

## Chaos Session II – Forcing chaos

Goals for the session:

1. Modify the pendulum simulation to create a driven pendulum simulation
2. Use the driven pendulum simulation to investigate chaos
3. Spend some time working on your own simulation project

### 1. Creating a driven pendulum simulation (30 minutes)

Start by loading into EJS the pendulum\_v4.xml file that is on the wiki site. You will need to download the file into the EJS folder system first – do this by right-clicking on the name of the file when you're on the wiki site, and saving the file into an appropriate folder in EJS. We will modify it to add a damping term and a driving term.

First, modify the simulation so the graph shows the angular velocity ( $\omega$ , on the y-axis) as a function of angular displacement ( $\theta$ , on the x-axis). Run the simulation to see how this graph behaves. Make sure you understand what it is showing.

Save the program as driven\_pendulum\_v1.xml

Now we need to add damping and an oscillating force term. We will also follow the lead of our chaos textbook and set  $m$ ,  $g$ , and  $L$  to 1.0.

We need some new variables.

$q$	2.0	damping parameter
$A$	1.2	amplitude of the driving force
$\omega_{\text{drive}}$	0.67	angular frequency of the driving force
$\phi$	0.0	angle of the driving force

We need to modify our evolution equations. Add a new equation at the top of the list that says:

$$d\phi / dt = \omega_{\text{drive}}$$

Then, modify the  $\omega$  equation to read

$$d\omega / dt = -(1/q)*\omega - \text{Math.sin}(\theta) + A * \text{Math.cos}(\phi)$$

Save the simulation as driven\_pendulum\_v2.xml

See if it runs – it should!

Now let's add a text box to the three that are already present. The text boxes should be linked, from top to bottom to:

theta

q

amp (the amplitude of the motion, A)

freq (the angular frequency of the driving force, omega\_drive)

Save the simulation as driven\_pendulum\_v3.xml

## 2. The Poincaré section, and investigating chaos (30 minutes)

The phase diagram, in which we plot the angular velocity as a function of the angle, is interesting but complicated. To see whether a motion is really chaotic, we can look at a Poincaré section, which plots a single point on the phase diagram once every period of the driving force. Converting our driven pendulum simulation to show the Poincaré section is not too difficult, but to save time simply download the Poincare.xml file from the wiki site, save it in the EJS folder system, and load it into EJS. Feel free to look at how the simulation is set up to see how the data for the Poincaré section is handled.

To run the program, it is even better to run the version Chris Gregg (a teacher in the course last year) posted on-line. Chris built in some nice features to see clean Poincare plots, and to make the program run fast. There is a link to it from our course web site, <http://physics.bu.edu/~duffy/NS548.html>.

Investigate different conditions to see when chaos occurs and when the driven pendulum is non-chaotic. Use various values of the amplitude of the drive force, and the frequency of the driving force, to see which sets of values result in chaos and which do not. You can also adjust the value of q, the damping parameter. Remember that the larger the value of q, the smaller the damping, because the evolution equation has a  $1/q$  in it.

Chapter 3 in our chaos textbook suggest trying the following amplitudes, if  $q = 2.0$  and the driving frequency is  $2/3$ . Compare your results to the plots on pages 46 – 52 in the book.

- (a) Amplitude = 0.9
- (b) Amplitude = 1.07
- (c) Amplitude = 1.15
- (d) Amplitude = 1.35
- (e) Amplitude = 1.45
- (f) Amplitude = 1.47
- (g) Amplitude = 1.50

## 3. Spend time working on your own simulation project (90 minutes)

We can brainstorm together regarding any issues you have.