

Writing a pendulum simulation

There is a pre-existing pendulum simulation, but let's write our own so we really understand how to do it.

I started with the spring.xml simulation that is in the Manual folder in examples

<u>Variable</u>	<u>Initial value</u>	<u>Description</u>
m	1.0	mass of the pendulum bob
L	1.0	length of the pendulum
g	9.8	acceleration due to gravity
x0	0.0	x-position of the support point
y0	0.0	y-position of the support point
theta	0.5	angle, from the vertical, in radians
omega	0.0	angular velocity
x	$L*\text{Math.sin}(\text{theta})$	x-position of the pendulum bob
y	$-L*\text{Math.cos}(\text{theta})$	y-position of the pendulum bob
t	0.0	time
dt	0.05	time increment

Evolution

$$d\text{theta}/dt = \text{omega}$$

$$d\text{omega}/dt = -g*\text{Math.sin}(\text{theta})/L$$

These equations are based on the angular variables, and the fact that the angular acceleration (which is domega/dt) is given by Newton's Second Law for rotation:

$$\alpha = \frac{\Sigma \tau}{I} = \frac{L \times mg \sin \theta}{mL^2} = \frac{g \sin \theta}{L}$$

Constraints

$$x = L*\text{Math.sin}(\text{theta});$$

$$y = -L*\text{Math.cos}(\text{theta});$$

Up to here is on the wiki site as pendulum_v1.xml

Now we need to work on the View

- In the Lower Panel, change the k textbox so it is called g, and is linked to g
- In the Drawing Panel, delete the Wall and the Spring
- In the Ball Properties, set Draggable to false and get rid of the entries in the OnDrag and OnRelease boxes
- In the DrawingPanel properties, I set the Maximum X and Maximum Y values to 1.2 and the Minimum X and Minimum Y values to -1.2. Then I re-sized the simulation window so that the pendulum bob was circular, instead of elliptical
- In the graphs, set the Input Y on the Displacement graph to theta, and Input Y on the Velocity graph to omega

At this point the simulation should run properly – give it a try.

Up to here is on the wiki site as pendulum_v2.xml

We should still modify a few things, though.

- In the TimeDisplayPanel, set Autoscale Y to true
- Add an arrow to the DrawingPanel, and call it string. For its properties, set X to x0, Y to y0, Size X to x-x0, and Size Y to y-y0. Also, set Style to SEGMENT
- Re-position the string so that it is in the View before the Ball. To do this, you can right-click on string and select Move Up.
- In the Evolution section of the Model, set FPS (frames per second) to MAX using the slider on the left

Up to here is on the wiki site as pendulum_v3.xml

Try doing these things yourself:

- In addition to the mass and g text boxes, add a theta box so that you can set the initial angle
- Add a graph to show the angular acceleration (you can use the equation we put in the Evolution section for $\text{d}\omega/\text{d}t$)
- Add a small circle to the View to represent the support point for the pendulum

Up to here is on the wiki site as pendulum_v4.xml

Another idea – try adding arrows to the ball to represent the various forces acting on it.

Let's examine the energy in the system

First, we need a couple more variables, PE and KE, to represent the potential and kinetic energy.

Then, we need to write the constraint equations for each one. If PE = 0 at the lowest point on the swing, we have:

$$PE = m * g * (y - (y_0 - L)); \quad \text{or} \quad PE = m * g * (y + L - y_0)$$

$$KE = 0.5 * m * (L * \omega)^2$$

That should give us $PE = mgh$, and $KE = 0.5 mv^2$, where we connect linear speed to angular speed using $v = r\omega$, where $r = L$.

I also added an EMax variable, which is initialized to $EMax = PE + KE$;

In the View, add a panel called RightPanel to the MainWindow, with its position set to Right.

To draw an energy bar diagram, we will use bars from the fourth tab in the Interface section.

Add a bar, called PE, to the RightPanel, and make its position left. I used the following settings:

Variable: PE

Minimum: 0.0

Maximum: EMax

Format: PE = 0.#

Orientation: VERTICAL

Size: 30,0

Foreground: BLUE

Similarly, add a bar, called KE, to the RightPanel, and make its position right. Use similar settings to those for PE, with a foreground color of RED.

Up to here is on the wiki site as pendulum_v5.xml

Note that the bar graphs can behave strangely if you change the numbers in the text boxes, depending on when you change them – you might want to think about how to get around that issue.