28-1 Line Spectra and the Hydrogen Atom

Figure 28.1 gives some examples of the line spectra emitted by atoms of gas. The atoms are typically excited by applying a high voltage across a glass tube that contains a particular gas. By observing the light through a diffraction grating, the light is separated into a set of wavelengths that characterizes the element.

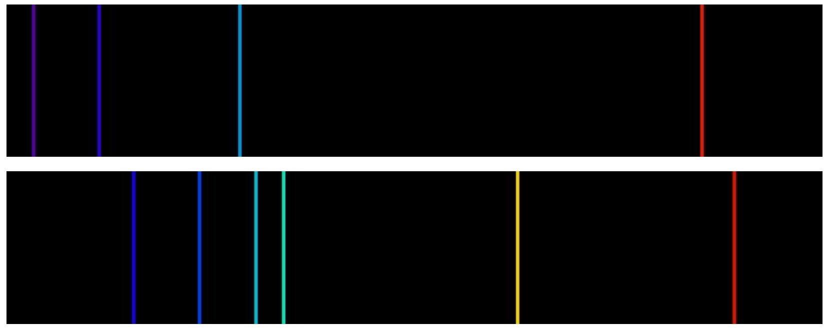

Figure 28.1: Line spectra from hydrogen (top) and helium (bottom). A line spectrum is like the fingerprint of an element. Astronomers, for instance, can determine what a star is made of by carefully examining the spectrum of light emitted by the star.

The spectrum of light emitted by excited hydrogen atoms is shown in Figure 28.1(a). The decoding of the hydrogen spectrum represents one of the great scientific mystery stories. First on the scene was the Swiss mathematician and schoolteacher, Johann Jakob Balmer (1825 – 1898), who published an equation in 1885 giving the wavelengths in the visible spectrum emitted by hydrogen. The Swedish physicist Johannes Rydberg (1854 – 1919) followed up on Balmer's work in 1888 with a more general equation that predicted all the wavelengths of light emitted by hydrogen:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$ (Eq. 28.1: **The Rydberg equation for the hydrogen spectrum**)

where $R = 1.097 \times 10^7$ m^{-1} is the Rydberg constant, and the two n's are integers, with n_2 greater than n_1.

Neither Balmer nor Rydberg had a physical explanation to justify their equations, however, so the search was on for such a physical explanation. The breakthrough was made by the Danish physicist Niels Bohr (1885 – 1962), who showed that if the angular momentum of the electron in a hydrogen atom was quantized in a particular way (related to Planck's constant, in fact), that the energy levels for an electron within the hydrogen atom were also quantized, with the energies of the electrons being given by:

$$E_n = \frac{-13.6 \text{ eV}}{n^2},$$ (Eq. 28.2: **Energies of the electron levels in the hydrogen atom**)

where n is any positive integer.

When the electron in a hydrogen atom drops down from a higher energy state to a lower energy state, a photon is given off that has an energy equal to the difference between the electron energy levels – thus, energy is conserved. Because the differences between the electron energy levels are limited, the photons that are emitted by excited hydrogen atoms are emitted at specific wavelengths, giving the few bright lines shown in Figure 28.1(a).

According to Bohr's model of the atom, the lines in Figure 28.1(a) correspond to photons emitted when electrons drop down to the second-lowest energy level in hydrogen from the levels with n = 3, 4, 5, 6 or 7. Bohr predicted, however, that photons should be observed at wavelengths corresponding to electrons dropping down from an excited state to the ground (n = 1) state. We don't see these wavelengths with our eyes because they are in the ultraviolet region of the spectrum. When scientists using detectors sensitive in the ultraviolet region found light emitted by hydrogen at the exact wavelengths predicted by Bohr, it was a tremendous validation of the Bohr model of the hydrogen atom. Bohr received the Nobel Prize in Physics in 1922 for his work.

EXPLORATION 28.1 – Building the energy-level diagram for hydrogen

Creating a diagram of the energy levels for hydrogen can help explain how the photon energies arise. We will start with the three lowest levels.

Step 1 – ***Using Equation 28.2, determine the energies of the three lowest energy levels for hydrogen. Then create an energy-level diagram, which looks like a ladder with rungs that are unequally spaced.*** The three lowest energy levels correspond to n = 1, 2, and 3. Substituting these values of n into Equation 28.2 gives E_1 = –13.6 eV, E_2 = –3.40 eV, and E_3 = –1.51 eV. The corresponding energy-level diagram is shown in Figure 28.2. Note that all the energy levels for n > 3 fall between E = 0 and the -1.51 eV of the n = 3 level.

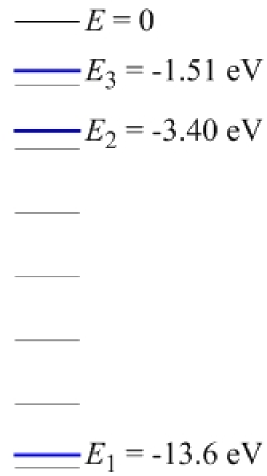

Figure 28.2: An energy-level diagram, showing the three lowest energy levels for hydrogen.

Step 2 – ***Let's confine ourselves to electrons that make transitions between only the three energy levels shown in Figure 28.2. Mark these transitions on the energy-level diagram with downward-pointing arrows from one level to a lower level. How many different photon energies are associated with these transitions? Determine the energies of these photons.*** With three energy levels, we can get three different electron transitions, and thus three different photon energies. The transitions are shown on the energy-level diagram in Figure 28.3. In decreasing order, by photon energy, the photon energies are:

$$E_{3\to1} = E_3 - E_1 = -1.51 \text{ eV} - (-13.6 \text{ eV}) = 12.1 \text{ eV};$$

$$E_{2\to1} = E_2 - E_1 = -3.40 \text{ eV} - (-13.6 \text{ eV}) = 10.2 \text{ eV};$$

$$E_{3\to2} = E_3 - E_2 = -1.51 \text{ eV} - (-3.40 \text{ eV}) = 1.89 \text{ eV}.$$

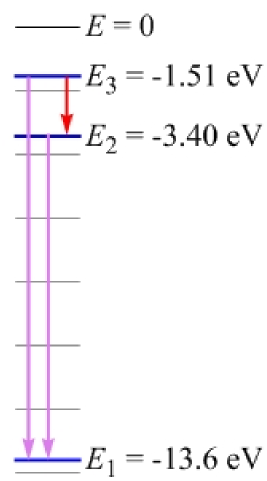

Figure 28.3: The energy-level diagram is modified to show the electron transitions that are possible between the lowest three energy levels in hydrogen. The photons emitted in the two transitions that end at the n = 1 level are in the ultraviolet region, while the photon associated with the n = 3 to n = 2 transition is red.

Key idea: The energy of a photon emitted by an electron that drops down from one energy level to a lower energy level is equal to the difference in energy between those two energy levels.
Related End-of-Chapter Exercises: 1, 2, 4, 13 – 18, 33 – 38.

Note that atoms can also absorb energy, in the form of photons, but they only absorb photons with an energy equal to the difference in energy between two of the atom's electron energy levels. In this case, the electrons make a transition from a lower energy level to a higher level.

Essential Question 28.1: Imagine that there is an atom with electron energy levels at the following energies: –31 eV, –21 eV, –15 eV, and –12 eV. Assuming that electron transitions occur between these levels only, (a) how many different photon energies are possible? (b) what is the (i) minimum and (ii) maximum photon energy?