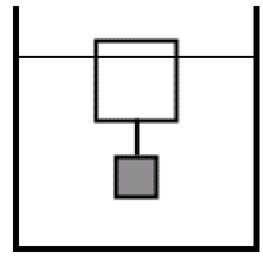
Newton's Second Law for Rotation: $\sum \vec{\tau} = I\vec{\alpha}$

Angular Momentum: $\vec{L} = I\vec{\omega}$ $L = r(mv)\sin\theta$

Rotational Kinetic Energy: $KE = \frac{1}{2} I\omega^2$

Energy Conservation: $U_1 + K_1 + W_{nc} = U_2 + K_2$

PROBLEM 1 – 20 points



A wooden cube measuring 20.0 cm on each side floats in water with 80.0% of its volume submerged. Suspended by a string below the wooden cube is a metal cube. The metal cube measures 10.0 cm on each side and has a specific gravity of 5.00.

[2 points] (a) Which cube has a larger buoyant force acting on it?

[ ] the wooden cube [ ] the metal cube [ ] neither, they're equal

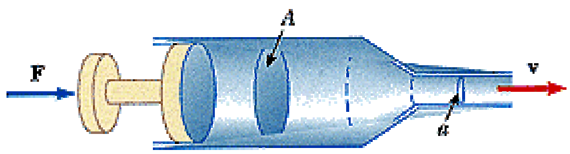
[8 points] (b) Taking the density of water to be 1000 kg/m^3, what is the density of the wooden cube?

[4 points] (c) What is the tension in the string between the cubes? Assume the string itself has negligible mass and volume.

[6 points] (d) The pair of blocks is now placed in a different liquid. In this new liquid the buoyant force acting on the wooden cube is exactly the same as the buoyant force acting on the metal cube. What is the density of this new liquid?

PROBLEM 2 – 10 points

A syringe containing liquid with a density ρ sits on a horizontal surface. Initially the pressure throughout the syringe is 1 atmosphere, so no liquid squirts out. The cross-sectional area of the needle is a, while the cross-sectional area of the syringe is A. The acceleration due to gravity is g.



[8 points] (a) With the syringe held at rest a force F is then applied to the plunger as shown above. In terms of the variables stated in the problem (ρ, a, A, g, and/or F), what is the speed v of the liquid emerging from the needle?

[2 points] (b) If the syringe is held vertically instead, with the needle up, and the same force is applied to the plunger, would the speed of the liquid emerging from the needle be larger than, smaller than, or the same as the speed when the syringe is horizontal?

[ ] the speed is larger when the syringe is vertical

[ ] the speed is smaller when the syringe is vertical

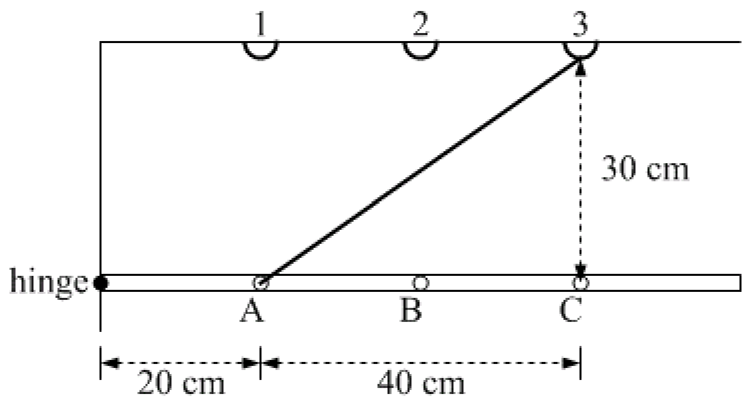
[ ] the speed is the same

PROBLEM 3 – 12 points

A rod with a length of 80 cm and a mass of 6.0 kg is attached to a wall by means of a hinge at the left end. The rod's mass is uniformly distributed along its length. A string will hold the rod in a horizontal position; the string can be tied to one of three points, lettered A-C, spaced at 20 cm intervals along the rod, starting with point A which is 20 cm from the left end of the rod. The other end of the string can be tied to one of three hooks, numbered 1-3, in the ceiling 30 cm above the rod. Hook 1 is directly above point A, hook 2 is directly above B, etc. **Use $g = 10\ m/s^2$.**

A string is attached from point A to hook 3. Remember that point B is 40 cm from the hinge.

[6 points] (a) What is the tension in the string?

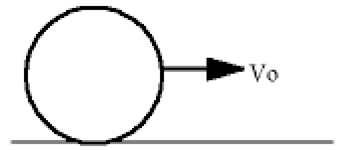


[3 points] (b) What is the horizontal component of the hinge force? (Clearly state whether this is directed left or right.)

[3 points] (c) What is the vertical component of the hinge force? (Clearly state whether this is directed up or down.)

PROBLEM 4 – 25 points

A bowling ball of mass M and radius R = 20.0 cm is released with an initial translational velocity of v_o = 14.0 m/s and an initial angular velocity of $\omega_o = 0$. The ball is a uniform solid sphere with moment of inertia $I = (2/5)MR^2$.



The coefficient of kinetic friction between the ball and the surface is $\mu_k = 0.200$. The force of kinetic friction causes a linear acceleration, as well as a torque that causes the ball to spin.

The ball slides along the horizontal surface for some time, and then rolls without slipping at constant velocity after that. **Use g = 10 m/s²**.

[4 points] (a) Draw the free-body diagram of the ball showing all the forces acting on it while it is sliding.

[4 points] (b) What is the acceleration a of the ball while it is sliding?

[4 points] (c) What is the angular acceleration α of the ball while it is sliding?

[6 points] (d) How far does the ball travel while it is sliding?

[2 points] (e) What is the constant speed of the ball when it rolls without slipping?

[5 points] (f) When the ball is rolling without slipping at constant velocity, which way is the force of friction acting on the ball, and what kind of friction is it?

[ ] a force of kinetic friction acts to the left
[ ] a force of kinetic friction acts to the right
[ ] a force of static friction acts to the left
[ ] a force of static friction acts to the right
[ ] none of the above, there is no force of friction acting

Briefly justify your answer: