Transforming from one Frame of Reference to Another
Let's take the Spacetime Diagram as far as we can

At our first session we talked about typical x-y coordinate systems, what is invariant, and what is not. The invariant quantity is the distance, *d*, of a point from the origin.

The Pythagorean theorem applies, so if a point is located at coordinates of (x, y) in one coordinate system and (x', y') in another coordinate system we have:

$$x^2 + y^2 = d^2 = (x')^2 + (y')^2$$

As we rotate the axes to move from one coordinate system to another the point traces out a circle of radius d around the origin. Since the geometry is based on the circle and on right-angled triangles, the equations that allow us to transform from one coordinate system to another involve sines and cosines of the angle between the coordinate axes.

If the primed coordinate system is rotated by an angle θ with respect to the unprimed coordinate system, we can transform from one frame to the other. How?

What's the analogous process for the geometry of spacetime?

If an event has spacetime coordinates of (ct, x) in one coordinate system and (ct, x) in another coordinate system we have:

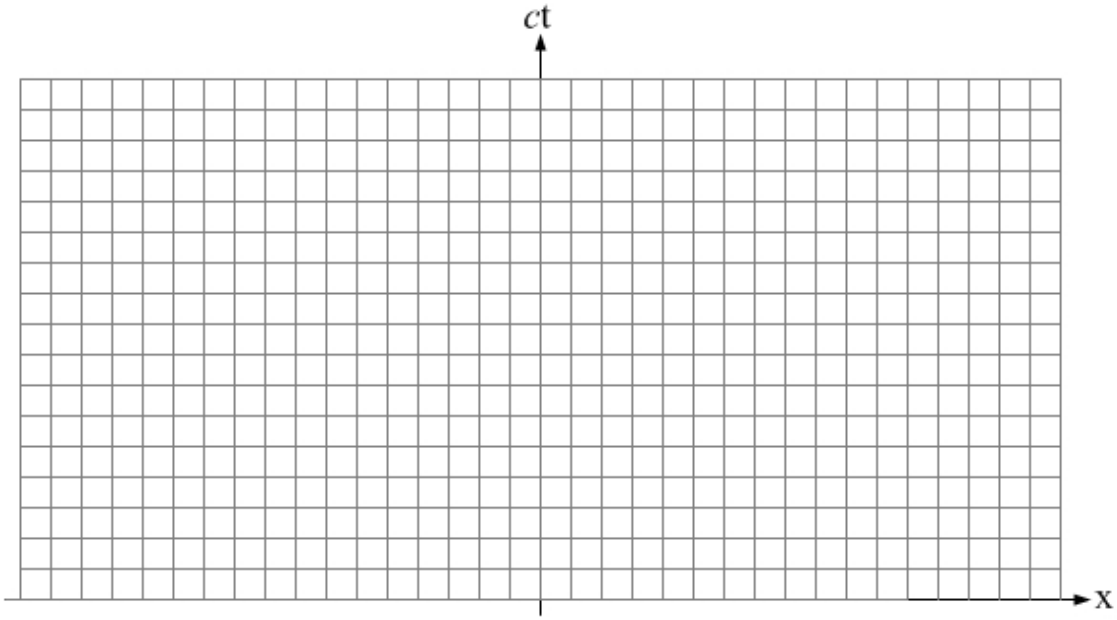
$$(ct)^2 - x^2 = \text{interval}^2 = (ct')^2 - (x')^2$$

This does not give us the circle we had earlier! What does it get us?

Consider the special case of the interval being 0. Plot that on the spacetime diagram.

Consider the interval being 1. Plot these points on the spacetime diagram:

ct	x
1	0
2	$\sqrt{3} = 1.73$
3	$\sqrt{8} = 2.83$
4	$\sqrt{15} = 3.87$
5	$\sqrt{24} = 4.90$
6	$\sqrt{35} = 5.92$



Suggestion – use a large scale, such as every two boxes = 1 lightyear.

Transformations

How do we transform from one coordinate system to another? The transformations involve a constant called gamma:

$$\gamma = \frac{1}{\sqrt{1-v^2}}, \quad \text{where v is expressed as a fraction of c.}$$

Transformations from one frame of reference to another are given by:

$$x' = \gamma\left[x - v(ct)\right] \qquad x = \gamma\left[x' + v(ct')\right]$$
$$t' = \gamma\left(ct - vx\right) \qquad t = \gamma\left(ct' + vx'\right)$$

What is the value of γ for the Isabelle-You-Yan situation?

A particular event we considered was Isabelle passing Yan. With the origin at Earth, and starting clocks when Isabelle passed Earth, we found this event to happen (according to You) at x = 20 light-years, and at a time t = 25 light-years. Use the transformation equations to calculate the values of x' and t' (primed coordinates represent Isabelle's reference frame).