The nucleus

A nucleus consists of protons and neutrons; these are known as nucleons. Each nucleus is characterized by two numbers: *A*, the atomic mass number (the total number of nucleons); and *Z*, the atomic number (the number of protons). The number of neutrons is *N*, so $A = N + Z$.

Any nucleus can be written in a form like this:

$$ {}^{A}_{Z}\mathrm{X} \qquad\qquad {}^{27}_{13}\mathrm{Al} $$

The X stands for the chemical symbol. On the right is a particular isotope of aluminum, aluminum-27. Isotopes of an element have the same *Z*, but a different number of neutrons.

How big is a nucleus?

How big is a nucleus? We know that atoms are a few angstroms, but most of the atom is empty space. The nucleus is much smaller than the atom, and is typically a few femtometers. 1 fm = 1 x 10^{-15} m.

The nucleus can be thought of as a bunch of balls (the protons and neutrons) packed into a sphere, with the radius of the sphere being approximately:

$r = (1.2 \times 10^{-15}\ \text{m})\ A^{1/3}$

The atomic mass unit

The mass of a tiny object like a neutron or an atom is often stated in terms of the atomic mass unit (u). The atomic mass unit is defined in terms of the mass of:

1. an electron
2. a proton
3. a neutron
4. a hydrogen atom
5. a helium-4 atom
6. a carbon-12 atom
7. a carbon-14 atom

The carbon-12 atom

One atomic mass unit (u) is defined as 1/12th the mass of a carbon-12 atom. What makes up the carbon-12 atom? Does anything seem odd here?

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
1 atomic mass unit	1.660540×10^{-27}	1.000	931.5
neutron	1.674929×10^{-27}	1.008664	939.57
proton	1.672623×10^{-27}	1.007276	938.28
electron	9.109390×10^{-31}	0.00054858	0.511

The mass defect

Each carbon-12 atom is made up of 6 neutrons, 6 protons, and 6 electrons, which separately have a mass of:

$$\text{six neutrons: } 6 \times 1.008664 \text{ u} = 6.051984 \text{ u}$$

$$\text{six protons: } 6 \times 1.007276 \text{ u} = 6.043656 \text{ u}$$

$$\text{six electrons: } 6 \times 0.00054858 \text{ u} = 0.00329148 \text{ u}$$

$$\text{Sum} = 12.098931 \text{ u}$$

When these are combined into a carbon-12 atom, the atom has a mass of precisely 12.000000 u. The missing 0.098931 u worth of mass is the mass defect.

The most famous equation in physics

$$E = mc^2$$

The missing mass is the binding energy of the atom (almost all of which is in the nucleus).

At the rate of 931.5 MeV per u, the mass defect of 0.098931 u corresponds to 92.15 MeV worth of binding energy in the carbon-12 atom.

Nuclear processes

In a typical nucleus, the binding energy is measured in MeV, considerably larger than the few eV associated with the binding energy of electrons in the atom. Nuclear processes (such as nuclear reactions or decays) involve changes in the nuclear binding energy, which is why nuclear reactions give you much more energy than chemical reactions; those involve changes in electron binding energies.

Holding the nucleus together

A typical nucleus has a number of protons and neutrons packed together in a small space. We've discussed a lot of forces - which of these forces is primarily responsible for holding the nucleus together?

1. gravity
2. tension
3. love
4. electrostatic
5. magnetic
6. none of the above

Holding the nucleus together

The nucleus is tiny, so the protons are all very close together. The gravitational force attracting them to each other is much smaller than the electric force repelling them, so there must be another force keeping them together. This other force is known as the strong nuclear force.

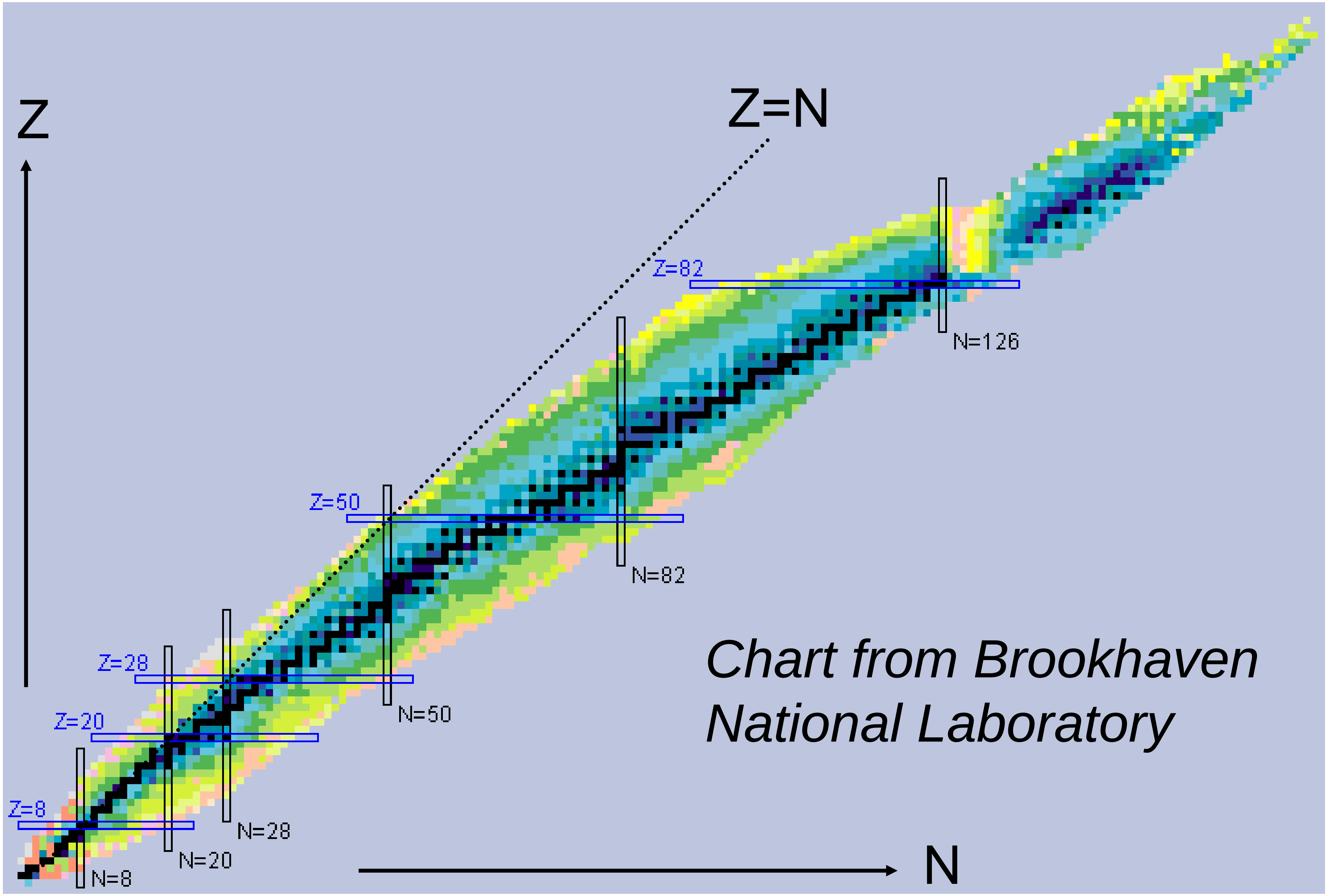
The strong nuclear force is a very strong attractive force for protons and neutrons separated by a few femtometers, but it is basically negligible for larger distances.

The strong force

The tug-of-war between the attractive strong force and the repulsive electrostatic force between protons has implications for the stability of a nucleus. Atoms with very low atomic numbers have about the same number of neutrons and protons. As Z gets larger, stable nuclei need more neutrons than protons.

Eventually, a point is reached beyond which there are no stable nuclei: the bismuth nucleus with 83 protons and 126 neutrons is the largest stable nucleus. Unstable nuclei eventually break up - this is known as radioactive decay.

Chart of the nuclides



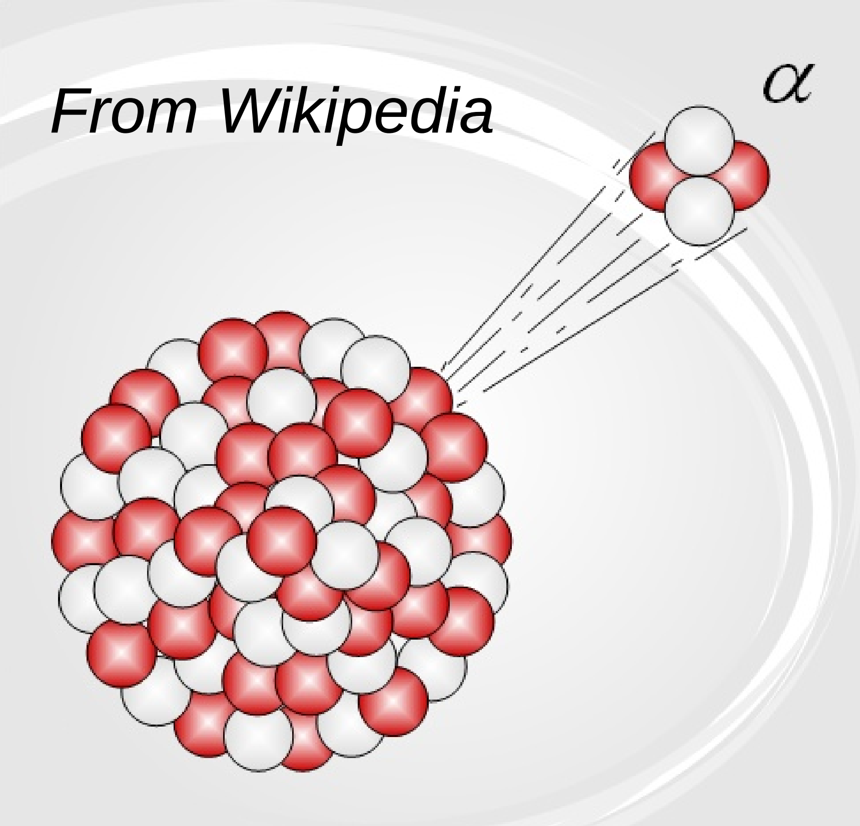
Alpha decay

In alpha decay, the nucleus emits an alpha particle. An alpha particle is a helium nucleus (two protons and two neutrons), which is very stable. Alpha particles do not travel far in air before being absorbed; this makes them safe for use in smoke detectors.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

An example of alpha decay involves uranium-238:

$$^{238}_{92}U \rightarrow ? + {}^{4}_{2}He$$



From Wikipedia

Alpha decay

$$^{238}_{92}\mathrm{U} \rightarrow {}^{A}_{Z}? + {}^{4}_{2}\mathrm{He}$$

What is the atomic mass number, A, of the unknown decay product on the right?

A = 234, by conservation of nucleon number.

What is its atomic number, Z?

Z = 90, by conservation of charge.

What is the chemical symbol for this element?

The atomic number 90 ➔ thorium, Th.

Alpha decay

$$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}$$

Looking up the data we need here:

Atomic mass of U-238 is 238.050786 u
Atomic mass of Th-234 is 234.043596 u
Atomic mass of He-4 is 4.002603 u

The total mass on the left side of the equation is 238.050786 u.

The total mass on the right side of the equation is 234.043596 u + 4.002603 u = 238.046199 u.

The left side of the equation has more mass, by 0.004587 u. Where did the extra mass go?

Alpha decay

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

The left side of the equation has more mass, by 0.004587 u. Where did the extra mass go?

The missing mass was converted to 0.004587 u × 931.5 MeV/u = 4.273 MeV of energy. This shows up in the kinetic energy of the two atoms after the reaction.

Reactions occur spontaneously when the total mass afterwards is less than the total mass before.

Alpha decay

Alpha decay is related to the strong nuclear force.

The fact that a helium nucleus can sneak out of the nucleus, breaking the attraction of the strong force, is evidence of a quantum phenomenon known as tunneling.

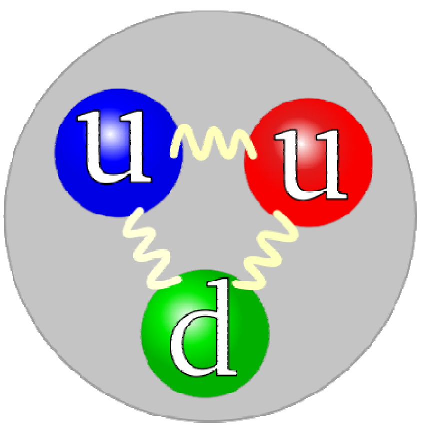
If you hit a tennis ball repeatedly against a wall, it will never get over the wall if you never give it enough energy to reach the top.

A helium nucleus, on the other hand, has a wave nature, and its wave extends to the far side of the "wall" – thus, it has a non-zero probability of getting out of a large nucleus, and eventually does so.

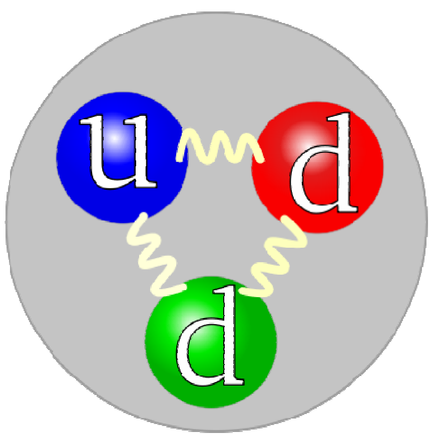
Beta decay, and quarks

Beta decay is related to the weak nuclear force.

The weak nuclear force involves quarks, changing from one kind of quark to another. There are six kinds of quarks, but all we need are two kinds (up and down) to make neutrons and protons.

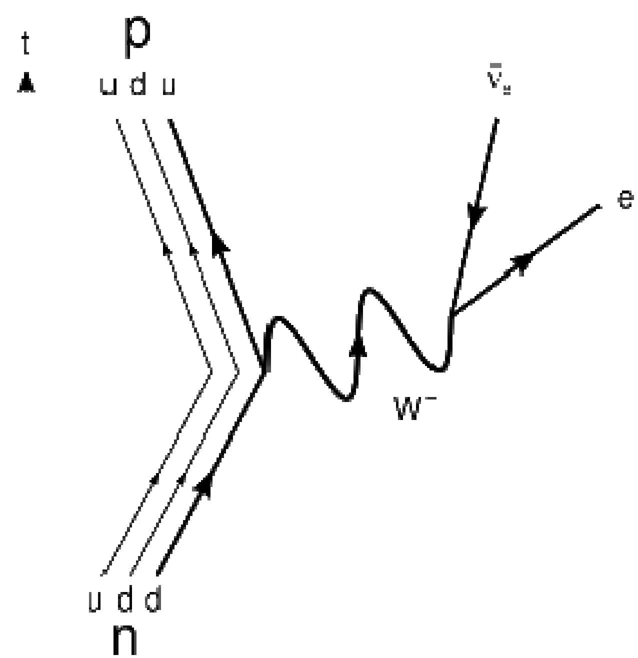


proton



neutron

(Images from Wikimedia Commons)



Beta decay

A beta particle is often an electron, but can also be a positron, a positively-charged particle that is the anti-matter equivalent of the electron. In terms of safety, beta particles are more penetrating than alpha particles, but less than gamma particles.

An important process in beta decay involves a neutron turning into a proton by giving up an electron:

$$^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\mathrm{e}^{-} + \overline{\nu}_{e}$$

Beta-minus (β^-) decay

$$^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\mathrm{e}^{-} + \overline{\nu}_{e}$$

The extra particle on the end is an antineutrino, with the e subscript denoting that it is an electron antineutrino. The existence of the neutrino was proposed by Wolfgang Pauli in 1930 to explain what seemed like violations of the laws of conservation of energy and conservation of momentum in the beta decay process.

Beta decay

An example of beta-minus decay is:

$$^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa}^{+} + {}^{0}_{-1}\mathrm{e}^{-} + \overline{\nu}_{e}$$

What is the atomic mass number, A, of the unknown decay product on the right?

A = 234, by conservation of nucleon number.

What is its atomic number, Z?

Z = 91, by conservation of charge.

What is the chemical symbol for this element?

The atomic number 91 ➔ Pa

Beta decay

An example of beta-minus decay is:

$$^{234}_{90}\mathrm{Th} \rightarrow \, ^{234}_{91}\mathrm{Pa}^{+} + \, ^{0}_{-1}\mathrm{e}^{-} + \overline{\nu}_{e}$$

Use the data from the appendix:

The atomic mass of Th-234 is 234.043596 u
The atomic mass of Pa-234 is 234.043302 u

We don't have to add the mass of the electron because the mass for Pa-234 includes 91 electrons, which is how many we have on the right side. It's already built into the mass for Pa-234. The mass of the anti-neutrino is non-zero, but negligible.

Beta-plus (β^+) decay

$$ {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e^{+} + \nu_e $$

In beta-plus decay, a proton turns into a neutron, positron, and an electron neutrino. In general, we get:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z\text{-}1}Y^{-} + {}^{0}_{+1}e^{+} + \nu_e $$

Beta decay

An example of beta-plus decay is:

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne}^{-} + {}^{0}_{1}\mathrm{e}^{+} + \nu_{e}$$

What is the atomic mass number, A, of the unknown decay product on the right?

A = 22, by conservation of nucleon number.

What is its atomic number, Z?

Z = 10, by conservation of charge.

What is the chemical symbol for this element?

The atomic number 10 ➔ Ne (neon).

Beta decay

An example of beta-plus decay is:

$$^{22}_{11}\mathrm{Na} \rightarrow \,^{22}_{10}\mathrm{Ne}^{-} + \,^{0}_{1}\mathrm{e}^{+} + \nu_{\mathrm{e}}$$

The atomic mass of Na-22 is 21.994436 u
The atomic mass of Ne-22 is 21.991385 u

We have to add the mass of one extra electron and the positron because the mass for Ne-22 includes 10 electrons, and we have the equivalent of 12 on the right side. The total mass on the right is:

21.991385 u + 2(0.00054858 u) = 21.992482

Gamma decay

The third class of radioactive decay is gamma decay, in which the nucleus changes from a higher-level energy state to a lower level. The nucleus has energy levels, similar to electron levels.

When an electron changes levels, the energy involved is usually a few eV, so a visible or ultraviolet photon is emitted. In the nucleus, energy differences between levels are much larger, typically a few hundred keV, so the photon emitted is a gamma ray.

Gamma decay

Gamma rays are very penetrating; they can be most efficiently absorbed by a relatively thick layer of high-density material such as lead.

A gamma decay is written in the following way:

$$ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma $$

The asterisk indicates that the nucleus is in an excited state. Note that this is the only decay in which the atom does not become another element.

Radioactivity

It is impossible to predict when an individual nucleus will decay. However, radioactive decay is governed by statistics, so we can predict the decay pattern of a large number of radioactive nuclei.

The rate at which nuclei decay is proportional to *N*, the number of nuclei there are.

"Activity"

$$R = -\frac{\Delta N}{\Delta t} = \lambda N$$

where λ is the decay constant.

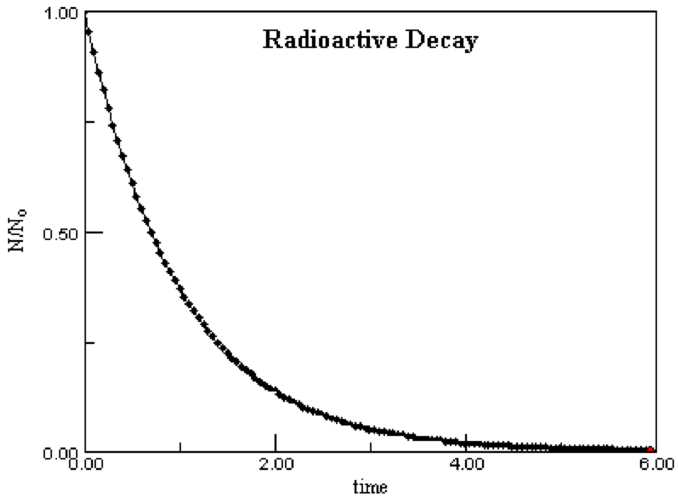
Radioactivity

Whenever the rate at which something occurs is proportional to the number of objects, the decay happens exponentially.

$$N = N_0 e^{-\lambda t}$$

Number remaining after a time t

Initial number at t = 0



The decay constant is closely related to the half-life, the time for half the material to decay.

$$T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

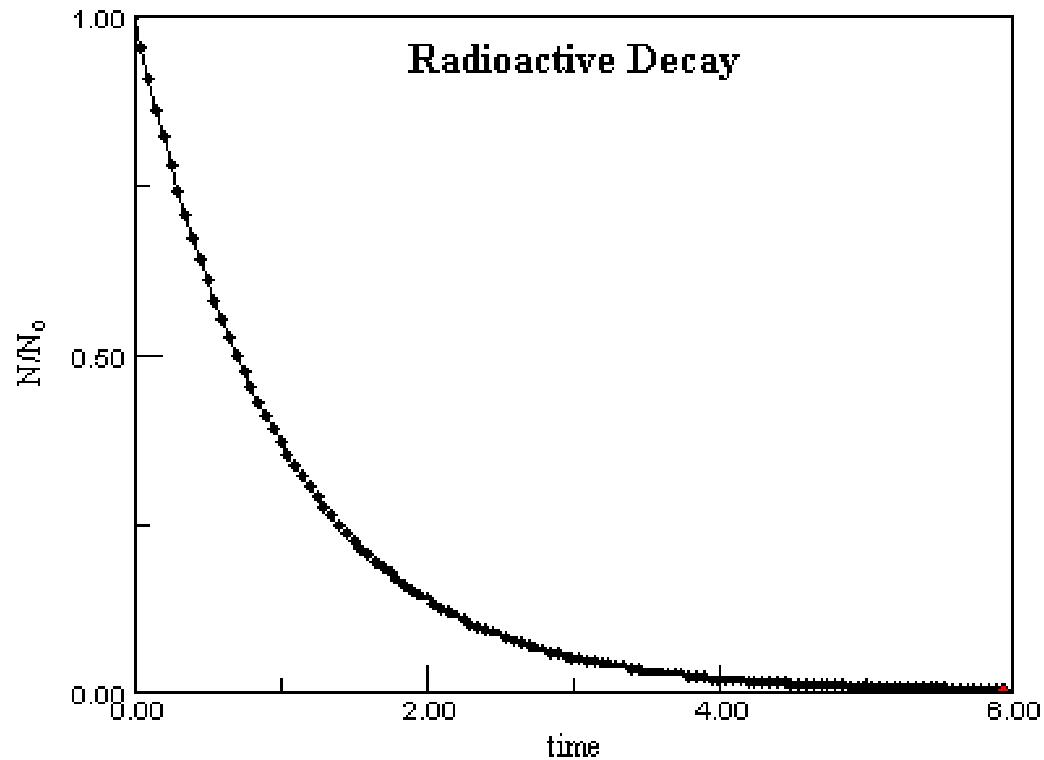
Radioactivity

$$N = N_0 e^{-\lambda t} \quad \text{or} \quad N = \frac{N_0}{2^{t/T_{1/2}}}$$

Number remaining after a time t

Initial number at t = 0

A graph of the number of undecayed nuclei as a function of time.



Units

The activity of a sample of radioactive material is measured in disintegrations per second, the SI unit for this being the becquerel (Bq).

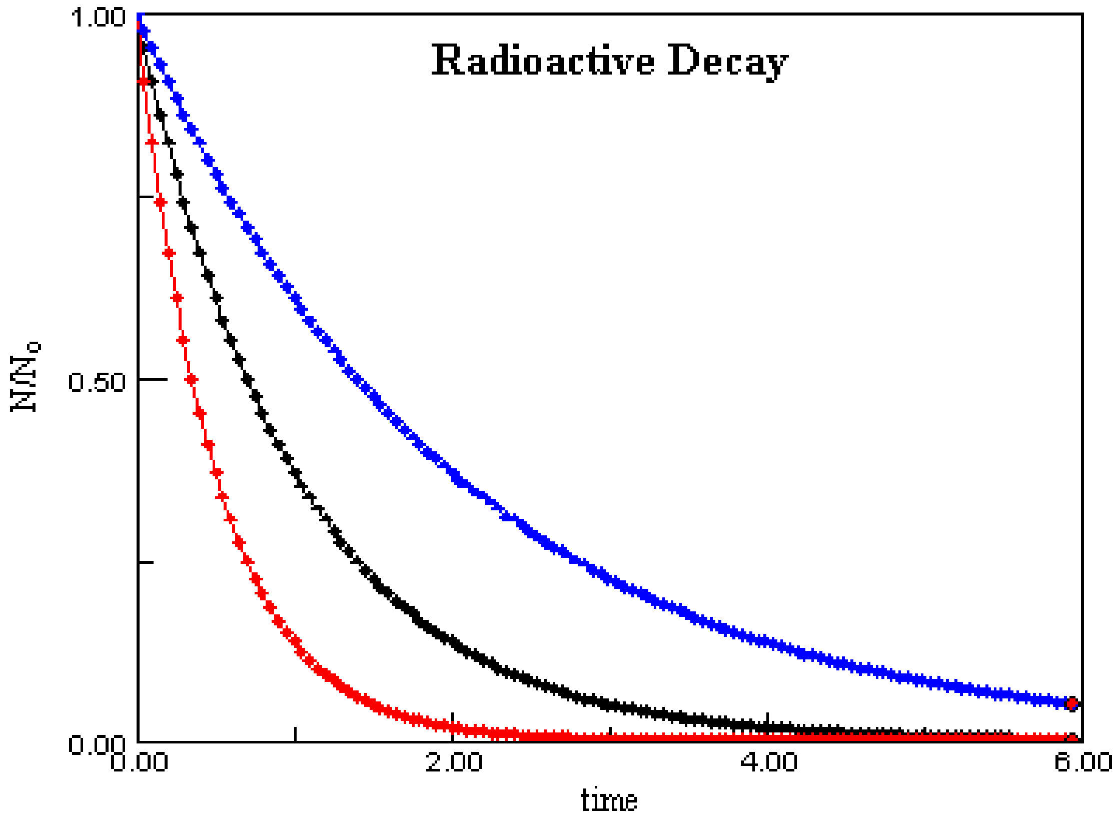
A more common unit is the curie (Ci).

1 Ci = 3.7×10^{10} decays/s = 3.7×10^{10} Bq

Increasing the decay constant

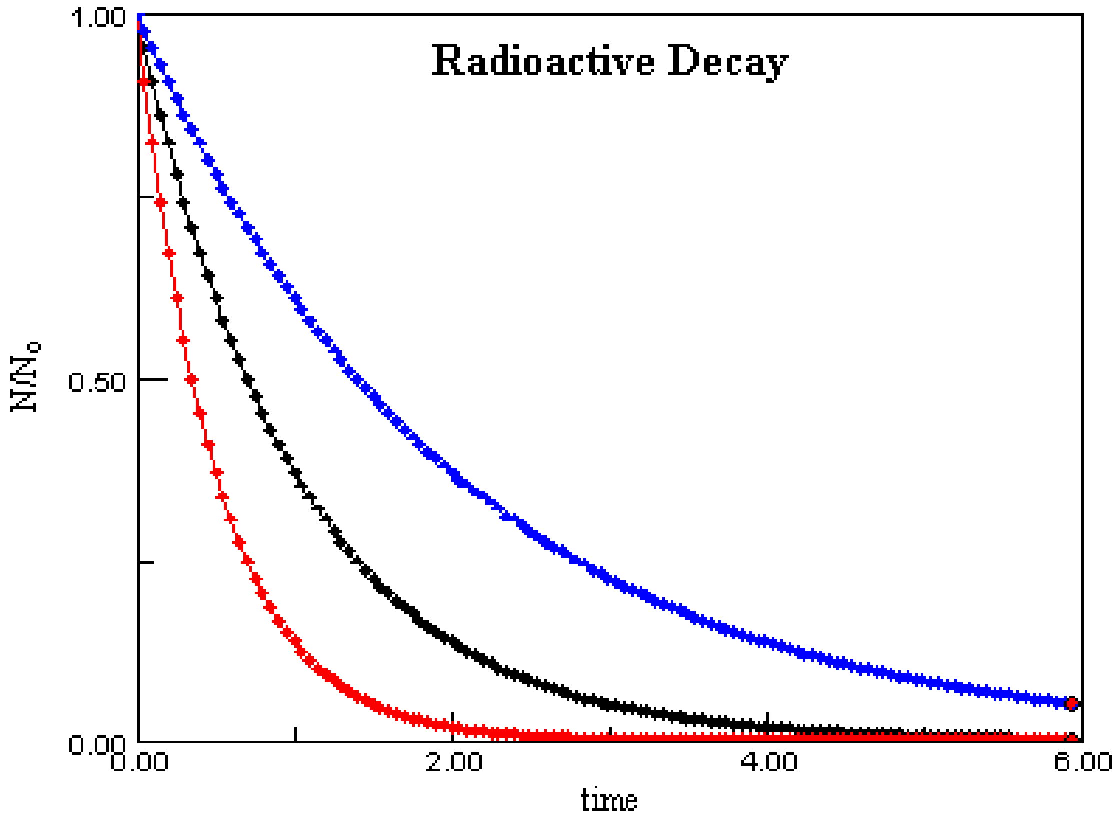
The black curve represents the decay curve for a particular radioactive sample. Which curve represents the decay of a second sample that has a larger decay constant?

1. the red curve
2. the blue curve



Increasing the decay constant

A larger decay constants means the decay happens more quickly. The red curve represents a larger decay constant, and a shorter half-life.



Radioactive dating

Radioactivity can be used to determine how old something is. When carbon-14 is used, the process is called radiocarbon dating, but radioactive dating can involve other radioactive nuclei. The trick is to use a half-life which is of the order of, or somewhat smaller than, the age of the object.

Radiocarbon dating

Carbon-14 is used because all living things take up carbon from the atmosphere, where there is about 1 atom of C-14 for every 8.3×10^{11} atoms of carbon.

When an organism dies, the carbon-14 decays. Carbon-14 has a half life of 5730 years, so it is useful for measuring ages of objects that are several hundred years, to several tens of thousands of years, old.

Applications of radiocarbon dating include dating of the shroud of Turin (to the 13th-14th centuries), and of Ötzi the Iceman (3300 BC), found in the Alps in 1991.