Momenergy

Combining momentum and energy into a single quantity, momenergy.

Once again, we find that the classical equations produce unreasonable results in situations when objects are moving at a significant fraction of the speed of light. In collisions involving very fast-moving objects, in fact, momentum and energy are not conserved according to the classical definitions of these quantities. Using relativistic equations, however, shows that momentum and energy are actually conserved in such situations.

Let's start with a little background.

Thus far one of the most useful relativistic expressions we have used has been the equation for the invariant quantity we called the interval:

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = (\text{interval})^2$$

In the special case of an observer who is present at the two events that mark the beginning and end of the interval, $\Delta x = 0$ and so, for that observer, $c\Delta t = \text{interval}$.
The time interval measured by this observer between two events is special, so we give it a special name – the proper time – and a special symbol, $\Delta\tau$.

In other words, the interval is essentially the proper time.

Example: Isabelle passes You on Earth, and then 15 years later (according to Isabelle) she passes Yan. You and Yan, however, think Isabelle's trip took 25 years. What is the proper time for Isabelle's trip?

A four-vector. An example of a four-vector (a vector with four components) is a displacement in spacetime. Since space has three dimensions (which we usually call x, y, and z) a displacement in spacetime has three spatial components. The fourth component of a dimension in spacetime is time. So far we have confined ourselves to displacements in two dimensions (one spatial dimension and the time dimension) but we could generalize to four dimensions:

$$(\text{interval})^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2$$

Now let's turn to a relativistic view of momentum and energy.

First, as we did with time and space, we will express momentum and energy in the same units. We can do this by defining speed as a dimensionless quantity, which we have also done before, by expressing speed as a number less than or equal to 1, representing the fraction of the speed of light an object is traveling.

The connection between this dimensionless speed v and the conventional speed v_{conv}, which has units of m/s, is:

$$v = \frac{v_{conv}}{c} \qquad \text{or} \qquad v_{conv} = vc\,.$$

If v is dimensionless, what are the units of momentum? What are the units of energy?

Because they have the same unit we can, as we did with time and space, combine them into one quantity called momenergy. Momentum (p) represents the spatial part of momenergy (3 components) and energy (E) represents the time part of momenergy (1 component). Thus momenergy is also a 4-vector.

The direction of an object's momenergy is the same as the direction of an object's worldline at that instant.

Once again we have an invariant quantity:

$$m^2 = E^2 - p^2$$

We have the following definition of momenergy:

$$\text{momenergy} = \text{mass} \times \frac{\text{spacetime displacement}}{\text{proper time for that displacement}}$$

Note that the spacetime displacement is a vector, while the proper time represents the magnitude of that vector, so momenergy is a vector with a magnitude equal to the mass.