

# ELECTRIC FIELD LINES AND EQUIPOTENTIAL SURFACES

The purpose of this lab session is to experimentally investigate the relation between electric field lines of force and equipotential surfaces in two dimensions. You will construct various conducting surfaces (metal electrodes) and study the electric field and equipotential patterns associated with them. Although you will study only several particular configurations, you should come away with a basic understanding of fields and equipotentials which is true for any geometry. This write-up begins with some basic facts about fields and equipotentials. For more details, you should consult the appropriate chapters in your physics textbook.

## FACTS ABOUT FIELDS AND POTENTIALS

Electric field  $\mathbf{E}$  is a vector quantity; it is defined as the force per unit charge that would act upon a unit positive test charge if the charge were placed at the point in question. A line of force or electric field line is the path a charge would follow if it were allowed to move along in exactly the direction of the force acting on it. By convention, the sense of  $\mathbf{E}$  is the direction of the force on a *positive* test charge.

The potential difference,  $\Delta V_{AB}$  (between two points  $A$  and  $B$ ) is defined as the work that would have to be done in moving a unit charge from point  $A$  to point  $B$ . The sign of  $\Delta V$  may be positive or negative. Work can be expressed as force times distance, where the force acting on the charge is due to the electric field at the location of the charge,  $\mathbf{F} = q\mathbf{E}$ , and the distance is measured along the direction of the force.

The calculus and non-calculus form of the equations which relate  $V$  and  $\mathbf{E}$  are given below. The electric field at a point is

$$\mathbf{E} = -\frac{dV}{dl}\mathbf{r} \quad \text{or approximately} \quad \mathbf{E} \approx -\frac{\Delta V}{\Delta l}\mathbf{r} \quad (1)$$

where  $\mathbf{r}$  is a unit vector in the direction of greatest change in  $V$ . If Eq. (1) is integrated over  $l$  (or both sides are multiplied by  $\Delta l$ ), the potential difference between points  $A$  and  $B$  is found:

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} \quad \text{or approximately} \quad V_B - V_A = -\sum \vec{E} \cdot \Delta \vec{l} \quad (2)$$

where  $d\vec{l}$  or  $\Delta \vec{l}$  are infinitesimal or finite, but small, line elements, respectively, along a given path. Since the work done in moving a test charge from  $A$  to  $B$  (and consequently the potential difference between  $A$  and  $B$ ) is independent of the path for a conservative force,

you can choose a path which makes the integral or sum of Eq. 2 easy to evaluate. If you have studied calculus, it is preferable to use the line integral

$$- \int_A^B \mathbf{E} \cdot d\mathbf{l}$$

For a constant  $E$ , as in the case of the parallel plate capacitor,  $V = E l \cos \theta$ .

Potentials are often defined relative to some standard reference point that is taken to be zero. The reference point is often referred to as *ground*. The MKS unit of potential is joule/coulomb that has been given the name *volt*. If a test charge is moved in a direction perpendicular to  $\mathbf{E}$ , no work is done (see Eq. 2). Therefore, there is no change in electric potential. Such a path is an *equipotential surface*. Since there is a potential and electric field value at *every* point in space, there is no limit to the number of field lines and equipotential surfaces. In this experiment you will work in two dimensions to acquire a *family* of lines spaced at convenient intervals.

## APPARATUS

- |   |  |
|---|--|
| <input type="checkbox"/> DC power supply (25-40V max) | <input type="checkbox"/> Multimeter                                |
| <input type="checkbox"/> Conductive paper             | <input type="checkbox"/> Copper foil tape with conductive adhesive |
| <input type="checkbox"/> Scissors                     | <input type="checkbox"/> Cork board                                |
| <input type="checkbox"/> Assorted leads               | <input type="checkbox"/> Push pins                                 |

The apparatus consists of a sheet of carbon-impregnated paper with a relatively high resistance, metallic tape used as the electrodes, push pins for connecting the power supply to the electrodes, and a multimeter with various probes for making measurements of  $V$  and  $\mathbf{E}$ .

You will construct two electrodes (dots, lines, circles, combinations, etc.) with the metallic tape and connect these shapes to a power supply to create a potential difference between them. Because they are good conductors, the taped regions build up a net charge, which sets up an electric field in the paper between them. You will be able to measure the potential difference (voltage) between any two points using two probes that are connected to a digital multimeter. You will use the multimeter as a *voltmeter*, which measures the potential difference between its terminals. It is calibrated in volts with several scale settings. By using a double probe (two probes connected together so that their tips are 1 cm apart), you can directly measure the average electric field in volts/cm in the region between the probes by measuring  $\Delta V$ ; see Eq. 1). When you touch the probe to your carbon-impregnated paper, you are actually reading the work in joules required to move a coulomb of charge from one position on the carbon sheet to the other. A negative reading means that instead of having to do work to transport a charge from one prong to the other, you would get work out.

It is important to understand that you are modeling three-dimensional systems in only two dimensions. The paper and conductors represent the cross-section of objects that are cylindrically symmetrical rather than spherically symmetrical. For example, the field of a dot and a circle on the paper will be identical to that of a line and a cylinder in space, not of a point charge and a sphere.

## PROCEDURE

It is assumed that 2 lab partners are working together to make measurements of various electrode configurations as detailed below, but lab reports and analysis of the data are done independently. The following electrode configurations should be measured:

1. Two point electrodes (Figure 1a)
2. Two parallel bar electrodes (Figure 1b)
3. A point electrode inside a circular electrode (Figure 1c).

Note: It is not advised that more than 2 students work together in lab, since more measurements would be needed than time allows.

Making Electrodes: Plan your electrode configurations by outlining them in pencil on the carbon impregnated paper. Use the 20 x 28 cm grid markings as a guide for dimensions. Generally, it is best to have your electrodes spaced 15-20 cm apart so that measurements are more easily made. You will need to make measurements on both sides of the electrodes, so be sure to leave a few centimeters of space between your electrodes and the edge of the paper. Some suggested dimensions for the required electrodes are:

2 point electrodes: 1 cm squares or dots separated by 20 cm.

2 parallel bar electrodes: 10-12 cm in length separated by 20 cm.

Construct your electrodes using the metallic tape. Be sure the tape is tightly adhered to the paper and free of wrinkles by rubbing with the edge of your fingernail.

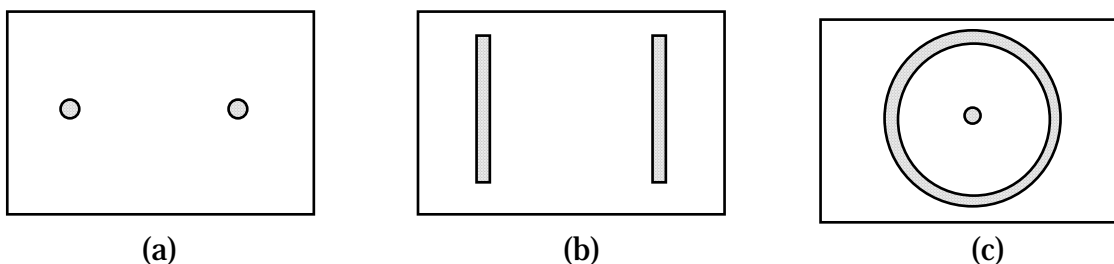


Figure 1: Electrode patterns

Follow the procedure presented below for each of the electrode configurations.

**Effective Electrode Potential:** Since the adhesive on the back of the tape insulates the metal from the paper somewhat, it is necessary for you to determine the *effective electrode potential*. To do this, insert push pins through each metal electrode and connect them with wires to the (+) and (−) terminals of the power supply respectively. Label the positive (+) and negative (−) electrodes on the carbon impregnated paper. Increase the power supply to 25-30 volts. Setup the multimeter as a voltmeter on the appropriate scale. Connect the ground side (−) of the multimeter to the ground terminal (−) of the power supply (See Figure 2). You are now ready to make potential measurements. First measure the potential of each of your electrodes by touching the multimeter probe directly to the metallic tape. The potential difference you measure should be very close to the voltage setting of the power supply. If not, check that you have connected everything correctly and that your push pins are making contact with the metal part of the tape. These potentials would be the electrode potentials if the tape made perfect contact with the carbon impregnated paper, but it does not. Hence, determine the *effective electrode potential* by measuring the potential on the paper very near to (~ 1 mm) but not touching the tape. Make several measurements along each electrode and use the average value as the *effective electrode potential*. Write the effective potentials next to each electrode.

You will probably observe a 5-8 volt difference between the potential measured directly on the tape and 1 mm away from the tape on the paper. The important factor is to achieve a 10-12 volt difference in your effective electrode potentials. If you do not, then adjust the power supply voltage or rub the tape more tightly to the paper until you get the minimum 10-12 volt difference.

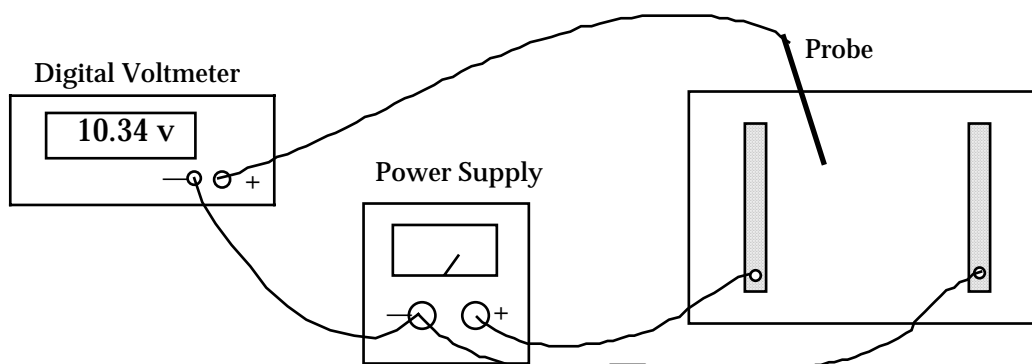


Figure 2. Arrangement for measuring potentials.

**Equipotential Lines:** Using the arrangement shown in Figure 2 you will first plot the equipotential lines -- lines on which the electric potential is constant. Touching the probe of the meter to the paper, find a place where the meter reads a whole number of volts (e.g. 15 not 15.4, etc.) which is approximately half way between the effective potentials. Mark this point with a pencil. Now move the probe around the paper to find other points at the same potential. Mark each of these points with your pencil. Find enough points so that you can connect them together to form an equipotential line between and/or around the electrodes. Now plot lines at intervals of 2, 4 and 6 volts etc. on each side of your first equipotential line. Also plot the equipotential lines close to but not touching the metallic electrodes. Label

all the lines with their voltages. Make enough measurements to determine the complete equipotential pattern. If you are pretty fast at plotting, you may want to make measurements in steps of 1 volt for better accuracy.

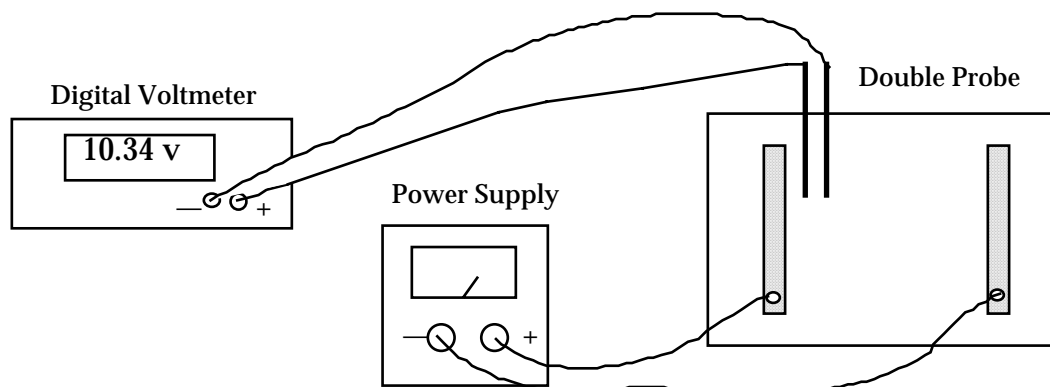


Figure 3. Arrangement for measuring electric field in volts/cm.

**Electric Field Lines:** Next you will plot the electric field lines on the same drawing. A different probe will be used as shown in Figure 3. This probe measures the difference in potential between two points separated by 1 cm. The electric field is defined as the difference of potential divided by the separation. The separation of the points is approximately 1 cm so that the meter readings will be the electric field  $E$  in volts/cm if the probe is oriented to give the maximum reading.

Start with one tip of the probe close to one electrode. Hold it nearly vertical. Carefully rotate the probe through  $360^\circ$  about the point farther from the electrode. You will notice that the meter goes through 0 and changes sign. Experiment with this until you understand *polarity*. Next rotate the probe and stop where the meter reads the maximum. The position for maximum meter reading has to be found carefully as it is not very sharp. It is best to rock the probe back and forth a little when you are near the maximum to close in on it from either side. Read the meter, then rotate the probe  $180^\circ$  and put it back in the same spot. You may note a slight asymmetry in the readings. If so, keep the probe oriented in the direction that gives the higher reading.

The magnitude of  $E$  is the maximum reading and its direction is toward the negative electrode. After you find the maximum, draw a pencil line between the two points of the probe. Label it with the meter reading. Indicate the sense of  $E$  with an arrowhead. Now move the probe 1 cm so that the other tip is on the tip of the arrow and repeat the process. Keep doing this and plot out a complete field line from one electrode to the other with no gaps. (See Question 3) Plot at least 5 lines at representative places along the electrodes. Keep in mind that the field lines should have a symmetry which reflects the symmetry of the electrode arrangement.

Repeat the above measurements for each of your electrode configurations. A minimum of 2 per student are required. If you have time, you may want to experiment with other electrode arrangements.

## QUESTIONS

1. What can you say about the angle at which field lines cross equipotential lines? What about the angle at which field lines start out from the electrodes? Can two field lines cross? Two equipotential lines?
2. Did you observe two field or equipotential lines crossing each other? Is this possible according to the theory? Explain.
3. For one of your electrode configurations, what were the effective potentials of the electrodes? The effective potential difference? Do the readings that you wrote next to the 1 cm segments equal the effective electrode potential difference when added up for an entire field line? Give numerical examples.
4. What relation do you observe between the values of  $E$  close to the bar conductors and the spacing of the equipotential lines near them? Explain.
5. What kind of line (field or equipotential) runs along, or "hugs", the boundary of a conductor (electrode)? What kind of line comes out perpendicular to a conductor?
6. Your textbook says that for parallel plate electrodes  $E = V/d$  where  $V$  is the (effective) potential between the electrodes. This formula suggests that  $E$  is uniform and constant between the plates. Is this what you observed? Explain any deviations.