Spacetime Diagrams

Let's now look at how we handle relativistic situations.

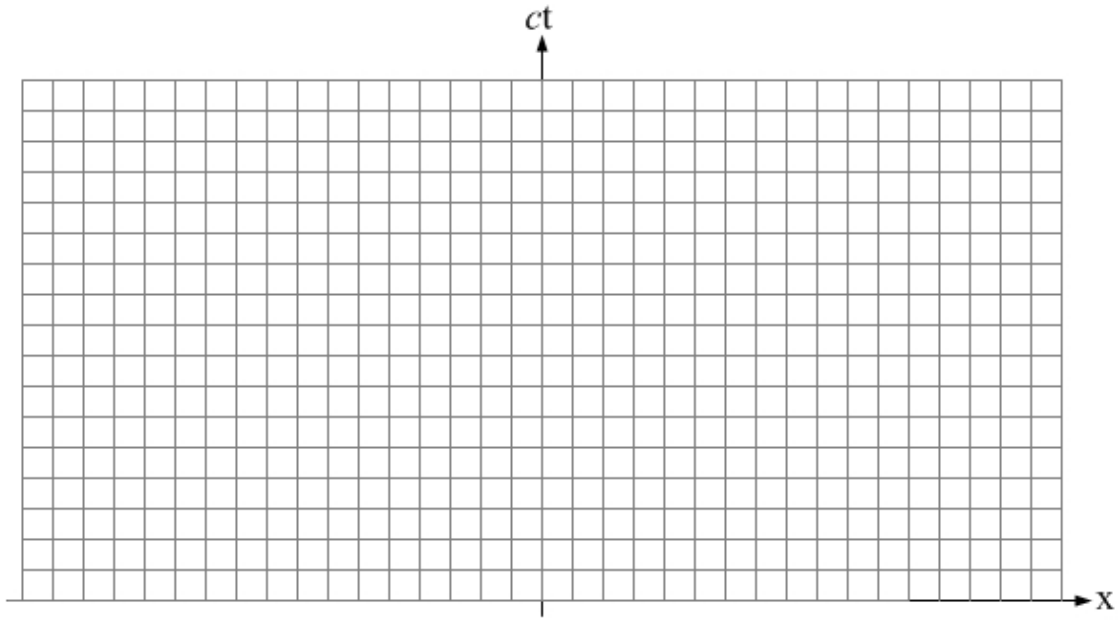
Now let's turn to relativistic situations, in which velocities are a reasonable fraction of the speed of light, $c = 3 \times 10^8$ m/s. In spacetime diagrams we generally multiply time by c to get length units on the "time" axis. If we measure time in years the length units are light-years.

One of Einstein's important realizations is that, no matter what your velocity, light always travels at the speed of light with respect to you.

Here's a typical relativity question. In the reference frame of the Earth a particular star is 20 light-years away. Isabelle is passing by the Earth in her rocket ship traveling toward the star at a constant velocity of 0.8 c. At the instant she passes by you send a light pulse toward the star.

According to you, on Earth, how long does the light pulse take to reach the star? How long does Isabelle take?

On the spacetime diagram plot, for the reference frame of the Earth, the worldlines of the star, the light pulse, and Isabelle in her rocket. The boxes on the graph measure 2 light-years wide by 2 light-years high.



It turns out that the following quantity is invariant: $(c\Delta t)^2 - (\Delta x)^2$. Thus we can say that: $(c\Delta t)^2 - (\Delta x)^2 = \text{constant}$. Work out what the value of the constant is for Isabelle's trip from the Earth to the star.

Now let's examine the trip from Isabelle's perspective. In Isabelle's perspective either the time interval $\Delta t'$ between passing the Earth and arriving at the star is zero, or the spatial separation $\Delta x'$ between these two events is zero. Which one is zero? Explain.

Since $(c\Delta t)^2 - (\Delta x)^2$ is invariant, Isabelle can use the constant we found above in the equation:

$(c\Delta t')^2 - (\Delta x')^2 = \text{constant}$

Solve this to find out how much time passes, in Isabelle's frame of reference, between her passing Earth and her arrival at the star.

What does Isabelle measure for the distance between the Earth and the star?

On the spacetime diagram, for Isabelle's frame of reference, plot worldlines for Isabelle, the Earth, and the star.