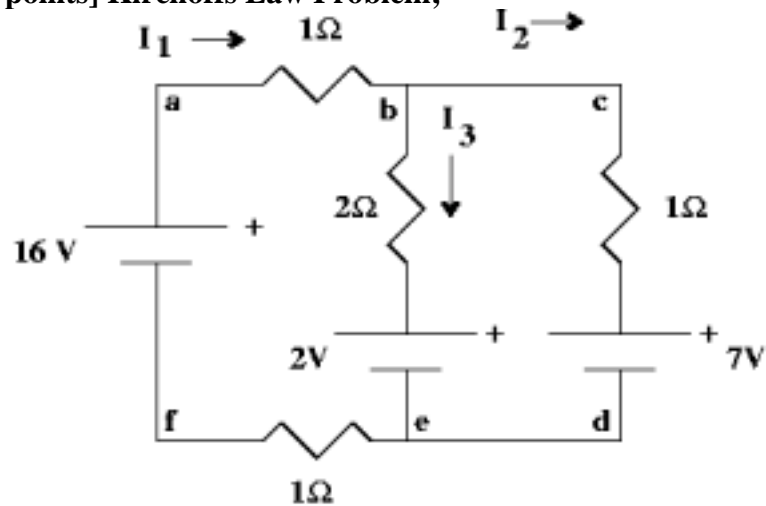


Problem 1: [15 points] Kirchoffs Law Problem;



- a. [2 pts] Write down the junction rule for the currents as marked

$$I_1 = I_2 + I_3$$

- b. [4 pts] Write down the loop equations for *abefa* and *bcdeb*

$$-I_1 - 2I_3 - 2 - I_1 + 16 = 0 \Rightarrow 14 = 2I_1 + 2I_3$$

$$-I_2 - 7 + 2 + 2I_3 = 0 \Rightarrow 5 = 2I_3 - I_2$$

- c. [6 pts] Use your equations to solve for the three unknown currents
plug junction eqnt into 1st equation, get

$$14 = 2I_1 + 2I_3$$

$$10 = 4I_3 - 2I_2 \text{ now add equations:}$$

$$24 = 8I_3 \text{ or } I_3 = 3A$$

or subtract equations to get:

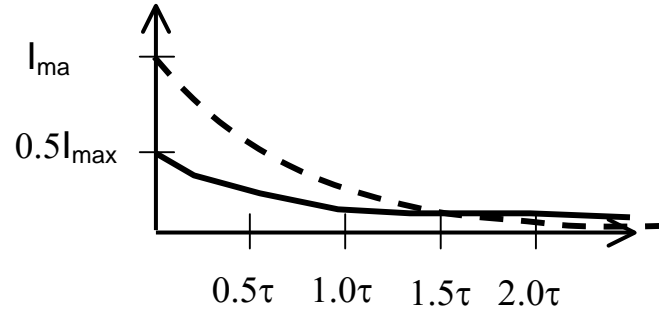
$$4 = 4I_2 \text{ or } I_2 = 1A \text{ which gives } I_1 = 4A$$

- d. [3 pts] What is the potential difference between the points *e* and *f* ($V_e - V_f$)?

$$V_e - V_f = 4A \times 1\Omega = 4 \text{ volts}$$

Problem 2 – 10 points

A battery, a resistor R_0 and a switch are placed in series with a capacitor. When the switch closes, the battery charges the capacitor through the resistor. In the following graph the dashed line plots the current across the resistor as a function of time following the closing of the switch. Now, the resistor is doubled to a value of $2R_0$. **Plot on the same graph, what the current through the new resistor as a function of time would be, otherwise starting at the same initial conditions.**



Justify your answer.

Key points to keep in mind about this are:

After a long time the capacitor voltage is essentially equal to the battery voltage, and the total charge stored on the capacitor is the same in the two cases.

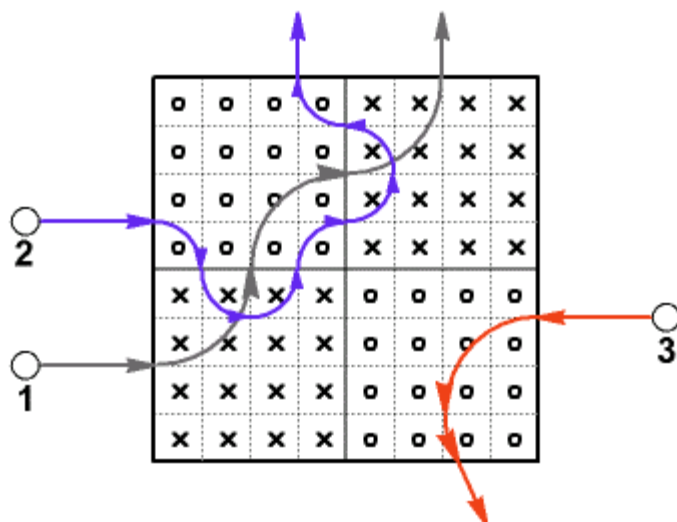
How does the total charge stored relate to the graph of current vs. time? The total charge is the area under the current vs. time graph.

Because the resistance is doubled the initial current in the circuit is reduced by a factor of two – this is why the second graph starts at a maximum value that is half the maximum value of the first graph. We can apply $V = IR$ here, and initially the voltage across the resistor is the battery voltage because the capacitor starts out uncharged and has an initial voltage of zero.

Doubling the resistance doubles the time constant, so the second graph changes more slowly than the first one. Combining this idea with the idea that the area under the curves must be equal, we can conclude that the second graph does cross the first graph at some time – it starts lower but is higher after a while.

PROBLEM 3 - 25 points

In the square region at right the magnetic field is uniform and directed out of the page in the upper left and lower right quadrants, and uniform and into the page in the upper right and lower left quadrants. The magnetic field has the same magnitude in all quadrants, and there is no magnetic field outside the square region. Three objects (1, 2, and 3) are fired into the square region – the objects travel on paths that are entirely in the plane of the page. The objects have equal masses and their charges have equal magnitudes. The path followed by object 1 is shown, and the quarter-circle path object 2 takes through the upper left quadrant is also shown.



[2 points] (a) What is the sign of the charge on object 1?

☒ positive ☐ negative ☐ the charge is zero ☐ insufficient data to determine

Use the right-hand rule. A positive charge moving right in a field into the page feels an upward force. That's what object 1 feels, because it bends up, so it must be positive.

[3 points] (b) On the diagram above, carefully complete the path followed by object 2 through the square region. Make sure the radius of curvature, and the point where the object exits the square region, are accurately drawn.

The radius of curvature is the same in each region, so each box is $\frac{1}{4}$ circle. The way it bends changes when you go from one quadrant to another.

[2 points] (c) How do the signs of objects 1 and 2 compare?

☒ they have the same sign ☐ they have opposite signs

Apply the right-hand rule again as in (a) to determine that object 2 is positive, or just think about what object 1 would do in that quadrant. It would bend down because the field is the opposite direction from what it is in the lower left quadrant. Object 2 bends down, so it must have the same charge as object 1.

[3 points] (d) How do the speeds of objects 1 and 2 compare?

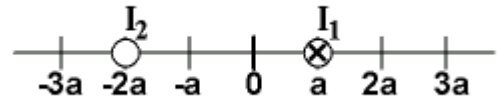
☐ $v_1 = 4v_2$ ☒ $v_1 = 2v_2$ ☐ $v_1 = v_2$ ☐ $2v_1 = v_2$ ☐ $4v_1 = v_2$

Compare the radii. $r = mv/qB$ and m , q , and B all have the same magnitude. The path of object 1 has twice the radius as that of object 2, so object 1's speed is twice as large as object 2's.

[3 points] (e) Object 3 has the same speed as object 1 but the sign of its charge is opposite to that of object 1. On the diagram above carefully sketch the path followed by object 3 through the square region. Make sure the radius of curvature, and the point where the object exits the square region, are accurately drawn.

Applying the right-hand rule shows that the force on object 3 is down. Its speed is the same as that of object 1 so the paths must have the same radius.

Two very long straight wires carry currents perpendicular to the page. The x axis is in the plane of the page. Wire 1, which carries a current I_1 into the page, passes through the x axis at $x = +a$. Wire 2, located at $x = -2a$, carries an unknown current.



The net field at the origin ($x = 0$) due to the current-carrying wires is:

$$B = 2 \mu_0 I_1 / (2\pi a)$$

[6 points] (f) I_2 has two possible values.

- (i) Determine one of the possible values of I_2 , stating both its magnitude and direction. You should be able to express the magnitude of I_2 in terms of I_1 .

Wire 1 creates a field at $x = 0$ that is up and has a magnitude $B_1 = \mu_0 I_1 / (2\pi a)$

If wire 2 also produces an upward field at $x = 0$ then its current is out of the page.

In this case the fields add:

$$B = B_1 + B_2 \text{ so } B_2 = B - B_1$$

$$\text{This gives } \mu_0 I_2 / (2\pi(2a)) = 2\mu_0 I_1 / (2\pi a) - \mu_0 I_1 / (2\pi a) = \mu_0 I_1 / (2\pi a)$$

Cancelling all the common factors gives $I_2 = 2 I_1$, out of the page.

- (ii) Determine the other possible value of I_2 , stating both its magnitude and direction.

The other possibility is that the net field is down, so wire 2's field is down and larger in magnitude than wire 1's field. The direction of I_2 is now into the page. In this case the fields subtract because they are in opposite directions but wire 2's field is larger so:

$$B = B_2 - B_1 \text{ so } B_2 = B + B_1$$

where B , B_1 and B_2 represent the magnitudes of the fields. The signs in the equations are taking care of the directions.

$$\text{This gives } \mu_0 I_2 / (2\pi(2a)) = 2\mu_0 I_1 / (2\pi a) + \mu_0 I_1 / (2\pi a) = 3\mu_0 I_1 / (2\pi a)$$

Cancelling all the common factors gives $I_2 = 6 I_1$, into the page.