Relative Velocity

Let's practice applying the relativistic Relative Velocity equation

A typical non-relativistic relative velocity situation might go like this:

You are at rest beside the Mass Pike. Isabelle is driving on the Mass Pike with a velocity of $\vec{v}_{IY} = 80$ miles/hour directed east, measured with respect to you. Jack is also on the Mass Pike, traveling with a velocity of $\vec{v}_{JY} = 80$ miles/hour west, measured with respect to you. Let's define $\vec{v}_{IJ}$ as the velocity of Isabelle with respect to Jack. Come up with an equation that expresses $\vec{v}_{IJ}$ in terms of $\vec{v}_{IY}$ and $\vec{v}_{JY}$.

Use your equation to work out $\vec{v}_{IJ}$.

A relativistic situation could be stated like:

You and Yan are separated by a distance of 20 light-years, as measured from your reference frame. Yan is at rest with respect to you. Define the positive direction as the direction pointing from you toward Yan. Isabelle is passing you with a velocity, according to you (and Yan), of $\vec{v}_{IY} = +0.8c$, while Jack is passing Yan with a velocity, according to Yan (and you), of $\vec{v}_{JY} = -0.8c$.

In this case we need to use the relativistic relative velocity equation: $\vec{v}_{IJ} = \dfrac{\vec{v}_{IY} - \vec{v}_{JY}}{1 - \dfrac{\vec{v}_{IY}\vec{v}_{JY}}{c^2}}$.

Use this equation to find the velocity of Isabelle with respect to Jack.

When Isabelle passes you, you release a pulse of light in the direction of Yan. Adjust the equation above to calculate the velocity of the light with respect to Jack.

Adjust the equation above to calculate the velocity of the light with respect to Isabelle.

The relativistic relative velocity equation is actually a general equation that can be used in all cases. Show that the non-relativistic equation is a special case of the relativistic equation in the limit where the velocities are small.

The relativistic equation is $\vec{v}_{IJ} = \dfrac{\vec{v}_{IY} - \vec{v}_{JY}}{1 - \dfrac{\vec{v}_{IY}\vec{v}_{JY}}{c^2}}$.

The speed of light is about 6.7 x 10^8 miles per hour. Use the relativistic equation to calculate Isabelle's velocity with respect to Jack in the non-relativistic situation we looked at originally:
You are at rest beside the Mass Pike. Isabelle is driving on the Mass Pike with a velocity of $\vec{v}_{IY} = 80$ miles/hour directed east, measured with respect to you. Jack is also on the Mass Pike, traveling with a velocity of $\vec{v}_{JY} = 80$ miles/hour west, measured with respect to you.

What is the approximate error we introduce by using the non-relativistic version of the equation? You might want to use the approximation:

$$\frac{1}{1+x} \approx 1 - x .$$