Problem 1.

What you will need: Potential energy of interaction of 2 point charges: $U = \frac{kq_1q_2}{d}$,

Potential, produced by a point charge: $V = \frac{kq}{r}$, electric field due to a point charge:

$E = \frac{kq}{r^2}$

a) Since potential energy of interaction of the 2 charges is positive then $q_1q_2 > 0$. Thus, since q_1 is positive then q_2 is also positive. If the second charge was located on the y-axis the vertical component of the total electric field at the origin would not be zero. So, the second charge is somewhere on x-axis. The field at the origin points towards the first charge. This is only possible if the second positive charge is placed on the positive x-axis.

b) Say the distance from the second charge q_2 to the origin is d_2. Taking into account the conclusions we made in part a) we can express the potential energy of interaction and the potential at the origin as follows:

$$U = \frac{kqq_2}{d + d_2} = \frac{2kq^2}{d}$$

$$V = \frac{kq}{d} + \frac{kq_2}{d_2} = \frac{4kq}{d}$$

From the second equation we get: $\frac{kq_2}{d_2} = \frac{3kq}{d}$, $q_2 = 3q\frac{d_2}{d} = 3qx$, where $x = \frac{d_2}{d}$.

Plugging this into the expression for energy we get $\frac{3q^2x}{d + d_2} = \frac{2q^2}{d}$, $\frac{3q^2x}{1 + x} = 2q^2$,

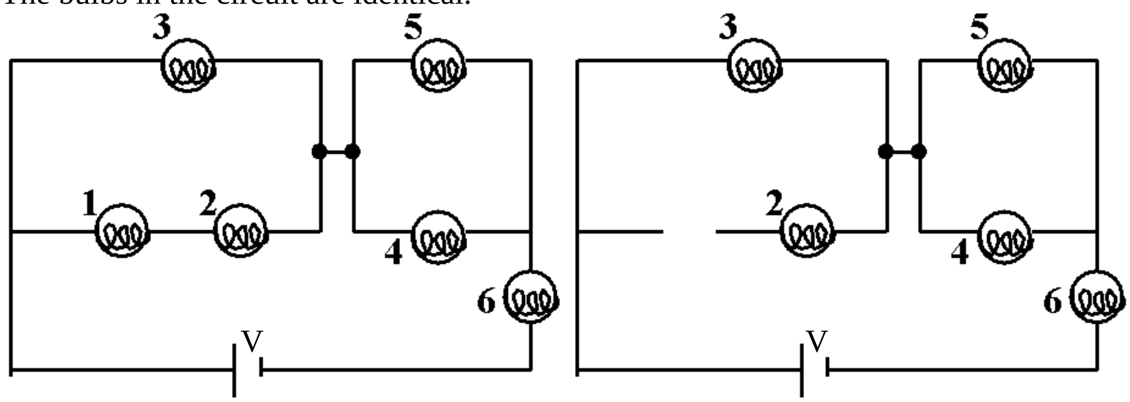
$\frac{3x}{1+x} = 2$, $\frac{3x}{1+x} = 2$, $x = \frac{d_2}{d} = 2$. Thus $d_2 = 2d$, $q_2 = 3qx = +6q$.

c) The field at the origin is: $E = \frac{kq}{d^2} - \frac{kq_2}{d_2^2} = \frac{kq}{d^2}\left(1 - \frac{6}{4}\right) = -\frac{kq}{2d^2}$.

As we can see the field direction is indeed in the negative x-direction.

PROBLEM 2. [20 points] – Light bulbs

The bulbs in the circuit are identical.



(1) [6 points] In the circuit on the left, rank bulbs 1-6 in order of **decreasing** brightness. Briefly explain your answers.

Brightness of a bulb is determined by the power dissipated in the bulb and is given by I^2R. Since R is the same for all the bulbs, the current passing through the bulb determines the brightness of a bulb. The larger the current passing through the bulb, the brighter it will be.

If the current drawn from the battery is I, then bulb 6 sees the full current passing through it. Bulbs 4 and 5 are in parallel and have the same resistance, so they share this current and half of the current passes through each one of them. Bulbs 1 and 2 are in series with each other and parallel to bulb 3, thus the path through 2 and 3 has twice the resistance compared to the path with bulb 3. Therefore the current splits with two-thirds passing through bulb 3, while one-third of the current flows through bulbs 1 and 2.

Ordering the bulbs according to increasing current (or brightness) we get:

6 > 3 > 5 = 4 > 1 = 2

(2) [6 points] If each bulb has a resistance of 12 ohms, what is the equivalent resistance of the circuit on the left?

Req = R6 + (R5 // R4) + (R3 // (R1+R2))
= R6 + (R5 * R4)/(R5+R4) + R3 * (R1+R2)/(R1+R2+R3)
= R + R/2 + 2 R/3 (use the fact that all resistances are the same)
= 13 R /6 = 13*12/6
= 26 Ohms.

(3) [8 points] When bulb 1 is removed, leaving a break in the circuit….

a) The brightness of bulb 3 ___________

[**X**] increases [ ] decreases [ ] stays the same

Briefly explain your answer:

When bulb 1 is removed all of the current now flows through 3, increasing its brightness

b) The brightness of bulb 6 ___________

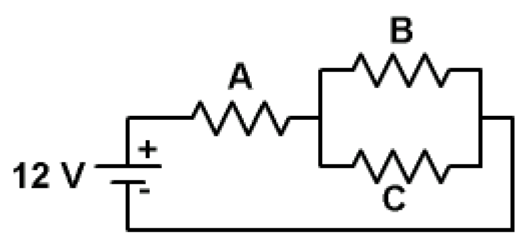
[ ] increases [**X**] decreases [ ] stays the same

Briefly explain your answer:

The overall resistance of the circuit increases, therefore the current decreases, leading to a decreased brightness of bulb 6

PROBLEM 3 – 20 points

Three resistors are connected as shown in a circuit with a 12 volt battery. The resistances are:
For resistor A, $R_A = 4\ \Omega$
For resistor B, $R_B = 3\ \Omega$
For resistor C, R_C is an unknown value



[3 points] (a) Which resistor has the most current passing through it?

[X] A [ ] B [ ] C [ ] It depends on the value of R_C

Briefly justify your answer:
A gets all the current. The current then splits, with B getting some and C the rest.

[5 points] (b) Rank the resistors based on the potential difference across them, from largest to smallest.

[ ] A > B > C [X] A > B = C [ ] B > C > A [ ] B = C > A
[ ] It depends on the value of R_C

Briefly justify your answer:

B and C have the same potential difference because they're in parallel. V = IR for resistor A is bigger than that for B because A has both more current and more resistance.

[8 points] (c) If R_C, the resistance of resistor C, is increased, what happens to the currents through the different elements in the circuit?

i) The current through the battery: [ ] increases [X] decreases [ ] stays the same

ii) The current through A: [ ] increases [X] decreases [ ] stays the same

iii) The current through B: [X] increases [ ] decreases [ ] stays the same

iv) The current through C: [ ] increases [X] decreases [ ] stays the same

If the resistance of C increases the overall resistance of the circuit increases so the current is reduced. The current through the battery is the same as the current through A so they are both decreased. A now gets less voltage so the voltage across the B-C combination increases. Increasing the voltage across B increases B's current. The total current into the B-C combination has gone down, so if the current through B increases the current through C decreases.

[4 points] (d) If the current through the battery is 2.0 amps, what is R_C, the resistance of resistor C?

R = V/I = 12/2 = 6 ohms is the equivalent resistance of the circuit. This is the sum of A (4 ohms) and the B-C combination, so the B-C combination must be 2 ohms. They're connected in parallel, so:

$1/2 = 1/3 + 1/R_C$, so $1/R_C = 1/2 - 1/3 = 3/6 - 2/6 = 1/6$

Therefore $R_C = 6$ ohms.